Common Core Standards: Math
How The CCSS Will Impact Teaching, Learning And Assessment In Mathematics

David Foster
Silicon Valley Mathematics Initiative
www.svmimac.org
"Optimism is an essential ingredient for innovation. How else can the individual welcome change over security, adventure over staying in safe places? A significant innovation has effects that reach much further than can be imagined at the time, and creates its own uses. It will not be held back by those who lack the imagination to exploit its use, but will be swept along by the creative members of our society for the good of all. Innovation cannot be mandated any more than a baseball coach can demand that the next batter hit a home run. He can, however, assemble a good team, encourage his players, and play the odds."

Robert N. Noyce
## Silicon Valley Mathematics Initiative

### 83 Members - School Districts, Charter School Networks, and Schools

| Albany USD                      | Fairfield-Suisun USD                      | Sacramento City USD                      |
| Alvor SD (Riverside County)    | Fremont USD                               | Salinas City Schools                     |
| Antioch Unified SD             | Forsyth County School (GA)               | San Carlos Charter Learning Center       |
| Aspire Charter School Network  | Hamilton County (Tn)                     | San Francisco USD                        |
| Assumption School (San Leandro)| Hayward USD                               | SMFC (Park School)                       |
| Bayshore SD                    | Jefferson ESD                             | San Jose Unified SD                      |
| Belmont-Redwood Shores SD     | Jefferson HSD                             | San Leandro USD                          |
| Berryessa SD                   | Las Lomitas SD                            | San Ramon Valley USD                     |
| Bolinas – Lagunitas SD         | La Honda-Pescadero Sd                     | Santa Clara USD                          |
| Brisbane SD                    | Livermore USD                             | Santa Cruz City Schools                  |
| Buckeye SD                     | Los Altos SD                              | Saint Michael’s School (Poway)           |
| Cambrian SD                    | Los Gatos SD                              | Saint Patrick’s School (San Jose)        |
| Castro Valley USD              | Menlo Park SD                             | Saratoga                                 |
| Charter School of Morgan Hill | Monterey Peninsula USD                    | Scotts Valley USD                        |
| Chicago Public School          | Moreland SD                               | SCCOE County Court Schools               |
| Creative Arts Charter (SF)     | Mountain SD                               | Sequoia HSD                              |
| CSU San Bernardino             | National Council of La Raza              | SMCOE County Court Schools               |
| Cotati – Rohnert Park          | New York City PS                         | South Cook Service District              |
| Cupertino SD                   | New Visions for Public Schools           | South San Francisco USD                  |
| Dade County Schools (GA)       | Oakland Unified SD                        | Sumter County (GA)                       |
| Discovery Charter School      | Pacifica SD                               | The Nueva School                         |
| Dioceses of Santa Clara        | Pajaro Valley USD                         | University of Illinois, Chicago          |
| Dublin USD                     | Palo Alto USD                             | Valley Christen (Dublin)                 |
| East Side UHSD                 | Pittsburgh USD                            | Valdosta City (GA)                       |
| Edmonds Community College      | Portola Valley SD                         | Walnut Creek SD                          |
| Emery SD                       | Ravenswood City SD                        | Woodside SD                              |
| Etiwanda SD (San Bernardino Co)| Riverside COE                             |                                         |
| Gilroy (Brownell MS)           | Redwood City Schools                      |                                         |

### Supporting Teaching and Learning of Mathematics Since 1996
The state of mathematics education in America
After a decade of high-stakes accountability promising to equalize performances among students of all demographic classifications, what is happening to the “Gap”?
## The Achievement Gap

<table>
<thead>
<tr>
<th>NAEP 2009</th>
<th>US 4th Grade</th>
<th>US 8th Grade</th>
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</thead>
<tbody>
<tr>
<td>All Students</td>
<td>240</td>
<td>283</td>
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<tr>
<td>Black</td>
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<tr>
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<tr>
<td>Hispanic</td>
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<td>300</td>
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<tr>
<td>Low Income</td>
<td>228</td>
<td>266</td>
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<td>Mid-High Income</td>
<td>250</td>
<td>293</td>
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<tr>
<td>English Learner</td>
<td>218</td>
<td>243</td>
</tr>
<tr>
<td>English Fluent</td>
<td>242</td>
<td>284</td>
</tr>
</tbody>
</table>

Source: U.S. Department of Education


Approximately 10 scale points is equivalent to a grade level of learning
The achievement gap between white and minority students has not narrowed in recent years, despite the focus of the No Child Left Behind law on improving black and Hispanic scores, according to results of a federal test considered to be the nation’s best measure of long-term trends in math and reading proficiency.
Where you live and your background correlates to how you score on tests.
Common Core Standards: A New Direction linking Instruction and Assessment
Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
46** States, DC, and US Virgin Islands
States have joined Assessment Consortiums funded by RttT

**PARCC States**

Partnership for Assessment of Readiness of College and Careers (PARCC) is being managed by Achieve, Inc., a Washington-based non-profit. There are 23 states and DC in PARCC.

**SMARTER-Balanced Assessment Consortium**

The SMARTER-Balanced Assessment Consortium is being managed by San Francisco-based WestEd and its senior program director, Stanley Rabinowitz. SMARTER-Balanced enlisted 31 states.

At this point, both consortia are targeting the first test administration by 2014-15. Both say they will integrate summative or end-of-the-year tests with interim and formative assessments that can guide instruction during the year. Both are promising to include performance-based tasks, such as conducting a science experiment and writing short answers to questions, that are intended to show deeper levels of learning and thinking than multiple choice questions supposedly can measure. Both indicate that technology will play a major role.
Goals of Assessment

“We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics.”

Everybody Counts
SMARTER Balanced States

States in the SMARTER Balanced Assessment Consortium (as of November 17, 2010):
## Current vs. CCSS

### Current STAR Assessments

- Grades 2-11, writing at 4<sup>th</sup> and 7<sup>th</sup>
- Only paper & pencil option
- Taken around 85% of the instructional days
- Only multiple choice
- Part of the state and federal accountability system

### Proposed CCSS Assessments

- Grades 3-8 and 11, Grades 9 and 10 available for states that choose to use them
- Delivered via computer (Paper and pencil option available for 3 years) and are computer adaptive
- Taken during the final 12 weeks of school
- Performance tasks and comprehensive end-of-year computer adaptive assessment which will some selected response items
- Accountability system has not been established yet
The System
(Possible Scenario)

Digital Clearinghouse of formative tools, processes and exemplars; released items and tasks; model curriculum units; educator training; professional development tools and resources; scorer training modules; and teacher collaboration tools.

Scope, sequence, number, and timing of interim assessments locally determined.

Performance Tasks
- Reading
- Writing
- Math

Retake option
Claim-Evidence-Warrant

A Model for Analyzing Arguments

(adapted from the work of Stephen Toulmin)
Content Specifications
for the Summative assessment of the
Common Core State Standards for Mathematics

DRAFT TO ACCOMPANY GOVERNING STATE
VOTE ON ASSESSMENT CLAIMS

March 20, 2012

Developed with input from content experts and Smarter Balanced Assessment
Consortium Staff, Work Group Members, and
Technical Advisory Committee
Acknowledgements

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  Former Head of Mathematics and Statistics, Bell Laboratories
W. James Popham, Emeritus Professor, University of California, Los Angeles
Cathy Seeley, Senior Fellow, Charles A. Dana Center, The University of Texas at Austin
Malcolm Swan, Professor of Mathematics Education, Centre for Research in Mathematics Education,
  University of Nottingham
Four Major Claims for the SMARTER Balanced Assessment
Consortium’s assessments of the
Common Core State Standards for Mathematics

Claim #1 - Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Claim #2 - Students can frame and solve a range of complex problems in pure and applied mathematics.

Claim #3 - Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Claim #4 - Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.
Types of Tasks in Mathematics

**Novice** — short items focused on skills and routines

**Apprentice** — medium performance tasks with scaffolding

**Expert** — long tasks with high cognitive load and/or complexity.
Novice
Task
Example

5. Water Tank
A water tank has shape and dimensions as shown in the diagram.
At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.

Click on the graph that shows how the height of the water surface changes over time.

Click on the graph that shows how the height of the water surface changes over...
The total cost \((c)\) in dollars of renting a sailboat for \(n\) days is given by the equation

\[ c = 120 + 60n. \]

If the total cost was $360, for how many days was the sailboat rented?

A 2
B 4
C 6
D 8
Performance Assessments
To Inform Instruction And Measure Higher Level Thinking

The Baker
This problem gives you the chance to:
+ choose and perform number operations in a practical context

The baker uses boxes of different sizes to carry her goods.
Cookie boxes hold 12 cookies.
Donut boxes hold 4 donuts.
Muffin boxes hold 2 muffins.
Bagel boxes hold 6 bagels.

1. On Monday she baked 24 of everything.
How many boxes did she need? Fill in the empty spaces.
cookie boxes __________
donut boxes __________
muffin boxes __________
bagel boxes __________

2. On Tuesday she baked just bagels. She filled 7 boxes.
How many bagels did she make?
Show your calculations.

3. On Wednesday she baked 42 cookies.
How many boxes did she fill?
How many cookies were left over?
Explain how you figured this out.

4. On Thursday she baked 22 of just one item and she filled 8 boxes.
What did she bake on Thursday?
Show how you figured this out.

The Mathematics Assessment Resource Service (MARS) is an NSF funded collaboration between U.C. Berkeley and the Shell Centre in Nottingham England.

The Assessments target grades 2-Geometry and are aligned with the State and NCTM National Math Standards.
Apprentice Task

CR 4: Baseball Jerseys

Bill is going to order new jerseys for his baseball team. The jerseys will have the team logo printed on the front. Bill asks 2 local companies to give him a price.

1. ‘Print It’ will charge $21.50 each for the jerseys. Using \( n \) for the number of jerseys ordered and \( c \) for the total cost in dollars, write an equation to show the total cost of jerseys from ‘Print It’.

2. ‘Top Print’ has a Set-Up cost of $70 and then charges $18 for each jersey. Using \( n \) to stand for the number of jerseys ordered and \( c \) for the total cost in dollars, write an equation to show the total cost of jerseys from ‘Top Print’.

3. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to order for the price from ‘Top Print’ to be less than from ‘Print It’. Explain how you figured it out.

4. Bill decides to order 30 jerseys from ‘Top Print’. How much more would the jerseys have cost if he had bought them from ‘Print It’? Show all your calculations.
Baseball Jerseys

This problem gives you the chance to:
• work with equations that represent real life situations

Bill is going to order new jerseys for his baseball team. The jerseys will have the team logo printed on the front. Bill asks two local companies to give him a price.

1. ‘Print It’ will charge $21.50 each for the jerseys.
   Using \( n \) for the number of jerseys ordered, and \( c \) for the total cost in dollars, write an equation to show the total cost of jerseys from ‘Print It’.
   \[ c = 21.50n \]

2. ‘Top Print’ has a one-time setting up cost of $70 and then charges $18 for each jersey.
   Using \( n \) to stand for the number of jerseys ordered, and \( c \) for the total cost in dollars, write an equation to show the total cost of jerseys from ‘Top Print’.
   \[ c = 70 + 18n \]
3. Bill decides to order 30 jerseys from ‘Top Print’.
   How much more would the jerseys cost if he buys them from ‘Print It’?
   Show all your calculations.

4. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to buy for the price from ‘Top Print’ to be less than from ‘Print It’.
   Explain how you figured it out.
Performance Exams
40,000 – 70,000 students per year since 1999

Students in grades 2 through 10th/11th grade are administered performance exams (5 apprentice tasks per exam).

Random sample of student papers are audited and rescored by SJSU math & CS students. (Two reader correlation >0.95)

Student tests are hand scored by classroom teachers trained and calibrated using standard protocols.

District scoring leaders are trained in using task specific rubrics

Educational Data Systems

Student results are collected, analyzed, and reported by an independent data contractor.
# Spring 2011 Trends Grade to Grade

<table>
<thead>
<tr>
<th>Grade 2</th>
<th>MARS 1</th>
<th>MARS 2</th>
<th>MARS 3</th>
<th>MARS 4</th>
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<tr>
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<td>0.6%</td>
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<table>
<thead>
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<th>MARS At or ^</th>
<th>Total</th>
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<td>6.5%</td>
<td>20.2%</td>
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<tr>
<td>CST AT or ^</td>
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**Spring 2011 Trends Grade to Grade**

<table>
<thead>
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<td>79.0%</td>
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<tr>
<td>Totals</td>
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<table>
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<td>5.8%</td>
<td>21.4%</td>
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<tr>
<td>CST AT or ^</td>
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## Spring 2011 Trends Grade to Grade

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<tr>
<td>Totals</td>
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<table>
<thead>
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</tr>
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<td>55.1%</td>
<td>2.8%</td>
<td>57.9%</td>
</tr>
<tr>
<td>CST AT or ^</td>
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<td>17.0%</td>
<td>42.0%</td>
</tr>
<tr>
<td>Totals</td>
<td>80.1%</td>
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<table>
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<tr>
<td>CST AT or ^</td>
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<td>15.4%</td>
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<tr>
<td>CST AT or ^</td>
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<tr>
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<td>51.4%</td>
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# 8th Grade Geometry
California’s Highest Achieving Students

<table>
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<th>Geometry</th>
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<tr>
<td>CST AT or Above</td>
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<td>84.7%</td>
</tr>
<tr>
<td>Totals</td>
<td>51.3%</td>
<td>48.7%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Expert Tasks

The main point in mathematics teaching is to develop the tactics of problem solving.

George Polya
Gas Bills, Heating Degree Days, and Energy Efficiency

Here is a typical story about an Ohio family concerned with saving money and energy by better insulating their house.

Kevin and Shana Johnson’s mother was surprised by some very high gas heating bills during the winter months of 2007. To improve the energy efficiency of her house, Ms. Johnson found a contractor who installed new insulation and sealed some of her windows. He charged her $600 for this work and told her he was pretty sure that her gas bills would go down by “at least 10 percent each year.” Since she had spent nearly $1,500 to keep her house warm the previous winter, she expected her investment would conserve enough energy to save at least $150 each winter (10% of $1,500) on her gas bills.

Ms. Johnson’s gas bill in January 2007 was $240. When she got the bill for January 2008, she was stunned that the new bill was $235. If the new insulation was going to save only $5 each month, it was going to take a very long time to earn back the $600 she had spent. So she called the insulation contractor to see if he had an explanation for what might have gone wrong. The contractor pointed out that the month of January had been very cold this year and that the rates had gone up from last year. He said her bill was probably at least 10% less than it would have been without the new insulation and window sealing.

Ms. Johnson compared her January bill from 2008 to her January bill from 2007. She found out that she had used 200 units of heat in January of 2007 and was charged $1.20 per unit (total = $240). In 2008, she had used 188 units of heat but was charged $1.25 per unit (total = $235) because gas prices were higher in 2008. She found out the average temperature in Ohio in January 2007 had been 32.9 degrees, and in January of 2008, the average temperature was more than 4 degrees colder, 28.7 degrees. Ms. Johnson realized she was doing well to have used less energy (188 units versus 200 units), especially in a month when it had been colder than the previous year.

Since she used gas for heating only, Ms. Johnson wanted a better estimate of the savings due to the additional insulation and window sealing. She asked Kevin and Shana to look into whether the “heating degree days” listed on the bill might provide some insight.
Grazing Area

A farmer tethers her goat to the corner of a 40-by-20-foot barn in a fenced lot that is 140-by-110 feet. She also has an herb garden next to part of the barn. The goat is tethered on a 50-foot rope to the corner of the barn farthest from the herb garden.

After leaving the goat out on the rope for one day, the farmer discovers that a large area of her herb garden has been nibbled to the ground! Where can she tether the goat so that her herb garden is not within reach of the goat, but without decreasing the grazing area of the goat? The original grazing area includes all the grass area the goat could reach including that section of the herb garden that the goat ate.

Open for Business

Malena is a student who wants to raise $5,000 to tour South America next summer. To raise the money, she decides to open her own business on 14th Street.

The owner of an electronics shop offers to sell Malena some of his products at the wholesale price. She needs to decide which items to sell and how to price those items in order to maximize her profit.

She does some market research and finds the information provided in the table below about some of the items she is considering selling. Her research results include the cost to buy these items from the wholesale supplier, the retail price at which different items were sold at different times, and the number of items sold at these different prices during the month.
Mathematics Claim #1

Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Mathematics Claim #2

Students can frame and solve a range of complex problems in pure and applied mathematics.

Mathematics Claim #3

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Mathematics Claim #4

Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.
SMARER Balanced Summative Assessment Development Overview

- **Released June 2010**: Common Core State Standards
  - Define the knowledge and skills students need for college and the workplace

- **Released for Review August 2011**: Content Specifications in ELA and math
  - Prototypes, items/tasks will be developed to inform test design, item specification, and test specifications

- **Begins December 2011**: Item Specifications
  - Test Design and Test Specifications

- **Begins March 2012**: Item Writing
  - Item writing materials will be developed using the item specification and content specifications

- **2014-2015 School Year**: SMARTER Balanced Assessment
Problem Sources

Part I: Short items
1: MARS
2: MARS
3: SBAC
4: MARS
5: PISA
6: MARS
7: PISA
8: MARS
9: MARS
10: MARS
11: SBAC
12: SBAC
13: SBAC

Part II: Selected Response Tasks
CR 1: SBAC
CR 2: MARS
CR 3: MARS
CR 4: MARS
CR 5: MARS
CR 6: MARS
CR 7: MARS
CR 8: MARS
CR 9: MARS
CR 10: MARS

Part III: Extended Performance Task

Ohio Department of Education and the Stanford University School Redesign Network
Teaching for Meaning
Erica is putting up lines of colored flags for a party.

The flags are all the same size and are spaced equally along the line.

1. Calculate the length of the sides of each flag, and the space between flags.
   Show all your work clearly.

2. How long will a line of $n$ flags be?
   Write down a formula to show how long a line of $n$ flags would be.
Algebra students had been working on system of linear equations for weeks.

\[ 6x + 5y = 170 \]

\[ 3x + 2y = 80 \]

\[ 6x + 5y = 170 \]

\[ -6x + -4y = -160 \]

\[ y = 10 \]
The Findings from Party Flags

• The task may be approached as a system of simultaneous equations, almost no algebra students used such an approach.
• 49% of algebra students had no success.
• 44% accurately found the two lengths (most commonly by an estimation strategy only using one constraint).
• 21% correctly used both constraints (the length of three flags is 80 cm. and the length of 6 flags is 170 cm.).
• 7% of the students were able to develop a valid generalization for n flags.
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Teaching for Understanding?

"Math Things Mingle"
Willard Middle School
Seventh Grade
January 22, 2009

Jacob Disston, Teacher
Discuss the Lesson

In small groups discuss:

• How did the students make sense of the mathematics?
• What mathematical ideas did student struggle to understand?
• What did student seem to learn?
• What role did the teacher play in the lesson?
• Which mathematical practices were the students engaged in during the lesson?
Press Esc to exit full screen mode.
Problems of the Month

A program to foster school-wide participation in math and problem solving.
Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.

George Polya, (1887 - 1985)
Father of Problem Solving;
“How to Solve It”, 1945
• George Polya, said, “A problem is not a problem if you can solve it in 24 hours.”
• Doing math is solving non-routine problems.
• Perseverance and learning from mistakes are important attributes of good mathematicians.
How are the POM be used?

• The POM are used school wide to promote problem solving.
• Each problem is divided into five levels, A-E, to meet the learning development needs of all students.
• A great tool for Differentiated Instruction.
• Students, teachers and parents learn to ask questions and persevere in solving non-routine problems.
• The whole school celebrates doing mathematics at school.
Celebrating Problem Solving

School Wide Use of POM’s
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Teachers facilitate problem solving -- asking good questions, encouraging perseverance, and probing for understanding.
The entire school does math together
Findings are shared through Group Collaboration, Individual Write-Up, Gallery Walks, and/or Presentations
Problem of the Month
Party Time

Level A

Cindy had a party. She invited two guests. Her guests each invited four guests, and then those guests each invited three guests.

How many people were at Cindy’s party?

Explain how you determined your solution.
Teacher Discussion about using the Party Time POM

Anna Yates Elementary School

Problem of the Month
Staff Discussion

April 2009
Problem of the Month
Party Time

Level C

Mia, Jake, Carol, Barbara, Ford and Jeff are all going to a costume party. Figure out which person is wearing what costume and when they arrived at the party.

- The person that arrived fourth was wearing bathing suit.
- Barbara was the last to arrive.
- Jake and Mia arrived and stayed together.
- The first person was dressed as a French Maid.
- Superman arrived right before Barbara.
- The Potato Heads were always together at the party.
- Ford was a Surfer Dude.
- The French Maid was not Carol.
- The Vampire arrived after Superman.
Problem of the Month

"Gallery Walk"

Anna Yates
Elementary School
“Mathematics is not a careful march down a well-cleared highway, but a journey into a strange wilderness, where the explorers often get lost.”

*Fermat’s Enigma, p. 71*
Math Talks

- A daily ritual with the entire class for the purpose of developing conceptual understanding of and efficiency with numbers, operations and other mathematics such as geometry and algebra. (no more than 10 minutes per day)

Math Talks are used to:
- Support active student discourse and discussions
- Review and practice procedures and concepts
- Introduce a concept before diving into the lesson of the day
- Support students in deepening their understanding of the number, operations and algebraic thinking.
- Explore mathematical connections and relationships
- Encourage students to construct viable arguments and critique the reasoning of others
- Support students in using precise mathematical language in sharing their different strategies and approaches
Today’s Number

36
Possible Solutions

\[ 18 + 18 \]
\[ 3^2 \cdot 2^2 \]
\[ 9 + 9 + 9 + 9 \]
\[ 25.65 + 10.35 \]
\[ 9 \div \frac{1}{4} \]
\[ -15 + 51 \]
\[ 3 \sqrt{144} \]

Today’s Number with Constraints

• More than one operation
• Using Two digit numbers (3 digits etc.)
• Using Fractions, Decimals, Percents
• Using sets of numbers and operations
• Using exponents, square roots
• Using integers (sign numbers)
• Using a set of numbers and different operations.
Today’s Number

"Number of the Day"
Stephanie Letson
Second Grade
Bayshore Elementary School
May 29, 2009
Today’s Number
Discuss the Number Talk Video

• Who did the math thinking during the number talk?
• What specific mathematics did the students demonstrate they understood?
• What did the teacher do to support the student discourse?
• What recording techniques did the teacher employ that supported learning in the class?
Curriculum inspired by the CCSS

MAP’s Formative Assessment Lessons and Professional Development Modules

Assessment For Learning
Formative Assessment Lessons (2 days) for High School and Middle School
Sorting Equations and Identities

Mathematical goals
This lesson unit is intended to help you assess how well students are able to:

• Recognize the differences between equations and identities.
• Substitute numbers into algebraic statements in order to test their validity in special cases.
• Resist common errors when manipulating expressions such as $2(x - 3) = 2x - 3$; $(x + 3)^2 = x^2 + 3^2$.
• Carry out correct algebraic manipulations.

It also aims to encourage discussion on some common misconceptions about algebra.

Common Core State Standards
This lesson involves mathematical content in the standards from across the grades, with emphasis on:

A-SSE: Interpret the structure of expressions.
A-REI: Solve equations and inequalities in one variable.

This lesson involves a range of mathematical practices, with emphasis on:

3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.
Pre-Assessment Task

Equations and Identities

1. Write down an example of an equation that has:
   (a) One solution.
   (b) Two solutions.
   (c) An infinite number of solutions.
   (d) No solutions.

2. For each of the following statements, indicate whether it is “Always true,” “Never true,” or “Sometimes true.” Circle the correct answer. If you choose “Sometimes true” then state on the line below when it is true. The first one is done for you as an example.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Always true</th>
<th>Never true</th>
<th>Sometimes true</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 2 = 3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Always true</td>
<td>Never true</td>
<td>Sometimes true</td>
</tr>
<tr>
<td></td>
<td>It is true when $x = 1$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x - 12 = x + 30$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Always true</td>
<td>Never true</td>
<td>Sometimes true</td>
</tr>
<tr>
<td></td>
<td>It is true when</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2(x + 6) = 2x + 12$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Always true</td>
<td>Never true</td>
<td>Sometimes true</td>
</tr>
<tr>
<td></td>
<td>It is true when</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3(x - 2) = 3x - 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Always true</td>
<td>Never true</td>
<td>Sometimes true</td>
</tr>
<tr>
<td></td>
<td>It is true when</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(x + 4)^2 = x^2 + 4^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Always true</td>
<td>Never true</td>
<td>Sometimes true</td>
</tr>
<tr>
<td></td>
<td>It is true when</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^2 + 4 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Always true</td>
<td>Never true</td>
<td>Sometimes true</td>
</tr>
<tr>
<td></td>
<td>It is true when</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Which of the equations in question 2 are also identities?

In your own words, explain what is meant by an identity.

........................................................................................................................................................................
........................................................................................................................................................................
........................................................................................................................................................................
<table>
<thead>
<tr>
<th>Common issues:</th>
<th>Suggested questions and prompts:</th>
</tr>
</thead>
</table>
| Student writes expressions rather than equations  
For example: The student writes \( y + 3 \) for an equation with an infinite number of solutions. | • What is the difference between an equation and an expression?  
• How can you change your expression to an equation? |
| Student fails to include a variable in their equation  
For example: The student has written \( 5 + 5 = 10 \) as an example of an equation with one solution. | • Can you include an unknown number or a variable in the equation so that we can look at all possible values of that unknown? |
| Student fails to provide an example of an equation with an infinite number of solutions | • What would an equation with an infinite number of solutions look like? |
| Student provides a quadratic with non-integer solutions as an example of an equation with no solutions  
For example: The student gives \( x^2 + 8x + 13 = 0 \) as an answer to Q1d. The student has assumed that, because it won’t factorize there are no solutions. | • Can a quadratic equation that will not factorize still have solutions/cross the \( x \)-axis? How can you check whether or not a quadratic equation has solutions? |
| Student assumes that \( -(x^2) \) is the same as \( (-x)^2 \)  
For example: The student classifies \( x^2 + 4 = 0 \) as true when \( x = -2 \). | • What does \( (-x)^2 \) mean? What kind of number do we get when we multiply two negative numbers together?  
• Is \( x^2 \) positive or negative? |
| Student correctly answers all the questions  
The student needs an extension task. | • Use algebra to justify one of your answers to Question 2.  
• Draw a diagram to justify one of your answers to Question 2. |
Is this Always, Sometimes or Never True?

\[(X + 2) (Y + 2) = XY + 4\]
Always, Sometimes, or Never True?

\[(x + 2)(y + 2) = xy + 4\]
### Card Set: Always, Sometimes, or Never True?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(x - 6 = 6 - x)</td>
<td>(x + 6 = y + 6)</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>(\frac{x}{6} = \frac{6}{x})</td>
<td>(6 + 2x = 8x)</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>(2(x - 3) = 2x - 3)</td>
<td>(2(x + 3) = 2x + 6)</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>(\frac{x + 6}{2} = x + 3)</td>
<td>(x^2 = 2x)</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>((x + 3)^2 = x^2 + 3^2)</td>
<td>((x - 6)^2 = (6 - x)^2)</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>((3x)^2 = 9x^2)</td>
<td>(x^2 - 1 = (x + 1)(x - 1))</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>(x^2 + 6 = 0)</td>
<td>((x + 1)(x + 4) = x^2 + 14)</td>
</tr>
</tbody>
</table>
Always, Sometimes, or Never True?

• In your groups, take turns to place a card in a column and justify your answer to your partner.

• If you think the equation is sometimes true, you will need to find values of $x$ for which it is true and values of $x$ for which it is not true.

• If you think the equation is always true or never true, you will need to explain how we can be sure that this is the case.

• Another member of the group should then either explain that reasoning again in his or her own words, or challenge the reasons you gave.

• When the entire group agrees, glue the card onto your poster. Write the reason for your choice of category next to the card.
New K-12 Math Curriculum Inspired by The Common Core State Standards
Middle School Curriculum
Inside Mathematics Website

http://www.insidemathematics.org

Mathematics Assessment Project
UC Berkeley & Shell Centre for Mathematical Education

http://map.mathshell.org/materials/lessons.php

Silicon Valley Mathematics Initiative

http://www.svmimac.org
"Don't be encumbered by history-- go off and do something wonderful."

Dr. Robert N. Noyce
Inventor of the Silicon Chip
Co-founder of Intel
Inside Mathematics Website

http://www.insidemathematics.org

Mathematics Assessment Project
UC Berkeley & Shell Centre for Mathematical Education

http://map.mathshell.org/materials/lessons.php

Silicon Valley Mathematics Initiative

http://www.svmimac.org
1. **Make sense of problems and persevere in solving them.**

**Traffic jam**

Last Sunday an accident caused a traffic jam 12 miles long on a freeway.

When the accident was cleared, the cars drove away from the front, one car every two seconds. Estimate how long it took before the last car moved.
2. Reason Abstractly and Quantitatively

Where are $a+b$, $b-a$ and $a-b$?

What can you say about where $a/b$ is?
Valerie shares some of the 12 candies. She gives Cindy 1 candy for every 3 candies she eats herself. How many does she give Cindy?
3. Construct viable arguments and critique the reasoning of others.

Jane and Tom are playing number card games. They have the four cards shown below.

\[
\begin{array}{cccc}
3 & 9 & 6 & 4 \\
\end{array}
\]

5. Show how they arrange the four cards to make the number that is nearest to 5000.

\[
\begin{array}{cccc}
\quad & \quad & \quad & \quad \\
\end{array}
\]

Explain how you figured it out.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
4. Model with mathematics

PLANNING AND ORGANIZING: A TABLE TENNIS TOURNAMENT

You have the job of organising a table tennis league.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.
4. Model with mathematics

SKELETON TOWER

1. How many cubes are needed to build this tower?
2. How many cubes are needed to build a tower like this, but 12 cubes high? Explain how you work out your answer.
3. How would you calculate the number of cubes needed for a tower n cubes high?
5. Use appropriate tools strategically

Which ropes are ‘Thin’?
Which ropes are ‘Medium’?
Which ropes ‘Thick’?

Explain your reasoning.
6. Attend to Precision

- Precision in Calculations
- Precision in Vocabulary
- Precision in Argumentation
Imagine that you have just discovered this ancient floor tiling pattern in Syria.

You telephone New York to tell them about this exciting discovery.

Describe the pattern as accurately as you can, so that someone else can draw it without seeing it.

Describe the shapes as completely as you can.
7. Look for and make use of structure

Sidewalk Patterns

In Prague some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns that are in various sizes.

Write formulas for the number $W$ of white tiles, and $G$ of grey tiles, in terms of the pattern number $n$. 
7. Look for and make use of structure

\[
\begin{align*}
\blacklozenge & + \, \odot = 7 \quad \text{and} \quad \blacklozenge & \times \, \odot = 12 \\
\_ & + \, \_ = 7 \quad \text{and} \quad \_ & \times \, \_ = 12 \\
\text{鶴} & + \, \, \text{犠} = 10 \quad \text{and} \quad \text{鶴} & \div \, \text{犠} = 4 \\
\_ & + \, \_ = 10 \quad \text{and} \quad \_ & \div \, \_ = 4 \\
\text{電話} & \div \, \, \text{心} = 3 \quad \text{and} \quad \text{電話} & - \, \text{心} = 6 \\
\_ & \div \, \_ = 3 \quad \text{and} \quad \_ & - \, \_ = 6
\end{align*}
\]
8. Look for and express regularity in repeated reasoning

Explain why the answer to:

\[
\frac{4}{5} \div \frac{2}{3}
\]

Makes sense knowing why \(45 \div 8\) makes sense.
8. Look for and express regularity in repeated reasoning

When Aaron plays the numbers game, he needs to decide which numbers belong in each set. Here is another game for you to play.

2. All of these numbers are Grogs.

```
123   789   456   345
```

None of these numbers are Grogs.

```
121   81    246   5678
```

Which of the numbers below are Grogs? Draw a circle around each Grog.

```
234   56    678   989
```

Explain how you know which numbers are Grogs and which are not.