Advantages of Using Manipulatives
Some Summary Statements from Research

Over the past few decades, researchers have studies the use of manipulatives in several different grade levels and in several different countries.

Majority of studies indicate:
• Mathematics achievement increases when manipulatives are put to good use
• Manipulatives improve children’s long-term and short-term retention of mathematics
• Using manipulatives improves environment in math classrooms
• Working with manipulatives and then reflecting on their experiences not only enhances mathematical learning but also reduces math anxiety
• Children better understand math

• The use of manipulatives is supported by both learning theory and educational research in the classroom
• “Manipulatives held students learn by allowing them to move from concrete experiences to abstract reasoning.”
• When students manipulate objects, they are taking the first steps toward understanding math processes and procedures
• “The effective use of manipulatives can help students connect ideas and integrate knowledge so they gain a deep understanding of mathematical concepts.”

• Students who use manipulatives in their mathematics classes usually outperform those who do not. This benefit holds across grade level, ability level, and topic, given that use of a manipulatives “makes sense” for that topic.
• Manipulative use also increases scores on retention and problem solving tests
• Attitudes towards mathematics are improved when students have instruction with concrete objects provided by teachers knowledgeable about their use
Aunt Alice gave each of her three nieces a number of silver dollars equal to their ages. The youngest felt that this was unfair. They agreed to redistribute the money. The oldest would split half of her silver coins evenly with the other two sisters. The middle sister would then give each of the others 3 silver coins. Finally, the youngest would give each of the others 1 silver coin. After exchanging money, each girl had 14 silver dollars. How old are the sisters?
Aunt Alice and the Silver Coins
Version 2

Aunt Alice gave each of her three nieces a number of silver dollars equal to their ages. The youngest felt that this was unfair. They agreed to redistribute the money. The youngest would split half of her silver coins evenly with the other two sisters. The middle sister would then give each of the others 4 silver coins. Finally, the oldest was to split half of her silver dollars equally between the two younger sisters. After exchanging money, each girl had 16 silver dollars. How old are the sisters?
Basic Model Drawing

1. Tyrone had $17 in his piggy bank. He added 8 more. What was his total savings?

2. Ray has 465 tractors and his brother Ben has 289. How many tractors do they have altogether?

3. Jennifer went shopping with $42. She came home with $9. How much money did she spend?

4. Hansel read 235 pages of his book over the weekend. Gretel read 198 pages of her book over the weekend. How many more pages did Hansel read than Gretel?
5. A total of 100 raffle tickets were sold over a 3-day period. If 21 raffle tickets were sold on Monday, and 67 tickets were sold on Tuesday, how many raffle tickets were sold on Wednesday?

6. There are 5 plates of cookies on the shelf. If there are 4 cookies on each plate, how many cookies are there in all?

7. There are 20 chairs. Kayla wants to put the chairs into 4 rows. How many chairs will be in each row?

8. Twelve students need rides to an after school event. If only 4 students can ride in each car, how many cars are needed to transport the students?
Fractions and Model Drawing

1. Harry cuts a pizza into 8 equal parts. If Harry eats 3 pieces of pizza, what fraction of the pizza does he eat? What fraction of the pizza is left?

2. Jonas divides a batch of cookies evenly into 9 groups. If he gives away 4 groups of cookies, what fraction of the cookies does he give away? What fraction of the cookies is remaining?

3. If \( \frac{2}{3} \) of a number is 4, what is the number?

4. In the school chorus, \( \frac{3}{5} \) of the students are girls. If there are 10 boys in the chorus, how many girls are there?

5. David spent \( \frac{2}{5} \) of his money on a storybook. The storybook cost $20. How much money did he have at first?

6. At the zoo, \( \frac{3}{7} \) of the visitors saw the panda exhibit. If 1,245 people saw the panda exhibit, how many people visited the zoo?

7. On Friday, 3,684 people visited the Statue of Liberty. \( \frac{5}{6} \) of the visitors climbed to the top of the Statue. How many visitors climbed to the top?

8. One number is one fifth of another number. If the difference between the two numbers is 20, find the two numbers.

9. Handy Mann bought some boards at the lumber store. He used \( \frac{1}{2} \) of them to make bookshelves, and \( \frac{1}{4} \) of the remaining boards to make a workbench. He had 6 boards left. How many boards did Handy buy at the lumber store?

10. \( \frac{2}{3} \) of Brett’s savings is equal to \( \frac{2}{5} \) of Andrew’s savings. If they have $160 in savings altogether, how much more money does Andrew have in savings than Brett?
HORSE TRADER

Once upon a time, there was a horse trader. One morning, the horse trader bought a horse for $60. Later that day, the horse trader sold the horse for $70. However, after thinking it over for a couple of days, he decided to buy the horse back for $80. But when someone offered him $90 for the horse, he couldn’t pass it up so he sold the horse again. How much money did the horse trader make or lose on this horse, or did he manage to just break even?
Model Drawing Level 1

1. Lu kicked 8 soccer goals. Mia kicked 9 soccer goals. How many goals did they kick in all?

2. The puppy chewed 5 old shoes and 6 new shoes. How many shoes did the puppy chew?

3. Simone read 6 books. Albert read 4 books. How many books did they read in all?

4. Kenny played the piano for 4 hours on Saturday and 3 hours on Sunday. How many hours did he play the piano over the weekend?

5. Mavis ate 5 candy hearts. Arturo ate 7 candy hearts. How many hearts did they eat in all?
6. The cook served 6 chicken dinners and 7 hamburger dinners. How many dinners did the cook serve?

7. Jana has 15 colored pencils at home and 10 colored pencils at school. How many colored pencils does Jana have in all?

8. Jarod caught 9 crickets, but 3 jumped away. How many crickets did he have left?

9. Tyler had $10. He spent $2 at the school store and $3 at the ice cream store. How much money does Tyler have now?

10. Dennis has 7 yo-yos. His sister has 2 fewer yo-yos than Dennis. How many yo-yos do they have in all?
11. There were 4 birds’ nests. Each nest had 3 eggs. How many eggs is that in all?

12. The 3 shelves in the ice cream store have 8 ice cream tubs on each shelf. How many ice cream tubs are on the 3 shelves?

13. There were 6 baskets. There were 3 pineapples in each basket. How many pineapples were in the 6 baskets?

14. Alexandra put 10 gingerbread men in each box. There were 6 boxes of gingerbread men. How many gingerbread men were there in all?
Model Drawing Problems Level 2

1. Thalia has 6 pencils at school and 8 pencils at home. How many pencils does Thalia have in all?

2. A teacher spent $4.48 on glue and $2.16 on crayons. How much money did the teacher spend in all?

3. A bag has 200 marbles in it. You look around the house and find another 30 marbles. How many marbles do you have altogether?

4. There are 456 students in a school. If 10 of the students move away, how many students will there be in the school?

5. My sister got $100 for her birthday. She spent some of the money and had $36 left. How much money did she spend?
6. Ryan had 69 pounds of potatoes. He used 55 pounds to make mashed potatoes. How many pounds of unmashed potatoes does he have now?

7. The girls have 26 red pencils. They have 42 purple pencils. How many more purple pencils than red pencils do the girls have?

8. Kris has 108 cents in his pocket. Martha has 171 cents in her pocket. How much more money does Martha have than Kris?

9. Aaron has wrapped 38 packages. He needs to wrap 62 in all. How many more packages does Aaron need to wrap?

10. Carlotta had 76 dishes to wash. She washed 46 of them. How many dishes does she still have to wash?
Model Drawing Problems Level 2

11. Bella went on vacation for 35 days. Her friend Emina joined Bella for 19 of those days. How many days was Bella alone?

12. Seven children went to the movies. Their tickets cost $3 each. How much did it cost all of them to go to the movies?

13. Two people can fit in a roller-coaster seat. If there are 6 seats, how many people can go on the roller coaster?

14. At the store, lollipops are sold in packages of 4. How many packages would you need to buy to have 24 lollipops?

15.
1. I dreamed that a large group of alligators munched on 40 fish for breakfast, 500 fish for lunch, and 8,000 fish for dinner. How many total fish did the alligators in my dream eat?

2. Niko had 63 word problems to solve. He solved 35 of them. How many word problems does Niko have left to solve?

3. Mr. Alvarez drove 233 miles to get 500 baseballs on Monday. On Friday Mr. Alvarez drove 987 miles. How many total miles did Mr. Alvarez drive?

4. My sister won $2,000. She spent some of the money and had $1,136 left. How much money did she spend?

5. Mirza has 3,345 goldfish. If 200 of Mirza’s fish get eaten, how many fish will be left?

6. Lisette baked 30 apple pies. Each pie took 4 apples. How many apples did Lisette use?

7. A cabinet has 439 blue blocks and 867 red blocks in it. How many fewer blue blocks than red blocks are in the cabinet?
8. There were 333 children in a spelling contest. Of those, 219 were girls. How many more girls than boys were in the spelling contest?

9. Bao-Yen made 123 apple muffins. She put them into boxes of 4. How many boxes did she use? How many muffins did she have left?

10. A zookeeper has 30 peanuts. He puts 5 peanuts in each bag. How many bags will the zookeeper need?

11. Zoe is 1 meter 56 centimeters tall, while her sister Kali is 1 meter 63 centimeters tall. If you put the 2 sisters together, how tall would they be?

12. A steak costs $18.27 while a lobster costs $25.00. How much less does the steak cost than the lobster?

13. Niko makes $22 for every lawn that he mows. If he mows 6 lawns, how much money will he make?

14. There are 18 jellybeans in a bag. If Kelly takes out 1/2 of the jellybeans, how many jellybeans will be left?
1. Donald has $595.95. He has $148.35 more than Daffy. How much money do they have altogether?

2. The distance around a track is 1 kilometer 600 meters. If Fabian ran around the track 5 times, how far did he run?

3. Mrs. Alvarez brought 5 kilograms 450 grams of Skittles into her class. She packed them equally into 5 bags. How much did each bag weigh?

4. Jonathan spends 45 minutes on his homework every night. Vignesh spends 30 minutes on his homework every night. How much time do the 2 boys spend altogether on homework each night?

5. Grace had 3 times as many strawberries as Tyler. After Grace eats 50 strawberries, she has half as many strawberries as Tyler. How many strawberries does Grace have left?

6. A piece of ribbon 1,624 inches long was cut into 2 parts. The longer piece was 312 inches longer than the shorter piece. What was the length of each piece of ribbon?

7. On the airplane, there are 258 adults and 65 children. How many more adults than children are on the airplane?

8. Frank has $1,578 in the bank. Marco has $269 more in the bank than Frank. How much money do they have in the bank altogether?
9. Holly had $5,000. She bought a table for $299. She also bought 4 chairs for $89 each. Now Holly wants to buy a cabinet for $1,500. After purchasing the other 5 items, does she have enough money?

10. Jonas divides a batch of cookies evenly into 9 groups. If he gives away 4 groups of the cookies, what fraction of the cookies does he give away? What fraction of the cookies is remaining?

11. Thomas counted 487 leaves on the tree in his backyard. Brittany counted 365 leaves on her tree. How many leaves did the children count on their trees in all?

12. At the flower shop, there are 5 times as many roses as sunflowers. If there are 252 roses and sunflowers altogether, how many roses are there? How many sunflowers are there?

13. Sheila was making place-value disks. She colored 2/6 of the disks red. She colored 1/4 of the disks yellow. If she colored 40 red place-value disks, how many disks did she color yellow? How many disks does she still have left to color?

14. Harry cuts a pizza into 8 equal parts. If Harry eats 3 pieces of pizza, what fraction of the pizza does he eat? What fraction of the pizza is left?
1. Angle DEF is a straight angle. Angle CEF is 37 degrees. What is the measure of angle CED?

2. The hot dog vendors at the state fair sell an average of 78 hot dogs every 30 minutes. At this rate, how many hot dogs can they expect to sell in 9 1/4 hours?

3. David’s birthday party was held at the bowling alley from 3:30 to 5:30. His family rented 2 lanes for that time. Lane rental for 1 lane is $10 an hour for 10:00 AM to 5:00 PM and $14 an hour from 5:00 PM to midnight. What was the total lane rental for 2 lanes for those 2 hours?

4. If 7/8 of a number is 63, what is 3 times that number?

5. A dozen watermelons weigh an average of 8.5 kilograms. If 8 of the watermelons weigh an average of 9.1 kilograms, what is the average weight of the other 4 watermelons?

6. At the farmer’s market there were 75 apples in each of a farmer’s 5 baskets. There were 200 apples still on his truck. How many apples in all did the farmer bring to the market?

7. Astronaut Alex has been in space 375 days. Astronaut Debi has been in space 190 fewer days than Alex. Astronaut Sue and Astronaut Lu have been in space an equal number of days; their total is 422 days. How many days has Debi been in space? How many days has Lu been in space? How many total days have these 4 astronauts been in space?
8. The folks at the concession stand sold 480 cans of soda during the tournament. They sold 150 more bottles of water than cans of soda. How many drinks did they sell in all?

9. The party store had many balloons. Of all the balloons, 10% were red, 25% were orange, 15% were green, and 20% were yellow. The other 30 were blue. How many balloons did the store have in all?

10. Tickets for animated to non-animated films at the Movie Star Theater sold last month at a ratio of 4:7. There were 14,700 movie-goers who saw non-animated films. How many saw animated movies? How many people total went to the movies at the Movie Star Theater last month?

11. Rocco’s Rock Shop has 725 igneous rocks on display. Rocco has divided the rocks up evenly among 5 shelves. The rocks on the bottom 2 shelves are $3 each. If he were to sell all the rocks on those 2 shelves, what would Rocco’s income be? The rocks on the other 3 shelves are $2 each. What would the total sales be if Rocco sold all of his rocks?

12. Of the people at the water park, 3/7 are children ages 3 through 12 and 2/7 are teens. Another 1/7 are children under the age of 3. The other people are adults. There are 87 adults at the park. How many children ages 3 through 12 are at the park? How many people in all are at the park?

13. Nidhi bought 3 baseball caps for $12.45 each and 4 jerseys for $27.75 each. What was her total bill?

14. The ratio of flutists to trumpeters to drummers in the school band is 3:4:1. If there are 5 drummers, how many flutists are there? How many trumpeters are there?
Problem-Solving Websites

Singapore Math – Model Drawing

  - Focused on Singapore math. It gives practice problems (and answers) from grades 3-6, with four levels of difficulty. It also has other problem solving methods and examples of those types of problems (i.e. draw a picture and work backwards).

- [http://www.risd.k12.nm.us/instruction/singaporemathbook.cfm](http://www.risd.k12.nm.us/instruction/singaporemathbook.cfm)
  - A book *Success with Word Problems* is a book put out by the Singapore Math Committee in the Roswell Independent School District. It has over 400 model drawing problems (and answers) arranged by topic. It can be used in many ways. For teachers in grade 5 and above, they can use this website in order (about 20 minutes a day) - 3 problems would be done whole class; 2 as assessment. The book gets progressively harder. Another way to use this site is by topic. A 3rd way is to find ones that are appropriate for your grade level (like addition problems for grades 1 and 2).

  - This is an interactive site, with videos, using model drawing with word problems. This site can be used as guided, or independent practice, and can be used whole class on an interactive whiteboard. The topics covered are addition, subtraction, multiplication, division, fractions, ratios, decimals, percents, and algebra.

  - This website goes over the steps of model drawing (with a copy of rules handout you can give to students). It also has 17 model-drawing examples, covering addition, subtraction, multiplication and division of whole numbers, fractions and decimals; plus ratio, rate, percentage, and “bridge to algebra” problems. No answers given – but they come from the book – *Step-By-Step Model Drawing*.

  - This website is an example of a 2/3 math lesson using model drawing. It includes teacher talk and examples.
Problem-Solving Strategies

- [http://library.thinkquest.org/4471/learn.htm](http://library.thinkquest.org/4471/learn.htm) This website was designed by two 4th graders, for 4th graders, who want to sharpen their problem-solving skills. The creators are into medieval history, so there is a theme with the problems. Students can try out these problems (or submit problems of their own to the site). Answers are given after they try the problems. Many different problem-solving strategies are shown on this site (i.e. estimation, look for a pattern, extra information, etc.).

- [http://mathlearnnc.sharpschool.com/cms/One.aspx?portalId=4507283&pa](http://mathlearnnc.sharpschool.com/cms/One.aspx?portalId=4507283&pa) This website is from the North Carolina State Board of Education. If you click on Instructional Tools on the left hand side of the website, there are many math resources covering grades K-12. There are math strategies, blackline masters, presentations, and more at this website – covering many math topics.

- [http://www.mathstories.com/](http://www.mathstories.com/) To use this website, you have to become a member, but once you do it gives you access to over 15,000 online and printable NCTM compliant math word problems for students grades 1-6. Word problems are available in both English and Spanish. Price ranges from $24-$48 for a year membership.

Virtual Manipulatives

- [http://nlvm.usu.edu/en/nav/vlibrary.html](http://nlvm.usu.edu/en/nav/vlibrary.html) This site has many interactive manipulatives that can be used to help visually teach math concepts. You can access many of these for free, but for an official license to use everything on the site it costs $39.95 for a one-computer license. Numbers and Operations, Algebra, Geometry, Measurement and Data Analysis/Probability are topics that are covered on this site.

- [http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-interface.html](http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-interface.html) On this site, you pick a grade level (K-8), pick a workmat (skill you want to work on), and then you pick a manipulative to use with the workmat. It is an interactive site that can be used on an interactive whiteboard or students can use it individually.

- [http://www.mathplayground.com/games.html](http://www.mathplayground.com/games.html) There are a lot of math, logic, and word problem games for students to use. It is colorful and has cartoon-like characters that students would really enjoy.

- [http://www.ct4me.net/math_manipulatives.htm](http://www.ct4me.net/math_manipulatives.htm) This website talks about what virtual manipulatives are, and then it gives many links/explanations to virtual manipulatives on the web.
1. Abdul put 3 apples into a bowl. He added 2 oranges. How many total pieces of fruit did Abdul put in the bowl?

2. Tyrone had $17 in his piggy bank. He added 8 more. What was his total savings?

3. Jennifer went shopping with $42. She came home with $9. How much money did she spend?

4. Carla has $19. She wants to buy a computer game that costs $36. How much more money will Carla need to buy the computer game?

5. Hansel read 235 pages of his book over the weekend. Gretel read 198 pages of her book over the weekend. How many more pages did Hansel read than Gretel?

6. A total of 100 raffle tickets were sold over a 3-day period. If 21 raffle tickets were sold on Monday, and 67 tickets were sold on Tuesday, how many raffle tickets were sold on Wednesday?

7. Mutt and Jeff both have money. Mutt has $15 more than Jeff. If Jeff has $30, how much money do they have altogether?

8. There are 5 plates of cookies on the shelf. If there are 4 cookies on each plate, how many cookies are there in all?

9. There are 20 chairs. Kayla wants to put the chairs into 4 rows. How many chairs will be in each row?

10. Twelve students need rides to an after school event. If only 4 students can ride in each car, how many cars are needed to transport the students?

11. James is half as old as Henry. Edward is twice as old as Henry. If Edward is 20 years old, how old is James?

12. If \( \frac{2}{3} \) of a number is 4, what is the number?

13. In the school chorus, \( \frac{3}{5} \) of the students are girls. If there are 10 boys in the chorus, how many girls are there?

14. One number is one fifth of another number. If the difference between the two numbers is 20, find the two numbers.
15. There are 240 students in the fifth grade. If 60% of them are girls, how many boys are in the fifth grade?

16. The ratio of boys to girls at a party was 2:3. If there were 9 girls, how many boys were at the party?

17. Together, Sally and Susie collected 32 seashells at the seashore. If Sally collected 6 more seashells than Susie, how many seashells did Susie collect?

18. Handy Mann bought some boards at the lumber store. He used $\frac{1}{2}$ of them to make bookshelves, and $\frac{1}{4}$ of the remaining boards to make a workbench. He had 6 boards left. How many boards did Handy buy at the lumber store?

19. Debbie, Eddie, and Fred have earned a total of 51 AR points. Eddie has twice as many points as Fred. Debbie has 15 more points than Fred. How many AR points did each person earn?

20. Amy, Betty, and Carla have a total of 67 marbles. Amy has 4 more than Betty. Betty has three times as many as Carla. How many marbles does each person have?

21. Tom has four times as many baseball cards as Greg. Together, they have a total of 40 cards. How many cards did Greg have?

22. Bill has 12 more than three times the number of baseball cards Chris has. Bill has 42 more cards than Chris. How many baseball cards does Chris have? How many baseball cards does Bill have?

23. The difference between two numbers is 24. If one number is three times the other number, find the sum of the two numbers.

24. David spent $\frac{2}{5}$ of his money on a storybook. The storybook cost $20. How much money did he have at first?
Smarter Balance Assessment Consortium

CLAIMS

Claim #1: Concepts and Procedures
“Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.”

Claim #2: Problem Solving
“Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.”

- understand (often in conjunction with one or more other relevant verbs),
- solve, apply, describe, illustrate, interpret, and analyze.

Claim #3: Communicating Reasoning
“Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.”

- understand, explain, justify, prove, derive, assess, illustrate, and analyze.

Claim #4: Modeling and Data Analysis
“Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.”

- model, construct, compare, investigate, build, interpret, estimate, analyze, summarize, represent, solve, evaluate, extend, and apply.
Using Model Drawing

Model Drawing provides a systematic approach to solving word problems.

1. **Read the entire problem**, first ignoring the actual numbers so we can “visualize” the problem conceptually (**reflective reading**) and then reading it again with all the numbers included.

2. **Rewrite the question** in sentence form leaving a space for the answer.

3. **Decide and write down (label)** who and/or what the problem is about.

4. **Draw unit bars of equal length** that we’ll eventually adjust as we construct the visual image of the problem.

5. **Chunk the problem** and **adjust the unit bars** to reflect the information in the problem.

6. **Determine exactly “what” we’re being asked to find** and place a **question mark** in the place on the model drawing that reflects the “what.”

7. **Compute** the problem to come up with an answer (show all work!)

8. **Write the answer in a complete sentence** that clearly states the solution.
Illustrative Mathematics Website
http://illustrativemathematics.org
- A work in progress
- Designed to provide examples that help to clarify what each CCSS means

Tools for the Common Core
http://commoncoretools.me
- Bill McCallum’s blog
- Contains articles, up to date information and links to viable sites related to the CCSS

Inside Mathematics website
www.insidemathematics.org
New resources on Common Core Standards for Mathematical Practice, including:
- Classroom video examples illustrating the math practice standards, including commentary
- Videos of exemplary lessons integrating multiple math practices
New resources on Common Core Standards for Mathematical Content, including:
- Common Core-aligned tasks, searchable either by grade level or by Common Core content area, such as “Operations and Algebraic Thinking,” or “Geometry - Congruence.”
Additional classroom videos of Number Talks
- Four new classroom videos of “Number Talks” showing students engaged in mental math exercises and conversations about math, including one from a bilingual Spanish-English classroom.

Mathematics Assessment Project
UC Berkeley & Shell Centre for Mathematical Education
http://map.mathshell.org.uk/materials/index.php
- Primarily middle school and high school materials
- Tasks
- Lessons aligned to CCSS

Smarter Balance Consortium Website
http://www.smarterbalanced.org
Smarter Balanced Assessments Tab – Sample Items and Performance Tasks
- Sample Items on Computer that model what students would see and do on the computer based assessments
Smarter Balanced Assessments Tab
- Scroll down on page until you see Item/Task Specifications
- Under that heading, keep scrolling down until you see Mathematics
- The zip files contain sample items for each of the 4 claims
California Mathematics Council
http://www.cmc-math.org/resources/main.html

New Resources and Webinars on Standards and Assessments
http://www.achievethecore.org/steal-these-tools/professional-development-modules

ASCD Common Core Resources Website
http://educore.ascd.org