CONSTRUCTING COLLECTIVE ALGEBRAIC OBJECTS IN A CLASSROOM NETWORK

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This paper presents a novel learning environment designed to support Algebra teaching and learning using a classroom network. We analyze a class session first at the level of the whole class, and then through more detailed examination of simultaneous activity in three student pairs. Classroom activity in this environment was organized around successive task goals that emerged from dynamic interactions among teacher, students and tools. The interplay between these emergent objectives and other features of the learning environment presented students with resources that they were able to organize into emergent solution strategies.

Introduction

A rich body of prior research has explored ways of using classroom networks of handheld calculators or computers to support classroom interactions (Hegedus & Penuel, 2008; Stroup, Ares & Hurford, 2005; White, 2006). Designs for learning activities featuring classroom networks often emphasize the collective construction of a set of mathematical objects—each student in the classroom group uses his or her device to contribute a distinct member of a family of functions, a locus of points, or a class of equivalent expressions to an aggregation on a teacher's computer publicly displayed via LCD projection. In each case, the collective construct illustrates a map between the set and its elements that mirrors the relations between the classroom group and each individual student member, using the social organization and structure of the classroom as a resource for directing student attention to the relations among these objects and guiding classroom discussions about patterns within and generalizations across the array.

This paper reports on an ongoing design-based research project that seeks to build on these previous studies by exploring ways classroom networks might support not only mathematically rich collective activities at the level of the whole class, but also mathematical conversations and interactions among pairs and small groups of students. To this end, our design approach uses network links among student devices to align pairs or small groups of students with mathematical objects that participants in a pair or small group must alternately or jointly manipulate through their networked devices. Such objects are collective to the extent that they or their attributes appear—and change—simultaneously on multiple devices or in a shared display as a consequence of contributions from multiple students. Broadly, we aim to structure tasks around these collective mathematical objects in order to make the successful solving of problems dependent on contributions from and coordination between all participants in a small group. A central aim of our current work is to investigate the potential for these collaborative classroom network designs to reorganize or reshape conventional classroom activity structures—to study designs that blur the boundaries between forms of instruction oriented toward individual students, small groups, or the whole class. Below, we briefly discuss theoretical perspectives involving mathematical objects and introductory algebra, and then describe an activity design intended to support student learning about those concepts. We then present an analysis of the kinds of whole- and small-group classroom mathematical activity supported by this environment.

Collective Algebraic Objects

The meaning of algebraic expressions likely comes from a variety of sources, including the structure of those symbolic expressions themselves as well as their other representational forms, the problem contexts in which they are enacted, and other learner experiences and interactions exterior to the mathematical situation (Kieran 2007). For present purposes, we focus on the first (structural) and last (external) of these sources. Studies from a structural perspective emphasize the complexity of cultivating learners' recognition of such expressions as mathematical objects (Sfard & Linchevski, 1994). From a perspective more attuned to exterior sources of meaning, coming to understand algebraic expressions as mathematical objects is part of a broader social process of objectification, in which learners become aware of objects as stable elements of sociocultural practice through interaction and reflection (Radford, 2006). In the design and study of the learning environment and activities described below, we sought to attend to both these structural and interactional elements of algebraic learning.

The *Terms and Operations* Design

The learning environment described in this paper was created using the NetLogo modeling platform (Wilensky, 1999) and HubNet architecture (Wilensky & Stroup, 1999) in concert with a classroom set of Texas Instruments graphing calculators connected through a TI-Navigator™ network. In the activity design called *Terms and Operations*, student pairs share responsibility for constructing a polynomial expression. Each student can use the directional arrow keys on a calculator connected to a network server to move an icon in a whole-class shared display (Figure 1) populated with a variety of floating monomial terms (of order, sign, scale and number set by the teacher prior to each activity). The right side of this display also includes a field that monitors the current state of the collective polynomial expression for each pair.

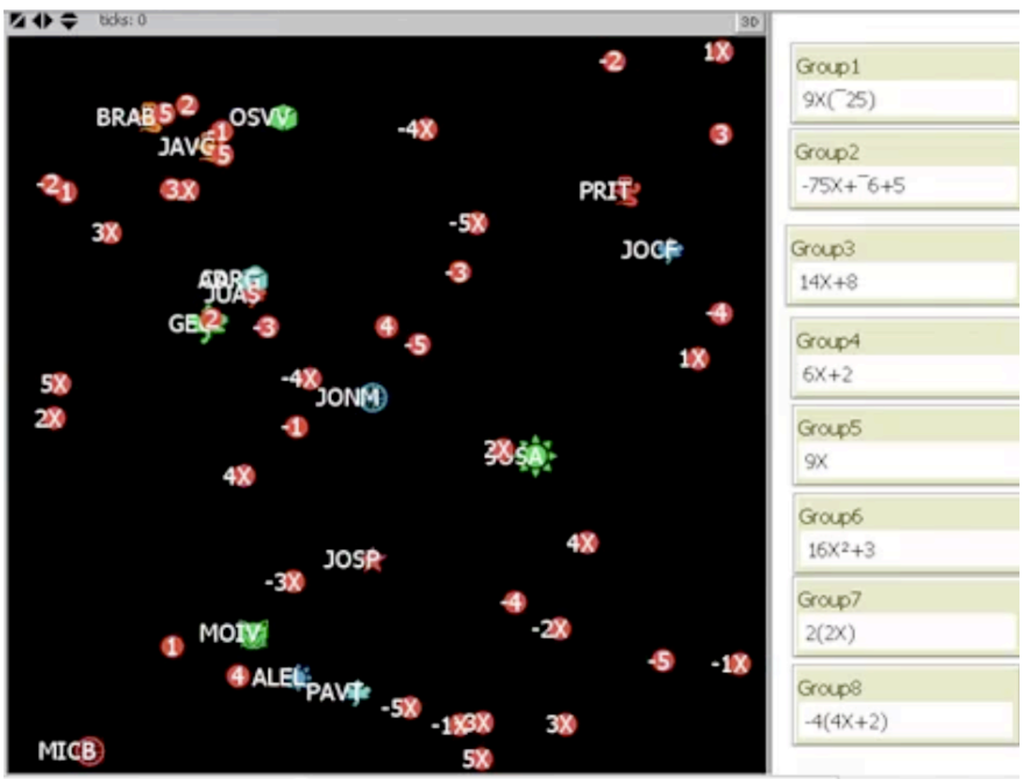
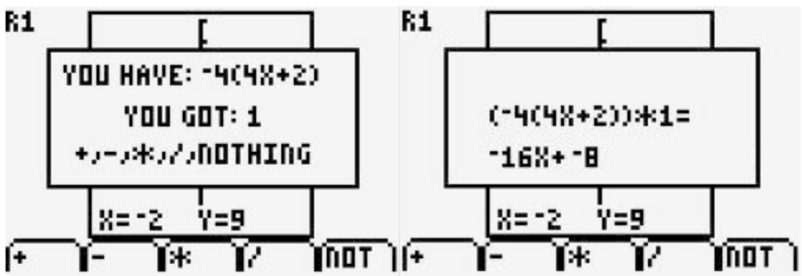


Figure 1. *Terms and Operations* shared display with floating terms and collective expressions.

In the activities for the present study, the members of each pair took turns capturing and operating on a monomial term to construct a new collective polynomial expression. Each time a student captures a term, she chooses an operation (Figure 2a) and enters the result of combining these with the pair's previous expression (Figure 2b). If this combination is equivalent to the new entry, the collective expression updates accordingly. Typical activities in this environment involve either asking different groups to construct the same expression in different ways, or to construct different expressions that all share particular characteristics. Having student pairs construct these expressions by aggregating terms under different operations is intended to use both pair-level and whole-class interactions as contexts for engaging learners in dynamic construction activities which emphasize both (structural) equivalences among successively more complex objects, and the (operational) consequences of actions on those objects.



Figures 2a and b. Student calculator screens featuring a) captured term and operation choices and b) a collective expression under the chosen term and operation and an equivalent student entry.

Method

The *Terms and Operations* design was implemented in a classroom-based design experiment with two groups of 16 9^{th} grade Algebra I students. Four days of *Terms and Operations* activities with each of these groups were part of a year-long project in which students participated in classroom network activities for a one-hour session each week as a supplement to their regular mathematics program. These sessions were taught by the first author; this arrangement reflects a researcher-teacher (Ball, 2000) approach in the larger design-based research project, and was intended to provide a context for nuanced investigation of forms of teaching practice supported by those designs. On the basis of informed consent, three student pairs in each class were selected as focus groups and videotaped during all activities. All screen states of the public computer display were recorded as a video file for each class session, and an additional camera with a wide zoom setting captured this projected display along with the whiteboard at the front of the room, as well as whole-class discussions and other teacher moves. Server logs recorded all terms and operations selected and expressions entered on student calculators. Below, we present data from the fourth session with one of these classes in order to examine the forms of whole and small-group classroom activity supported by the *Terms and Operations* environment.

Analysis

Table 1 summarizes the sequence of collective expressions constructed by each student pair during the fourth day of *Terms and Operations* activities. These successive expressions reflect an evolving set of goals that emerged over the course of the session, taking the shape of at least four distinct and overlapping tasks posed by the teacher and taken up to varying degrees by each student pair. In the next section, we describe the ways these emergent tasks unfolded through the interplay between students, teacher and tools in this learning environment.

Table 1: Summary of successive collective expressions constructed by each student pair.

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
2	5	-1	0	3	4	2	4X
2(4X)	5(5X)	5X	-3	3(3X)	4X*(4)	2(2X)	4X+(2)
8X+1X	25X+2	5X+4	6X	(3*3X)+5	4X		-4(4X+2)
9X(5)	-3(25X+2)	10X+8	6X+2	9X+9	4X		-16X+-8
9X(-25)	-75X+-6+5	14X+8	1.5+1/(2X)	9X+12	$16X^2$		-15X+-8
(9X*2X)(-25)	-1X(-75X+-6+5)	14X+8+2+2	3+2/(2X)	9X+7	$16X^2+3$		-14X+-8
$18X^2(-25)$	$75X^2+4X$	2X(14X+12)	6X+2	9X+2	$16X^2-1$		-7X-4
(9X*2X)(-25)	X(75X+4)	$28X^2+24X+5$		9X	$(16X^2-1)(-5X)$		
		$28X^2+24X+9$		4(9X)			
				4(9X*5)			
				4(9X*5/2)			
				4(9X*5/2)-3X			

Whole Class-Level Activity: Emergent Tasks

After beginning the class with a brief review of the distributive property from the previous session, the teacher opened the *Terms & Operations* activity by asking each student group to make a collective expression that featured parentheses. When Group 2 completed this task by writing 5(5x), the teacher pointed out what they had done to the whole class, and then asked Group 2 pick up a new term, and to write their next expression without parentheses. As the other groups finished writing expressions with parentheses they were likewise given the same subsequent task. Continuing to build a succession of expressions with and without parentheses became an ongoing goal for each group throughout the remainder of the session.

Emergent Task 1. As other groups continued to produce initial expressions that included parentheses, they tended to follow the format of Group 2's initial solution by writing a constant term in front of a set of parentheses containing a linear term (as in the second entries in Table 1 for Groups 1, 5 and 7). While several pairs were generating their next expressions without parentheses, the teacher rewrote their respective initial solutions (5(5x), 2(4x), 3(3x), 4x*(x), 2(2x), 4x+(2)) on the board and highlighted the similarity, and then encouraged these groups to build new expressions that included two terms and an operation within the parentheses.

Emergent Task 2. Eleven minutes into the activity, and as other groups were continuing to operate on new terms and write resulting expressions with and without parentheses, Group 6 created an expression with an x^2 term. Taking note of this expression as it appeared in the public display, the teacher pointed it out to the whole class and noted that there were no x^2's among the floating terms. The teacher then prompted Group 6 to write a new expression that again featured parentheses, and meanwhile challenged the other groups to see if they could likewise construct an expression with an x^2 term.

Emergent Task 3. Over the next twelve minutes, groups continued to work on some combination of the parentheses and quadratic term challenges as the teacher alternated between assisting individual groups and leading brief discussions with the whole group. Twenty-three minutes into the activity, the teacher noted the successes by Group 3 ($28x^2+24x+5$) and Group 2 ($75x^2+4x$) in constructing quadratic expressions, and presented a fourth task, prompting them to try factoring these expressions by rewriting with parentheses but no x^2 term. When both groups initially struggled with how to simultaneously factor their current expression and pick up a new term, the teacher told the whole class about a discovery made earlier in the session by Group 8 (reported below) that they could keep their expressions the same by choosing a 1 as their new term and multiplication as their operation. In the final remaining minutes of the session, Group 2 was able to use this approach to rewrite their expression as x(75x+4).

Discussion. The four main tasks (writing increasingly complex expressions alternately with and without parentheses, including two terms and an operator within the parentheses, writing an expressions with a quadratic term, rewriting to eliminate the square) undertaken by students during this session were all initiated by direction of the teacher. However, while the initial alternating parentheses task was the planned emphasis for the activity, both the revision of the task to include an operator within the parentheses, and the subsequent challenges involving quadratic terms and factoring, emerged as responses by the teacher to specific student constructions appearing in the public display. We take these emergent objectives as both a consequence and a characteristic of the kind of classroom activity supported by this designed learning environment—as resulting from the superposition of small-group engagement with dynamic objects and teacher-led whole class discussion about those group-level objects as they appeared in fairly rapid succession on the collective display. In the next section, we examine three student pairs during a portion of this session in order to consider the relations between whole-class and small-group level activity in the *Terms and Operations* environment.

Pair-Level Activity: Collective Objects and Emergent Solutions

Table 2 presents a set of transcripts that span the simultaneous dialogue of three student groups, as well as comments made by the teacher both to the whole class and through tableside conversations with one of the groups, over a three-minute segment of the Terms & Operations session described above. In particular, this segment begins with the teacher's observation that Group 6 had constructed an expression featuring a quadratic term, and follows the varying degrees to which these groups took up this new challenge or continued to work on other tasks. Below, we briefly examine the work of each pair during this segment.

Group 8. The start of this episode found Group 8 actively blurring the lines between small- and whole group activity, picking up on the teacher's efforts to call the class's attention to the public display by first looking around the room to figure out which of their classmates were in Group 6 (lines 1-5), and then finding Group 1's expression on the screen and calling across the room to ask how they made it (lines 8-10). As the teacher set other groups to work generating an x^2 term, Group 8 agreed that they were "lost" and should ask for help (lines 16-17). Having previously constructed -4(4x + 2), they had repeatedly attempted to pick another term and to remove the parentheses without success, each time failing to correctly distribute the -4 to rewrite their current expression before incorporating the new term. On arriving at their table, the teacher looked at Anna's calculator and noted that she had just picked up a 1 and selected multiply, but not yet entered a new expression (lines 21-22). The teacher asked them about the effect of multiplying by 1, and then encouraged them to use this circumstance as an opportunity to simplify their current expression without also having to incorporate a new term and operation (lines 22-35). As the teacher moved away, the students discussed how to multiply both 4x and 2 by -4, and soon correctly entered -16x-8. Thus while other groups were taking up the new task of constructing a squared term, the teacher and these students were using their coincidental choice of an identity term and operation to resolve some persistent confusion about distributing over parentheses. This emergent solution would later mark an opportunity (described above) for the teacher to showcase this pair being successful and to share this lesson about multiplying by one, and become an important resource for other groups as they worked to rewrite increasingly complex expressions.

Table 2. Overlapping teacher comments and dialogue of 3 student groups during a three-minute segment of the class session.

Line	Teacher	Group 8	Group 1	Group 2
1	T: Whoa. Group 6, what happened	D: [turns around, looks at groups seated behind him] Who's Group 6?		J: How'd they get a squared?
2	there? Nice.	A: [Turns and points to Group 6 in the back of the room]		B: Cause they got two x's. Cause they probably
3	T: Group 6, everybody see what	D: Yeah.		picked up, like, x and… 4x and 4x again.
4	group 6 has made up here?			J: Oh, two x's.
5	T: You guys have got an x^2 term,	A: Un-hnn.		J and B: No.
6	are there any x^2's out there?			
7	T: No, so you must have done something clever to make that. So, group 6, now I want to see if you can rewrite this with parentheses…			
8		D: [Calls to G in Group 1, points up at the screen and asks how his group got their current expression,		B: What do we do?
9		9x(-25).]		J: Yeah, what do we do?
10		G: [holds his hands up and shrugs in response to John's question.]		
11		A: Where am I, where am I, where am I? Go straight down.		
12	T: That's a good challenge actually.	D: Where are you going? Oh, I see you.	M: I guess you can try, um…. Try	B: What are you doing?
13		A: [picks up a 1] Hmmm….	9x. And then, in the parentheses,	J: I don't know. Just moving around.
14			put negative 22.	
15	T: The rest of you who are looking for some ways to challenge yourselves, see if you can get an x^2 into your expression.			
16		A: I'm hecka lost now. Hecka hecka lost.	G: [enters expression, realizes it is	B: Yeah.
17		D: [laughs] I know. Let's ask him for help.	incorrect when the collective	J: There's probably a, negative 1x or something.
18	T: Group 8, can you guys get n x^2 into your expression?		display does not update] Nope.	B: Yeah.
19	D: No, we already tried it like three times to get off the parentheses, but we can't.		M: No? [Goes to work on picking	J: Or 1x. I see negative 1x. I want to change the
20	T: Oh, you can't make them go away. Ok.		up a 2x and multiplying times	negative. I don't like negatives.
21	D: No. Somehow, well, she just got a number.		current expression. Changes	B: [as J picks up a -1x] Alright, try to multiply it.
22	T: Let's see what you've got. [looks at A's calculator] Ok, so, here's a suggestion. What do you think that		collective expression to (9x*2x)(-	J: Yeah, multiply. So you get two 75's… So then
23	that [(-4(4x+2))*1] is equal to?		25)]	everything's the same?
24	[D looks and A, neither answers]		G: Now Manuel's following you.	B: Wait, did you multiply? 1x by negative one?
25	T: Well, this is really interesting actually, because you picked up a one, right, and you multiplied times		M: There we go. I finally changed	J: Yeah.
26	one? So, what happens when you multiply, do you know what happens when you multiply something by		it.	B: So it'd be, no, you want to put it back in
27	one?		G: [looks closely at screen and	parentheses. So it would be…
28	D: It stays the same.		laughs] Oh.	J: Oh. [typing] x…parentheses…-75…
29	T: It stays the same. [A nods] So, your expression won't change, right?		M: [laughs] I know, it looks hard.	B: x, plus…er, plus negative six…and then
30	D: Sure.			minus five outside the parentheses.
31	T: What it's equal to won't change, but you can write it as something else that's equivalent. So how could			J: It's plus five.
32	you rewrite that expression without parentheses?			B: Plus five outside the parentheses.
33	A: Ohhhh, ok, I think I get it. So negative 4 multiplied…			J: Why outside?
34	D: It could be negative four multiplied by 4x+2, right?			B: You put 'em inside?
35	T: Sounds like it's worth trying.	D: Negative four?		J: I thought you would, cause like,
36	T: Whoa, Group 1. Can you rewrite	A: Negative, negative four, so it would be negative 16?	M: (smiles, nods)	B: Oh yeah, put it inside.
37	this?	D: No.	G: I'm trying.	J: [Enters new expression of -1x(-75x+-6+5)]
38		A: Yes, because you're multiplying them together… Is it? I'm sorry!		Dude, I have too many numbers.
39	T: So Group 1 has, check this out.	D: No, it's ok, yeah, it's right, you're right. It's right.		
40	Cause this might, If you didn't know	A: Plus? D: Yeah.		
41	this already, Group 6, how did you	A: Negative 8. [Collective expression updates to -16x-8.] Oh! Oh my	G: Can I?	
42	make x^2?	god, I got another one. [She and John high-five].	M: Hold on, go back.	

Group 1. In the moments preceding this segment, Group 1 had been struggling to pick up and operate on a new term that would allow them to rewrite 9x(-25) with two terms and an operation inside parentheses. Just before the teacher interrupted to show the class Group 6's expression, they had chosen to add a 3. As they returned to pair work, Melissa suggested adding this 3 to the -25 would generate 9x(-22) (lines 12-14). Just as the calculator showed that this new expression was incorrect (lines 16-18), the teacher encouraged all groups to try to construct an expression with a squared term. Melissa appears to have taken this new direction as an opportunity to sidestep their difficulties in inserting another term to their current set of parentheses using addition or subtraction; instead, in that moment she sought out a 2x, selected multiplication, and wrote a new set of parentheses in which she combined the new term and operation with their current 9x (lines 19-23). In this case, then, a newly emergent task (multiplying linear terms to construct a quadratic) also became an emergent solution to the previous task.

Group 2. As the teacher called the class's attention to Group 6's quadratic expression in the public display, the students in Group 2 discussed how Group 6 managed to "get a squared" (lines 1-6). As the teacher directed Group 6 to rewrite their quadratic expression using parentheses (line 7), the students in Group 2 wondered what they should be doing next (lines 8-14). Overhearing this comment, the teacher invited Group 2 and others looking for a new task to make an expression featuring x^2 (line 15). Group 2 immediately took up this challenge, seeking out a -1x to pick up and multiply times their current expression (-75x+-6+5) in order to both "get a squared" and "change the negative" (lines 16-26). In doing so, however, Ben felt they should also continue with the alternating parentheses task (lines 27-8). Similarly, in arguing that they should keep both uncombined constant terms inside the parentheses along with the linear term (lines 34-37), Jorge appears to have been continuing to attend to the emergent task constraint of writing multiple terms and an operator inside the parentheses. They merged these various tasks over the course of writing their next two collective expressions, first building -1x(-75x+-6+5) in lines 27-39, and then distributing and adding 3x to form $75x^2+4x$ in the minutes following this segment. Thus they were able to construct successively more complex objects and to integrate new challenges by successfully layering these with a progression of previous task constraints.

Discussion. These simultaneous episodes found each group negotiating an important set of challenges: because the environment required them to pick up and operate on a new term even as they undertook newly emerging tasks related to the construction of equivalent or more complex expressions, pairs had to simultaneously deal with the previous expression as object to be rewritten in different but equivalent form, and the chosen operation as process of combining old and new terms. In the first case, pair 8's choice of an identity operation enabled them to focus on collaboratively working through and resolving some confusion about the process of rewriting an equivalent expression without worrying about a new transformation. Similarly, in abandoning their unsuccessful attempts to add a constant term and instead multiplying by a linear, Group 1 turned the occasion of the second emergent task into a means of accomplishing the previous objectives. And Group 2 carefully selected a new term and discussed steps that would allow them to simultaneously accomplish several construction tasks. In each case, features of the learning environment—the properties of an arbitrarily chosen term and operation, system feedback via the calculator about the equivalence of expressions, the public display of another group's construction—provided resources that pairs were able to incorporate into emergent solution strategies.

Conclusion

We summarize two main themes from the analysis presented in this paper. Firstly, classroom activity in this learning environment was organized around successive and overlapping task goals that emerged from dynamic interactions among teacher, students and tools. In addition to those objectives initially set by the teacher, other tasks and constraints arose as both teacher and students attended to new expressions constructed by groups and publicly displayed via the network. Secondly, the interplay between these emergent objectives and other features of the learning environment presented students with a dynamic set of challenges reflective of the complexity of this classroom activity, and also with an emergent set of resources that they were able to organize into solutions to those challenges. We take the above episode to illustrate a novel form of classroom mathematics activity supported by this learning environment, one which blurs the boundaries between conventional instructional modes such as student-centered small group work and teacher-led whole class discussion. This hybrid activity structure is mediated by collective objects belonging to each group but publicly displayed for the whole class, thus providing emergent resources for the teacher to orchestrate, and for all students to actively and successfully participate in, simultaneous mathematical activity across multiple groups.

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