Solving Problems on a Number Line

1. I had 375 candies. I sell 90 of them. How many do I have left?

2. I was taking a trip to visit my sister. I drive 90 miles; then stop to rest. The total distance to my sister's house is 375 miles. How much farther do I have to go?

3. Mavis ate 5 candy hearts. Arturo ate 7 candy hearts. How many candy hearts did they eat in all?

4. Zayden's book has 91 pages. He has read 73 pages. How many pages will he have to read to finish the book?

5. A rope is 349 centimeters long. If 154 centimeters of the rope are cut off, how long is the piece that is left?

6. Kris drove for 3 hours at 70 miles per hour and then drove another 2 hours at 50 miles per hour. How far did he drive?

Measurement and Word Problems

2nd Grade – A rope is 83 centimeters long. If someone cuts off 45 centimeters of the rope, how long is the piece that is left?

3rd Grade – A fish tank holds 35 gallons of water. If you fill it using a 9-gallon bucket, how many buckets will it take to fill the fish tank?

4th grade – One grapefruit weighs 2 pounds 4 ounces. A second grapefruit weighs 3 pounds 12 ounces. How much do the 2 grapefruits weight together?

4th grade – JoAnne's ribbon is 52 inches long and Marie's ribbon is 4 feet long. Who has the longer ribbon?

5th grade – There are 3 identical boxes. Each one is 2 feet by 15 inches by 6 inches. What is the total volume of the 3 boxes?

We need _____ of trim.

_____ _____

Partner A Partner B

We agree that they need _____ of trim.

_____ _____

Table Group Member A Table Group Member B

We need _____ of trim.

_____ _____

Partner A Partner B

We agree that they need _____ of trim.

_____ _____

Table Group Member A Table Group Member B

We need _____ of trim.

_____ _____

Partner A Partner B

We agree that they need _____ of trim.

_____ _____

Table Group Member A Table Group Member B

Convincing Arguments

Criteria for Convincing Arguments

- **Convincing arguments leave nothing to inference.** A solid convincing argument is clear about assumptions being made, is based on facts, and has no gaps or holes.
- **Convincing arguments are tied to the original context of the problem.** If students use tables, diagrams, equations, or other tools to describe a problem situation, their justification must link the representation to the original context.
- **Convincing arguments stand up to any challenge.** A good convincing argument includes a justification for each step.

Fostering Convincing Arguments

- Takes time and practice
- Should be part of regular instruction
    - Teachers asking why
    - Students writing down their arguments in a coherent way

Questions to Help Develop Convincing Arguments

Audience	Sample Questions for Students to Ask Themselves
In preparing to convince a friend or person who thinks like you, you want to ask yourself:	• What do I know for sure? • What do I think may be true? • What do I want to be able to show? • What did I do to convince myself that I covered all cases? • Did I check for mistakes? • What will be some key things to emphasize in my explanation?
In preparing to convince a group that may include skeptics, presentation may play a greater role. You may want to ask some further questions:	• What examples can best make my case? • What will best communicate my argument: words? a picture? a table? an equation? • Am I prepared to have them test my rule on any case they select? • What counterarguments should I anticipate?

Common Core State Standards
Mathematics | Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students:

1. make sense of quantities and their relationships in problem situations.
2. bring two complementary abilities to bear on problems involving quantitative relationships:
   - ✓ *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents
   - ✓ *contextualize* – to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.
3. use quantitative reasoning entails habits of creating a coherent representation of the problem at hand:
   - ✓ considering the units involved
   - ✓ attending to the meaning of quantities (not just how to compute them), and
   - ✓ knowing and flexibly using different properties of operations and objects.

Do students:

- Mathematize the problem?
  - What are the given quantities?
  - How do they relate to each other?
- Represent the problem symbolically?
  - Can students explain what symbols mean and how relate to quantities, other symbols, representations, models?
  - Explain the context of problem?
- What properties and reasoning will support solutions? How can the problem be decomposed and recombined?
- What are the units needed while solving and reporting answer?

The Teacher's Role in Developing Mathematical Practice 2

- **Provide Distinct Opportunities for Students to Develop a Deep Number Sense**
  1. Express interpretations about number:
     - Count and understand quantity, know that quantities are equal to, greater than, or less than a given number
     - Make a given number in a variety of ways – 17 is 10+7 or 5+12 or 20-3
     - Understand that numbers are used in different ways – page 5, 3rd place, a 17-inch monitor
  2. Apply relationships between numbers:
     - Understand how numbers relate in a given context.
       - In K-2, understand that 5 and 10 are related because 5 + 5 = 10 or 2 fives = 10
       - By 4th and 5th, understand that they can rewrite $1/3$ as $3/9$ because 9 is a multiple of 3
  3. Recognize the magnitude of numbers and understand that context is important:
     - "Is 20 a big number?" depends – 20 eggs for breakfast is a large amount; 20 stars in the sky is not
     - 93 years old is pretty old, $93 may be a lot of money (to a 4th grade student), 93 blades of grass is barely anything
  4. Compute:
     - Reason through procedures and computations and determine if an answer is reasonable
     - Understand that there are 12 tens in 125, not 2
     - Understand how to compose and decompose numbers to make computation easier – changing 300-147 to 299-146 or multiplying 4 x 28 by using place value - 4 x 20 + 4 x 8
     - Understand that multiplication doesn't always make the number larger
  5. Make decision involving numbers:
     - Make sense of and understand the roles that numbers play in the real world. For example:
       - Suppose I need to turn at the 3rd street – being able to count to find the street
       - Suppose I need to catch the bus at 6:45. How much time do I need to get up and get ready? What time do I need to wake-up?
  6. Solve problems:
     - Reason through word problems
     - Translate given information (for example, from a word problem) into number sentences and algebraic representations
- **Draw Students' Attention to Numbers and Their Applications**
  - Help students see that numbers are everywhere and, as such, as a great source for explorations
  - Regularly have students solve interesting, contextualized problems and discuss the representations and strategies they use
  - Ask "How do you know?" and require students to justify their responses
  - Discuss reasonableness of responses in context of problems
- **Encourage Discussion That Promotes Reasoning**
  - Promote classroom discourse that facilitates not only teacher-student communication but also student-student communication
  - Consider student answers (right or wrong) as opportunities to stretch student reasoning
  - Provide opportunities for students to make and test conjectures about the math they're learning
  - Allow students to work collaboratively and share both their mathematical thoughts and their decision-making about the path their thinking takes.

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Timothy D. Kanold, Series Editor

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- compare the effectiveness of two plausible arguments
- distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.
  - ✓ elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades
  - ✓ later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Do students:

- Make conjectures?
  - Explore the problem to support or disprove their conjecture?
  - Refine or change their conjecture?
- Construct their justification? Use objects? Drawings? Diagrams? Examples and counter examples? Cases?
- Have opportunities to explain their conclusions and communicate their reasoning with others? What language is needed?
- Have opportunities to ask useful questions to seek clarity? Follow the arguments of others looking for flaws and explaining them?

The Teacher's Role in Developing Mathematical Practice 3

Does your class have a norm that requires students to provide an explanation with their solution? This norm, if present, places importance on ***how*** students solved a problem rather than just ***if*** they solved the problem.

- Establish Supportive Social Norms
    1. Model these sorts of behaviors.
    2. Share several correct solutions to the same problem, including various types and levels of explanations and justifications. Ask students to evaluate the acceptability of the solutions (either whole class or in small groups).
        a. Discussions should focus on completeness of solution and the *mathematical reasoning* involved.
    3. Share several solutions that have adequate explanations and justifications. Have students valuate the reasonableness of the solution and reasoning.
        a. Discussions should focus on the mathematics behind the solution instead of the completeness of the explanation and justification.

- Provide Opportunities for Students to Make and Evaluate Conjectures
    1. Provide opportunities for students to examine patterns and relationships and draw conclusions.

- Facilitate Meaningful Discussions of the Mathematics
    1. Prompt students to provide richer explanations and justifications, as needed.
    2. Prompt students to ask clarifying questions.
    3. Ask challenging and probing questions as needed to move the mathematics forward. (You need to have a deep understanding of the mathematics you are teaching so that you can ask questions that push students to deepen their understanding.)