Definitions of Balance

Conceptual Understanding – *"Why" the math works*

- Provides an understanding of the structure and logic of mathematics
- Identifies the necessary constraints on the types of procedures students use to solve mathematical problems
- Enables students to detect when they have committed a procedural error
- Facilitates the representation and translation phase of problem solving

Computational and Procedural Skills – *"How" the math works*

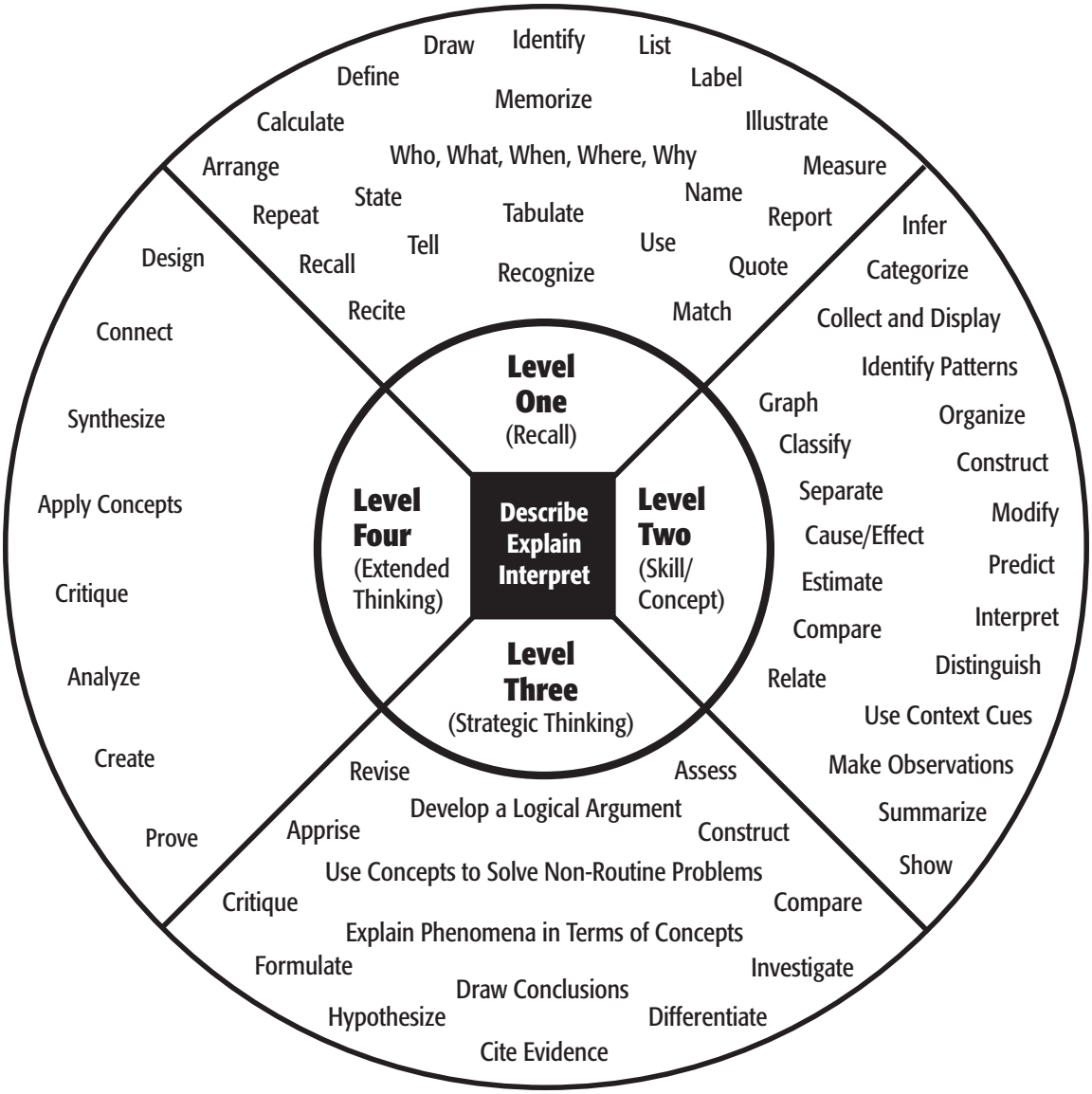
- Practice is required to become proficient
- Develops over time and increases in depth and complexity over several grades
- Distinguishes among different basic procedures by understanding what the procedures do

Problem Solving – *"Where" the math works*

The process of problem solving involves

- Determining mathematical hypothesis, making conjectures, recognizing existing patterns, searching for connections to known mathematical structures, and translating the gist of a problem into mathematical representation
- Putting together different pieces of information that are presented in complex problems, such as multi-step problems
- Developing a range of strategies to use in solving problems and verifying the correctness of the solution
- Applying skills that require and strengthen a student's conceptual and procedural competencies

Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting.	Identify and summarize the major events in a narrative.	Support ideas with details and examples.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.
Conduct basic mathematical calculations.	Use context cues to identify the meaning of unfamiliar words.	Use voice appropriate to the purpose and audience.	Apply mathematical model to illuminate a problem or situation.
Label locations on a map.	Solve routine multiple-step problems.	Identify research questions and design investigations for a scientific problem.	Analyze and synthesize information from multiple sources.
Represent in words or diagrams a scientific concept or relationship.	Describe the cause/effect of a particular event.	Develop a scientific model for a complex situation.	Describe and illustrate how common themes are found across texts from different cultures.
Perform routine procedures like measuring length or using punctuation marks correctly.	Identify patterns in events or behavior.	Determine the author's purpose and describe how it affects the interpretation of a reading selection.	Design a mathematical model to inform and solve a practical or abstract situation.
Describe the features of a place or people.	Formulate a routine problem given data and conditions.	Apply a concept in other contexts.	
	Organize, represent and interpret data.		

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>.

DOK Question Stems

DOK 1

- Can you recall_____?
- When did ____ happen?
- Who was ____?
- How can you recognize____?
- What is____?
- How can you find the meaning of____?
- Can you recall____?
- Can you select____?
- How would you write___?
- What might you include on a list about___?
- Who discovered___?
- What is the formula for___?
- Can you identify___?
- How would you describe___?

DOK 2

- Can you explain how ____ affected ____?
- How would you apply what you learned to develop ____?
- How would you compare ____? Contrast_____?
- How would you classify____?
- How are____alike? Different?
- How would you classify the type of____?
- What can you say about____?
- How would you summarize____?
- How would you summarize___?
- What steps are needed to edit___?
- When would you use an outline to ___?
- How would you estimate___?
- How could you organize___?
- What would you use to classify___?
- What do you notice about___?

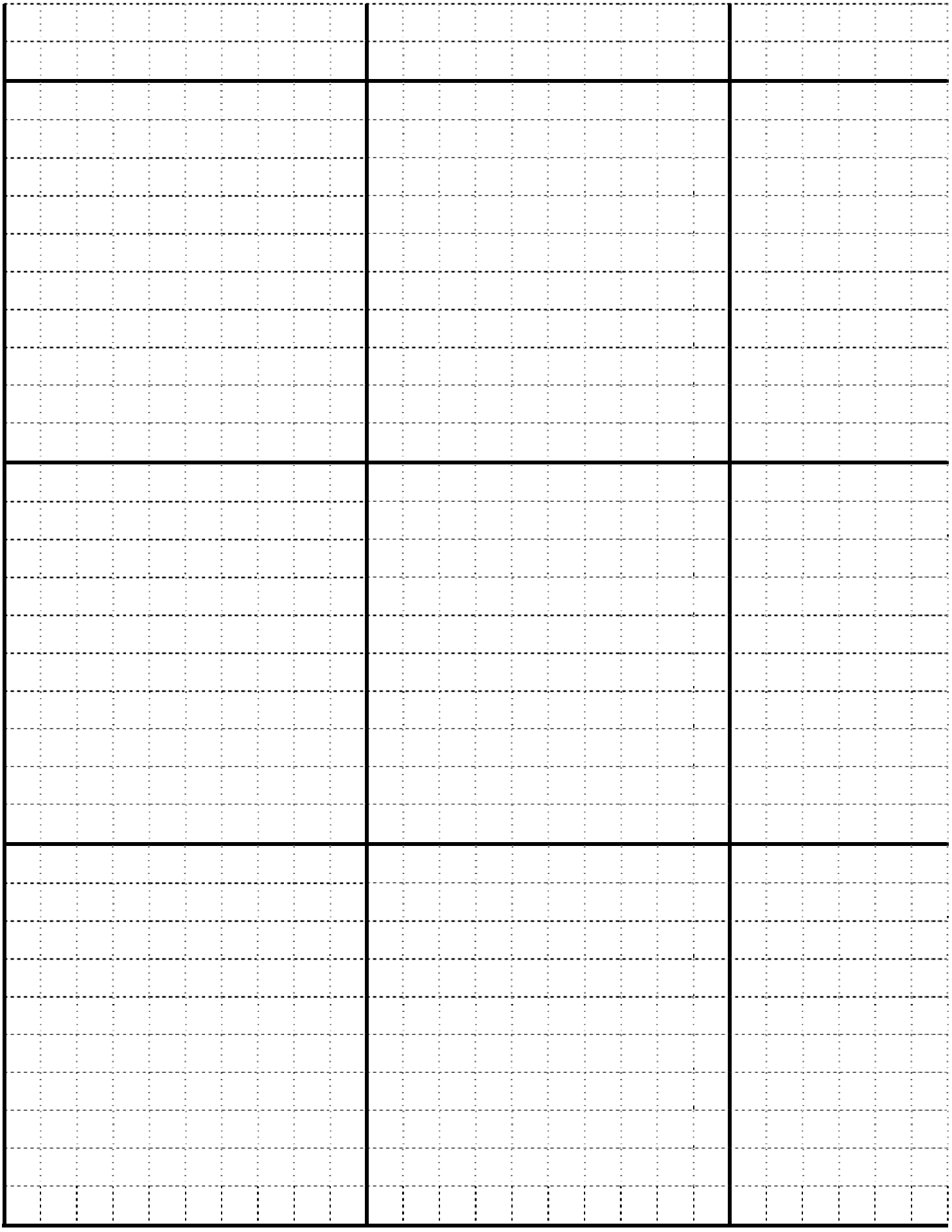
DOK 3

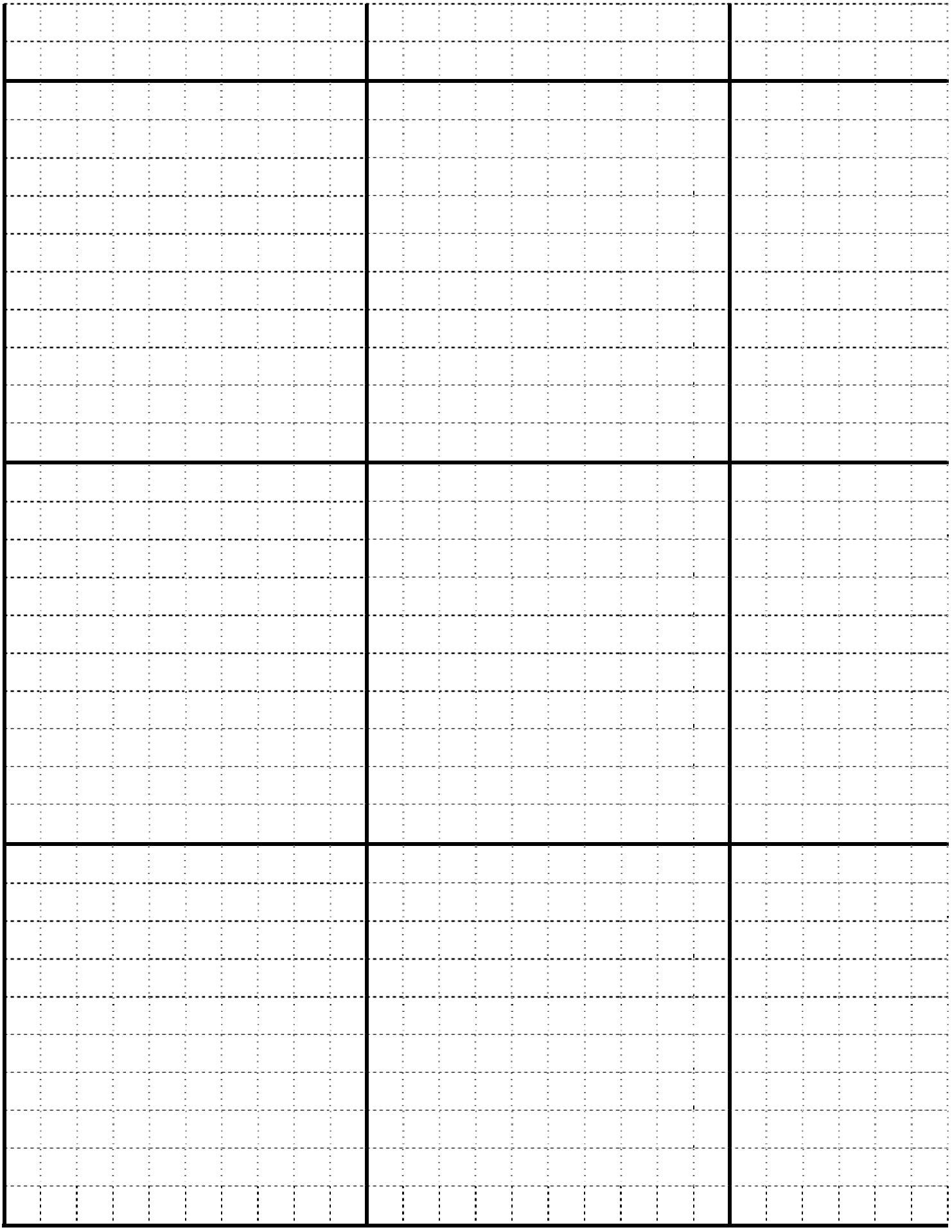
- How is ____ related to ____?
- What conclusions can you draw _____?
- How would you adapt____to create a different____?
- How would you test____?
- Can you predict the outcome if____?
- What is the best answer? Why?
- What conclusion can be drawn from these three texts?
- What is your interpretation of this text? Support your rationale.
- How would you describe the sequence of____?
- What facts would you select to support____?
- Can you elaborate on the reason____?
- What would happen if___?
- Can you formulate a theory for___?
- How would you test___?
- Can you elaborate on the reason___?

DOK 4

- Write a thesis, drawing conclusions from multiple sources.
- Design and conduct an experiment. Gather information to develop alternative explanations for the results of an experiment.
- Write a research paper on a topic.
- Apply information from one text to another text to develop a persuasive argument.
- What information can you gather to support your idea about___?
- DOK 4 would most likely be the writing of a research paper or applying information from one text to another text to develop a persuasive argument.
- DOK 4 requires time for extended thinking.

From Depth of Knowledge – Descriptors, Examples and Question Stems for Increasing Depth of Knowledge in the Classroom Developed by Dr. Norman Webb and Flip Chart developed by Myra Collins





1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81
10	20	30	40	50	60	70	80	90

10
20
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80
90
100

Problem of the Month
Squirreling It Away

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:
In the Problem of the Month *Squirreling It Away,* students use number operations, organized lists, and counting methods to solve problems. The mathematical topics that underlie this POM are knowledge of number sense, comparison subtraction, division, factors and divisibility, counting principles, systematic charting, and closed-form equations. The mathematics that includes counting principles and systemic charting is often referred to as discrete mathematics.

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In the first level of the POM, students are presented with a situation that involves making sense of totals and comparison differences. Their task involves making a number story about giving acorns to two different size groups of squirrels and then determining how many were left over from the original total acorns. In level B, students start to examine how the acorns can be partitioned into different sets. The students are told that different squirrels can carry different amounts of acorns on given trips. Students are asked to find the number of trips it takes to carry the acorn for each type of squirrel. In level C, students are asked how many ways three different types of squirrels can carry away 24 acorns. In level D, the student determines the number of ways 24 acorns can be divided between three specific squirrels. In level E, students are asked to find and justify a closed-form equation that will determine the number of ways that n acorns can be divided between three squirrels.

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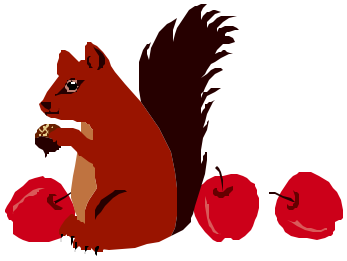
Problem of the Month



Squirreling It Away

Level A:

Austin has a bag of 17 acorns. Eight squirrels came up to him. He gave each squirrel an acorn. Then five more squirrels came up to him and he gave away one acorn to each of them. How many more squirrels can he still feed?

Show how you figured it out?

How do you know you have the right answer?

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Level B:

Austin likes to watch squirrels find and store acorns for the winter. Brown Squirrels can carry two acorns at a time. Gray Squirrels can carry three acorns at a time and Black Squirrels can carry five acorns at a time. There is a pile of 24 acorns.

How many trips would a Brown Squirrel need to make to store all of the acorns in the pile?

How many trips would a Gray Squirrel need to make to store all of the acorns in the pile?

How many trips would a Black Squirrel need to make to store all of the acorns in the pile?

If all three squirrels worked together to store the acorns how many trips would the squirrels need to make to store all of the acorns?

Explain your solution.

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Level C:

Brown Squirrels can carry 2 acorns at a time. Gray Squirrels can carry 3 acorns at a time.
Black Squirrels can carry 5 acorns at a time.

Suppose the three squirrels all wanted to store acorns for the winter. Depending on how motivated each squirrel was they would end up with different amounts. For instance suppose the Brown Squirrel took 4 trips, the Gray Squirrel took 2 trips and the Black Squirrel took 2 trips. The Brown Squirrel would end up with 8 acorns, the Gray Squirrel would have 6 acorns and the Black Squirrel would have 10 . Between them they took every one of the 24 acorns.

How many different ways could the three Squirrels divide up the 24 acorns and not leave any left over? Each Squirrel must carry their maximum load each trip.

How do you know that you have found all of the ways?

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Level D:

The Squirrels are rather smart. They realize that they can carry less than their maximum loads. How many different ways could the Squirrels divide up the 24 acorns.

Explain your solution.

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Level E:

Suppose there are a different number of acorns than 24. Determine a generalization for finding how 3 squirrels can divide up any given number of acorns.

Explain your solutions.

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Problem of the Month

Squirreling It Away

Primary Version

Materials: **A set of acorns or cubes (1-10) for each pair of students. Paper and pencil, crayons, or markers to use, write or draw.**

Discussion on the rug: "Here are some acorns. What animal likes to eat acorns?" (Teacher continues to ask children to name animals who like acorns). (Teacher holds five acorns in her hand) Suppose I have 5 acorns and one squirrel came up to me and I gave it an acorn, how many would I have left?" The teacher encourages the students to find answers for different amounts of acorns and ask the students to explain how they know.

In small groups: (Each group has a set of acorns or cubes) (Teacher asks the following questions. Go on to the next question if students have success)

1. You have 10 acorns. Four squirrels come to you. You give each squirrel one acorn. How many acorns did you give? How many are left? Now two more squirrels come to you and you give them each an acorn. How many did you give now? How many are left? How many more squirrels can you feed?

2. (Select a set of numbers that is reasonable for your class) You have __ acorns. __ squirrels come to you. You give the squirrel each one acorn. Now __ more squirrels come to you and you give them each an acorn. How many more squirrels can you feed? (At the end of the investigation, have students

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either draw a picture or dictate to you to represent their solution).

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Problem of the Month

Squirreling It Away

Task Description – Level A

This task challenges a student to reason about repeated subtraction in a word problem to find the number of acorns left after giving set amounts to two different sets of squirrels.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problems.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Problem of the Month

Squirreling It Away

Task Description – Level B

This task challenges students to solve problems involving subdividing a whole amount into sub-sets. Students need to think about equal groups or maximum size groups and interpret remainders in context.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1. Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares,, or as a number of objects in each share when 56 objects are partitioned into equal shares of 8 objects each.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and mathematical quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problems.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding.

Use four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem

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situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Problem of the Month

Squirreling It Away

Task Description – Level C

This task challenges a student to use division and equal-size groups to find all the possible ways the 3 squirrels can carry away exactly 24 acorns if they take the maximum amount every trip. Students are challenged to organize their thinking to develop a convincing argument about how they know they have found all the possibilities.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1. Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares,, or as a number of objects in each share when 56 objects are partitioned into equal shares of 8 objects each.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and mathematical quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problems.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding.

Use four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of the factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is a prime or composite.

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are

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comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y -2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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Problem of the Month

Squirreling It Away

Task Description – Level D

This task challenges a student to determine the number of ways 24 acorns can be divided between three specific squirrels, if they don't have to carry the maximum on every trip. Students must develop a convincing argument to explain their strategy and how they know they have every possibility.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1. Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

3.OA.2 Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares,, or as a number of objects in each share when 56 objects are partitioned into equal shares of 8 objects each.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and mathematical quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problems.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Ass the reasonableness of answers using mental computation and estimation strategies including rounding.

Use four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Problem of the Month

Squirreling It Away

Task Description – Level E

This task challenges a student to find and justify a closed form equation that will determine the number of ways that n acorns can be divided between three squirrels.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Analyze patterns and relationships.

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane.

Expressions and Equations

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems, include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y -2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain

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oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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Problem of the Month

Squirreling It Away

Task Description – Primary Level

This task challenges a student to think about subtraction and taking away. Students use cubes to help them act out the situation and record their ideas with paper and markers.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20 count out that many objects.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations verbal explanations, expressions or equations.

Represent and solve problems involving addition and subtraction.

1.OA.2 Solve word problems that call for addition of three whole numbers, whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.4 Model with mathematics.

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Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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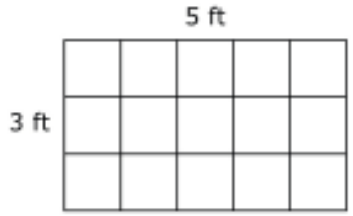
3rd Grade – ER

The top of Table A is a rectangle that is 5 feet long and 3 feet wide.

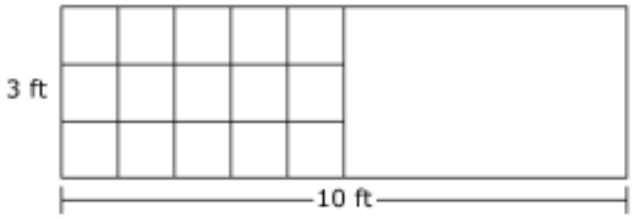
Table A



What is the area of Table A? _____ square feet

Table B is larger than Table A. Table B is also 3 ft wide, but it's 10 feet long.

Table B



Using what you already know about the area of Table A, show one way to find the area of Table B. You may use words and/or mathematical expressions/equations.

Another table, Table C, is added to Table B. The total area of these two tables is 54 square feet.

What could be the length and width of Table C?

Length: _____ feet

Width:_____ feet

Show how you got your answer. You may use drawings, mathematical expressions/equations, and words.

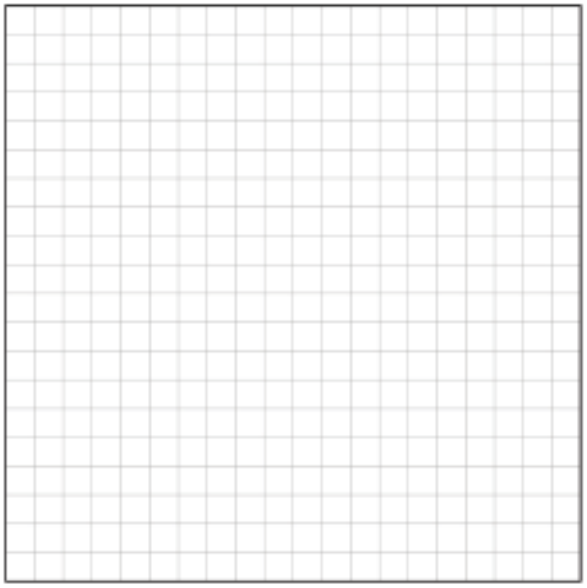
SMARTER BALANCED ASSESSMENT CONSORTIUM

CLAIM 2: "Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."

3rd Grade – ER

Jasper used the expression $5 \times (10 + 3)$ to find the area of a rectangular closet floor, in square feet.

On the grid, draw a rectangle that Jasper could have measured.



What is the area of the closet floor? _____ square feet

Jasper has 200 square feet of tile. He will use some of the tile to cover the closet floor. He will only use whole tiles.

How many square feet of tile will Jasper have left after covering the closet floor with tile?

_____ square feet

Jasper wants to use some of the remaining tile to cover the floor of a kitchen. The kitchen is 12 feet long and 12 feet wide.

Does Jasper have enough tiles to cover the kitchen floor? Circle your answer.

Yes No

Show how you got your answer. You may use drawings, mathematical expressions/equations, and words.

3rd Grade – CR

The digits in a three-digit number represent the amounts of hundreds, tens, and ones. Fill in the chart to show the amounts of hundreds, tens, and ones in the number 523.

Number	Hundreds	Tens	Ones
523			

In the box below, write a number that meets the following conditions.

- The number must be between 1 and 9.
- When the number is subtracted from 523, the digit in the ones place of the difference is **greater** than the ones place of 523.

4th Grade - CR

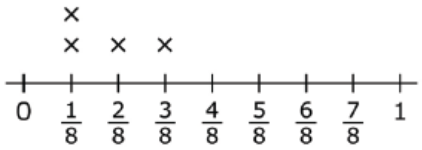
Click on the play button to view a video about evaporation.

Evaporation_Animation.mov

Judy conducted an experiment. She put a total of $2\frac{1}{8}$ cups of water into an empty container. Then, Judy recorded the amount of water that evaporated from the container each day for four days.

The line plot below shows the amount of water that evaporated from the container on each of the four days.

Amount of Water That Evaporated Each Day (cups)



Each × represents 1 day.

What mixed number represents the amount of water left in the container at the end of the fourth day?

☐ ☐/☐ Cups

4th Grade - ER

Mr. Torres sold a total of 30 boxes of sports cards at his store on Monday. These boxes contained only baseball cards and football cards.

- Each box contained 25 sports cards.
- He earned $3 for each sports card he sold.
- He earned a total of $1134 from the football cards he sold.

What amount of money did Mr. Torres earn from the baseball cards he sold? In the space below, use pictures, numbers, and/or words to show how you got your answer.

4th Grade – TE

There are 37 students in a class. Students go to a science lab in groups that contain no more than 7 students. Make a model to show the fewest number of science lab groups that will need to be formed with these 37 students.

Click on an oval to make a group. Continue as many times as necessary to make the correct number of groups.
[When an oval is clicked, an oval will be created in the working space.]

Click on a student and then click on an oval to put the student in a group. Continue as many times as necessary.
[When a student icon is clicked and then an oval is clicked, student icons snap to position in the oval to allow for multiple icons.]

What is the fewest number of science lab groups that will need to be formed with these 37 students?

☐ lab groups

5th Grade – CR

Three students created posters for a school election.

Part A

Carolyn created a square-shaped poster. The length of each side of this poster is $1\frac{1}{2}$ feet.

What is the area of this poster, in square feet?

☐ Square feet

Part B

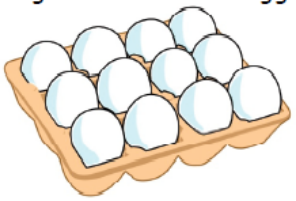
William created a rectangular poster with the same area as Carolyn's poster. His poster has different dimensions than Carolyn's poster.

What could be the dimensions of William's poster, in feet?

Length= ☐ feet Width= ☐ feet

5th Grade – SR

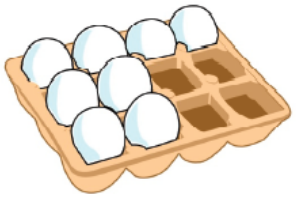
On Sunday, Travis bought the carton of eggs pictured below.



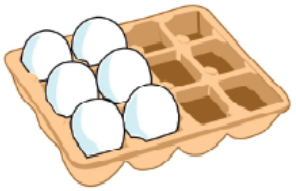
- On Monday, Travis used $\frac{1}{4}$ of the eggs in the carton.
- On Tuesday, Travis used $\frac{2}{3}$ of the eggs that **remained** in the carton after Monday.

Which picture represents the number of eggs remaining in the carton after Travis used eggs on Tuesday?

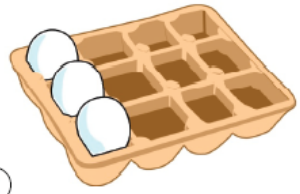
A



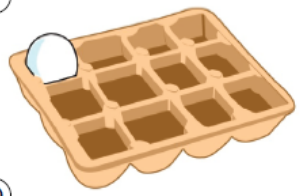
B



C



D



5th Grade – ER

Mrs. Phelps bought 4 boxes of crayons at the store to share with her students. Each box contained a total of 64 crayons.

Part A

What is the total number of crayons Mrs. Phelps bought at the store? Explain your answer using diagrams, pictures, mathematical expressions and/or words.

☐ crayons

Part B

Mrs. Phelps wants to give each of her students an equal number of the crayons she bought. There are 32 students in Mrs. Phelps' class. How many crayons should each student get?

☐ crayons

Part C

How many **more** boxes of crayons does Mrs. Phelps need if she wants each of her students to get 12 crayons? Explain your answer using diagrams, pictures, mathematical expressions and/or words.

☐ boxes of crayons

6th Grade - SR

In art class, Marvin painted tiles to use for a project. For every 5 tiles he painted blue, he painted 8 tiles green.

Identify the equivalent ratio(s) of blue tiles to green tiles. Select all that apply.

Ⓐ 20:23

Ⓑ 40:25

Ⓒ 50:800

Ⓓ 60:96

6th Grade – CR

Alia wants to buy pizza for a party.

- 40 to 50 people will be coming to the party.
- A large pizza from Paolo's Pizza Place serves 3 to 4 people.
- Each large pizza from Paolo's Pizza Place costs $11.50.

Part A

Alia wants to buy enough pizza so that people will not be hungry, and wants to have the least amount of pizza left over. How many large pizzas should Alia buy?

☐ pizzas

Part B

If Alia buys the number of large pizzas that you determined in *Part A*, how much money will she spend on pizza?

$☐

6th Grade – ER

Part A

Ana is saving to buy a bicycle that costs $135. She has saved $98 and wants to know how much more money she needs to buy the bicycle.

The equation $135 = x + 98$ models this situation, where x represents the additional amount of money Ana needs to buy the bicycle.

- When substituting for x, which value(s), if any, from the set $\{0, 37, 98, 135, 233\}$ will make the equation true?
- Explain what this means in terms of the amount of money needed and the cost of the bicycle.

Part B

Ana considered buying the $135 bicycle, but then she decided to shop for a different bicycle. She knows the other bicycle she likes will cost more than $150.

This situation can be modeled by the following inequality.

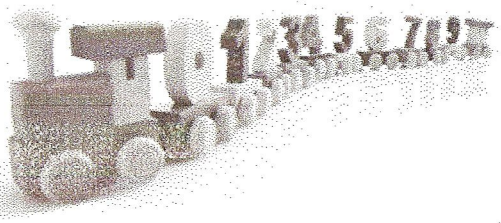
$$x + 98 > 150$$

- Which values, if any, from -250 to 250 will make the inequality true? If more than one value makes the inequality true, identify the least and greatest values that make the inequality true.
- Explain what this means in terms of the amount of money needed and the cost of the bicycle.

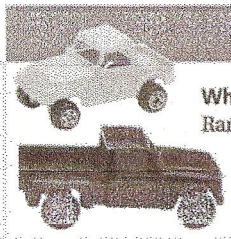
Lisa Englard

Toying with math

This month, consider how children's toys and games offer many opportunities to count, compare numbers, look for patterns, explore the coordinate plane, and investigate fractions and ratios.



Grades 5–6



WEEK 1

Who has more trucks? Ramon and Jared are playing with their toy trucks. Three-fourths of Ramon's trucks are dump trucks, and $\frac{2}{5}$ of Jared's trucks are dump trucks. Ramon says he has more dump trucks than Jared, because they learned in math class that $\frac{3}{4}$ is greater than $\frac{2}{5}$. Is Ramon correct? If Ramon has 8 trucks and Jared has 20 trucks, is Ramon correct? Why or why not?

WEEK 2

Susan and Ted are playing a board game with play money. Susan has 6 times as much money as Ted. She also knows that she has $250 more than Ted. How much money do Susan and Ted have altogether? On Susan's next turn, she uses $\frac{2}{5}$ of her money to make a purchase, and then loses $\frac{3}{4}$ of the remainder when she passes a space on the board. After Ted collects some money on his next turn, the ratio of his money to Susan's is 5:1. How much money did Ted collect in that turn? How do you know?

WEEK 3

Paul decided to donate some toys to a children's shelter. He noticed that the ratio of his toy soldiers to toy cars was 3:4. If he gave away $\frac{1}{3}$ of his soldiers and $\frac{1}{2}$ of his cars, he would have 24 soldiers and cars left. How many did he have at first? He also noticed that the ratio of the number of marbles to baseball cards was 4:7. If he gave away 30 baseball cards, he would have twice as many marbles as baseball cards. How many marbles did Paul start with? Finally, he counted and found that $\frac{2}{5}$ of his 126 puzzles had pictures of wild animals, $\frac{1}{3}$ of them were scenes from movies, and the rest were pictures of either famous buildings or bridges. He had $\frac{3}{4}$ as many building puzzles as bridge puzzles. If Paul gave away all his bridge puzzles and half his building puzzles, how many building puzzles would remain?

WEEK 4

Guess the location. *Ben and Jared are playing a game where they must guess the location of each other's pegs* on a coordinate grid. The grid starts at (0, 0) and extends 8 units up and 8 units to the right, with holes for the pegs at the whole-number coordinates. Ben has decided to place his 4 pegs at the vertices of a square. If he places 1 peg at (5, 3), what are some possible locations for the other 3 pegs? How many different squares are possible? How would this number change if he placed his peg at (4, 3)? At (3, 4)? Where should he place his peg to make the most possible combinations and therefore the hardest for Jared to guess correctly?

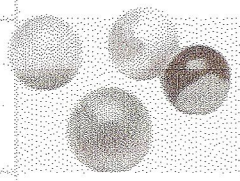
Lisa Englard, lisa@khanacademy.org, is a content specialist at Khan Academy. She enjoys challenging students and teachers with puzzles and problems that promote algebraic reasoning. Edited by Mark Amador, mamador@aventuracharter.org, who teaches seventh-grade mathematics at Aventura City of Excellence School in Aventura, Florida. E-mail problem collections for the editor to consider for future Math by the Month columns. See detailed submission guidelines at www.nctm.org/tcmdepartments. E-mail creative solutions and adapted problems to tcm@nctm.org for potential publication, noting Readers Exchange in the subject line.

Grades 3–4

WEEK 1

As they build towers with blocks, Sammy and Kendall use the following pattern to put a fence around one tower: cube, cube, cylinder, cube, cube, cylinder. What shape will the 8th block be? The 12th? The 19th? The 25th? Use the pattern to predict the 50th block and the 100th block. When Sammy reaches the 100th block, how many cylinders will be in the fence?

WEEK 2



Find the fractions. Joe and his cousin are playing a game with marbles. Joe has 3 red, 5 blue, and 11 green marbles. His cousin has 11 marbles, of which 6 are blue and the rest are red. What fraction of all their marbles is red? During the game, 3 green marbles roll away, and Joe cannot find them. What fraction of the remaining marbles is red?

WEEK 3

Kristen, Daniela, and Camilla are playing Lucky Seven. They toss a disk onto a number line, stand on the number where the disk lands, and then jump by 3s across the number line. They make 10 jumps each. The winner is the person who lands on the most multiples of 7 in 10 jumps. If Kristen starts on 0, Daniela starts on 1, and Camilla starts on 2, who will win the game? To win, on which number between 0 and 10 would they want to start? If they changed the game to jumping by 2s, where would they want to start? If they were to jump by 5s, where would they start?

WEEK 4

Margo's class has 3 times as many jump ropes as soccer balls. If they have 44 jump ropes and soccer balls altogether, how many jump ropes do they have? In her last soccer game, Margo scored 6 points. This was $\frac{3}{10}$ as many points as the number of jumps she can do in a row. How many jumps can Margo do in a row?

K–Grade 2

WEEK 1

On her computer, Gabriela's teacher has 2 math games, 3 reading games, 1 science game, and 2 music games. How many games does her teacher have in all? How many more reading games does she have than science games? If Gabriela's teacher buys 3 more math games, how many games will she have altogether?

WEEK 2

Eden and Dalia are playing a board game. Eden is on the 4th space, and Dalia is on the 2nd space. You win the game if you reach the 10th space. If you land on the 5th space, you must go back 1 space. Eden rolls a number cube that lands on 1, so she moves 1 space. Then Dalia rolls the number cube, and it also lands on 1. Which spaces are Eden and Dalia on now? Who is ahead, and by how much? On their next turn, Eden rolls a 5, and Dalia rolls a 4. Which spaces are they on now? Has anybody won yet? If not, what do they each need to roll to win on the next turn?

WEEK 3

Playing with cars during recess, Santiago invites Cindy to join him with her 13 toy cars. When they count up all their toy cars, they see that they have 21 altogether. How many cars was Santiago playing with before Cindy joined him? If each of the cars has 4 wheels, how many total wheels are there? How much larger is the number of wheels than the number of cars?

WEEK 4

To win the game that Gene and Talia are playing with tiles, they must slide a tile at least 25 cm farther than the other player. They slide tiles along the floor, and then measure to see how far the tiles slid. Gene slides his tile first and measures 103 cm. Talia's slide measures 79 cm. Did Gene win the game?

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones, & Walkup, 2009)

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	-Recall conversions, terms, facts			
Understand	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/ observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question -Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets
Evaluate			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
Create	-Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation