The Birthday Problem

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UCDMP Saturday Series #5

A. If there are *twenty-five* people in a room, what is the probability that two or more people share a birthday?

(a) Less than 10% (b) 10-20%, (c) 20-30% (d) 40-50% (e) over 50%

B. If there are ten people in a room, what is the probability that two or more people have a birthday in the same week?

(a) Less than 10% (b) 10-20%, (c) 20-30% (d) 40-50% (e) over 50%

C. If there are five people in a room, what is the probability that two or more people have a birthday in the same month?

(a) Less than 10% (b) 10-20%, (c) 20-30% (d) 40-50% (e) over 50%

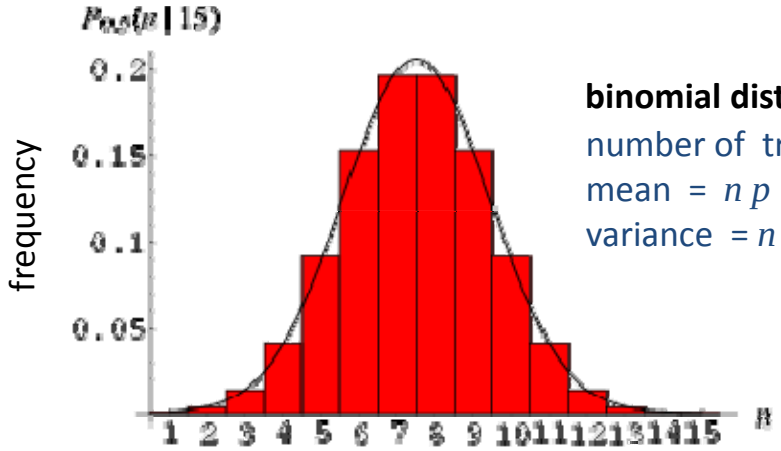
birthday survey results for workshop participants

# of people = 51; match = yes! (in fact, two matches)

class exercise: collect data from different classes; compile statistics.

birthday problem simulator

http://mste.illinois.edu/reese/birthday/intro.html



binomial distribution

number of trials with matches

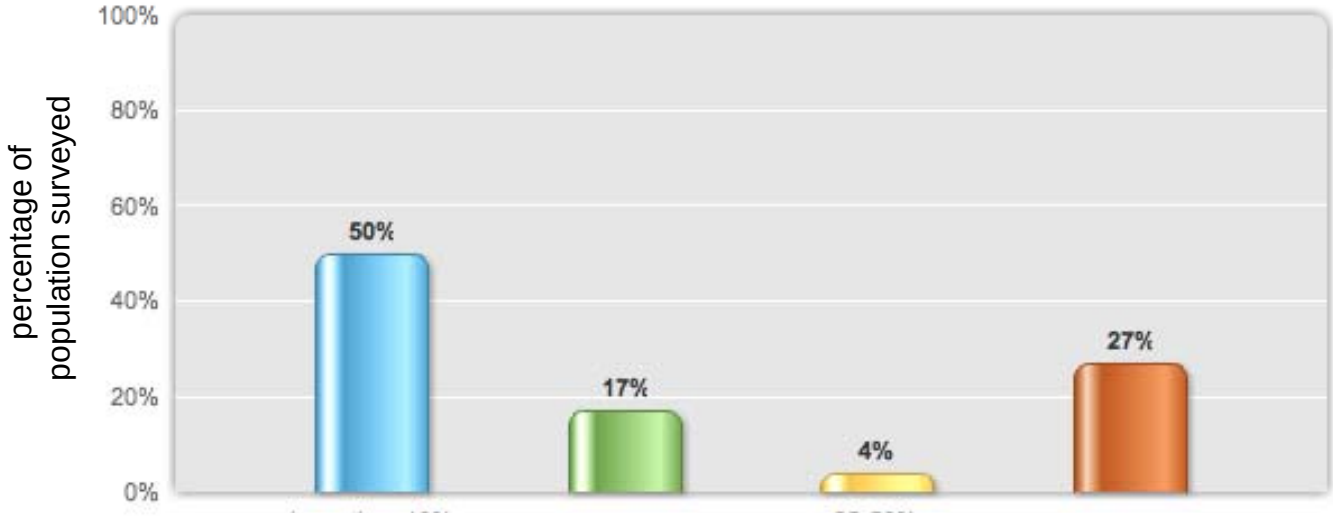
mean = $n\,p$

variance = $n\,p\,(1\text{-}p)$

If there are *thirty* people in a room, what is the probability that two or more people share a birthday?

(a) less than 10% (b) 10-25% (c) 25-50% (d) over 50%

(the answer is 71%)



survey of130+ executives and investment professionals in a LinkedIn poll
http://markturrell.wordpress.com/2010/11/17/the-spooky-maths-of-coincidence/

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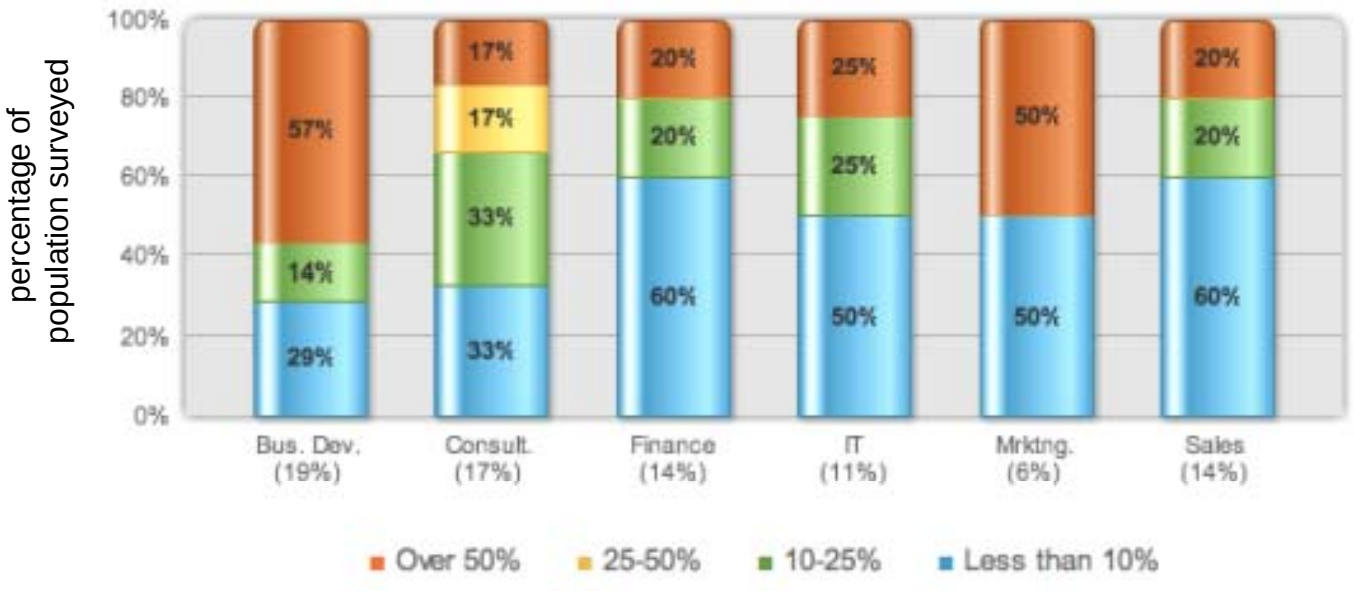

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The Birthday Problem for "N" people in a room

A. If there are ***N* people** in a room, what is the probability that two or more people share a birthday?

If there are *two* people in a room, what is the probability that two or more people share a birthday?

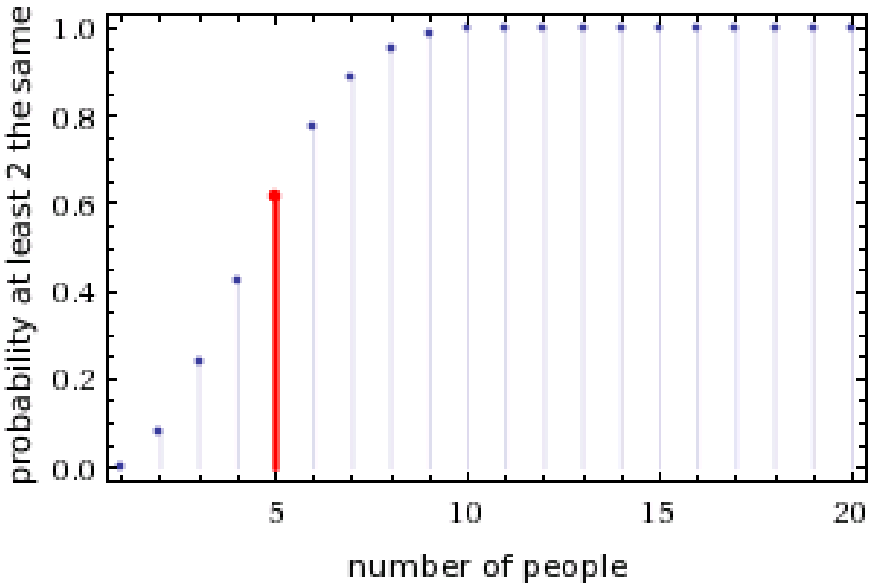
$$p(2) = 1/365$$

If there are *366* people in a room, what is the probability that two or more people have a birthday in the same month?

$$p(366) = 1$$

Birthday Problem Calculator (from Wolfram Alpha)

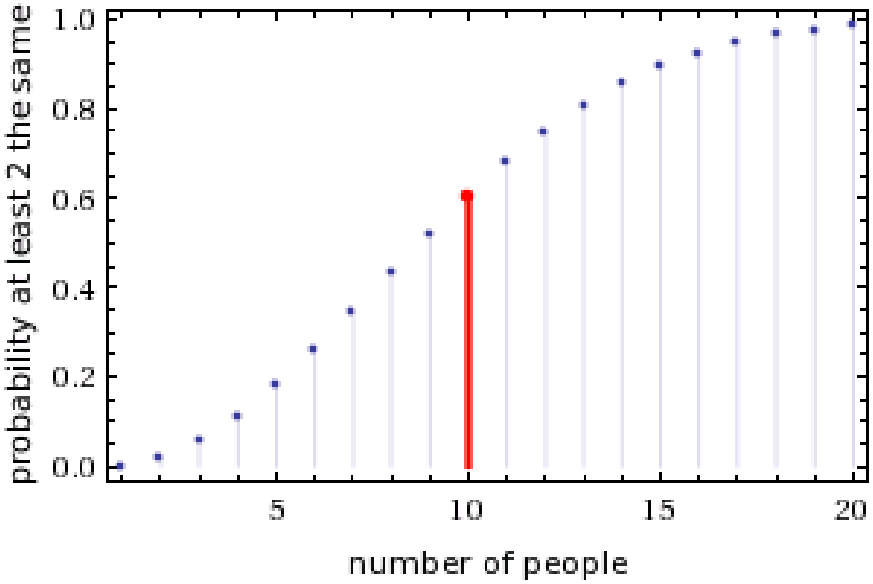
C. If there are ***N* people** in a room, what is the probability that two or more people have a birthday in the **same month**?



http://www.wolframalpha.com/input/?i=birthday+problem+calculator&a=FSelect_**BirthdayProblem-.BirthdayProblemWithLeapYear-&f2=23&f=BirthdayProblem.n_23&a=*FVarOpt-_**BirthdayProblem.pbds--

Birthday Problem Calculator (from Wolfram Alpha)

B. If there are ***N* people** in a room, what is the probability that two or more people have a birthday in the **same week**?



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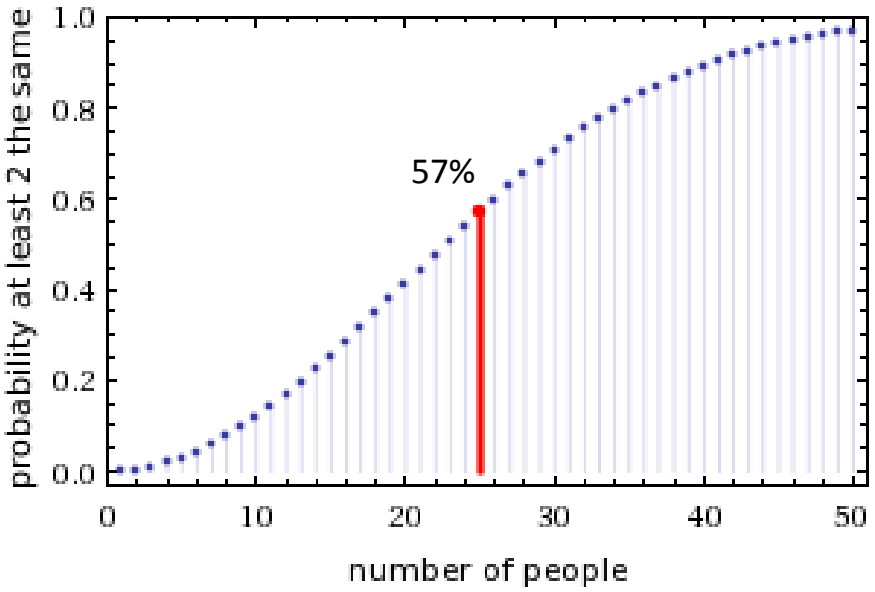
Birthday Problem Calculator (from **Wolfram Alpha)**

A. If there are ***N* people** in a room, what is the probability that two or more people share a **birthday**?



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The Birthday Problem

"The Tonight Show" (Feb. 6, 1980)
Johnny Carson and Ed McMahon
talk about the **birthday problem**,

http://www.cornell.edu/video/the-tonight-show-with-johnny-carson-feb-6-1980-excerpt

… what is wrong with Johnny and Ed's logic?

A1. If there are *N=25* people in a room, what is the probability that two or more people share a birthday?

p(N)?

A2. If there are *N=25* people in a room, what is the probability that someone else in the room has your birthday?

q(N)?

Equally probable outcomes:

If all possible outcomes of a random process are equally likely, then

(1) the probability that any given outcome occurs is 1/M, where M is the total number of outcomes.

(2) the probability that one of a set of m of the outcomes occurs is m/M

i. What is the probability that Bob's birthday is on May 4?

ii. What is the probability that Bob's birthday is in May?

iii. What is the probability that May 4 is not Bob's birthday?

a "counting" exercise: the multiplication rule

Suppose you are ordering pizza for a birthday party, and we had the following options:

crust: thick and thin (2 options)
sauce: tomato, pesto, white (3 options)
one topping: mushrooms, peppers, olives, pepperoni, garlic, onions, anchovies (7 options)

How many different pizzas could you order?

i. How many possible ways can N=5 people have birth dates?

ii. How many possible ways can N=4 people have a different birthday than you?

iii. How many possible ways can N=5 people have different birth dates?

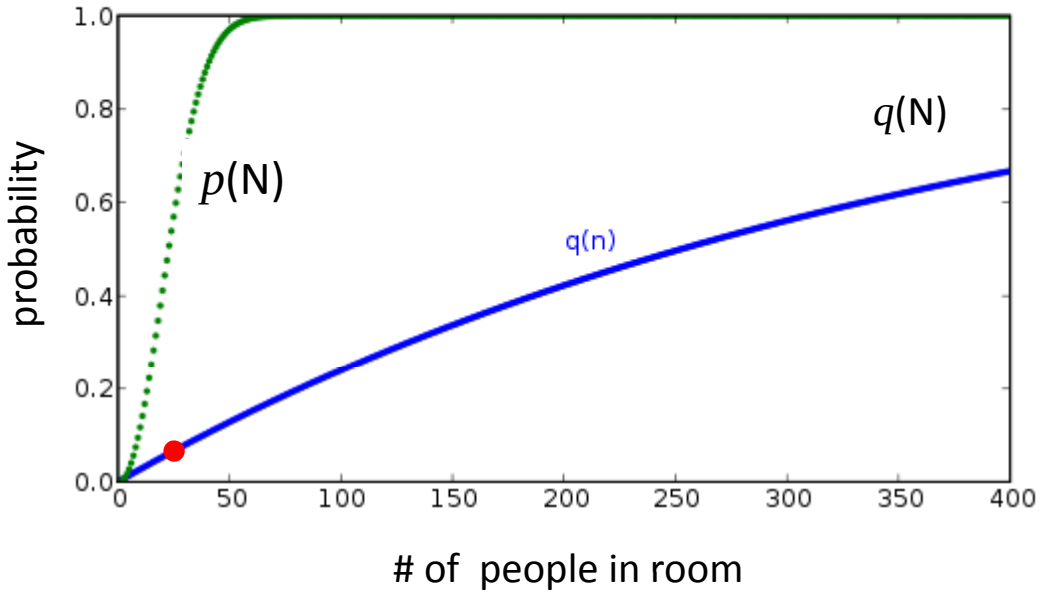
A2. If there are *N=25* people in a room, what is the probability that ***at least one other person*** in the room has your birthday?

$$q(N) = \text{prob}\,(\text{at least one match})$$

$$= 1 - \text{prob}\,(\text{no matches})$$

A2. If there are *N=25* people in a room, what is the probability that ***at least one other person*** in the room has your birthday?

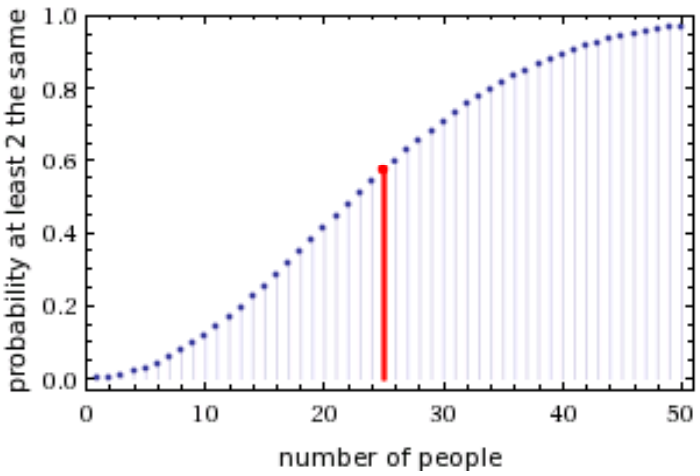
$$q(N) = 1 - \left(\frac{364 \times 364 \times 364 \times \cdots \times 364}{365 \times 365 \times 365 \times \cdots \times 365} \right) = 1 - \left(\frac{364}{365} \right)^N$$



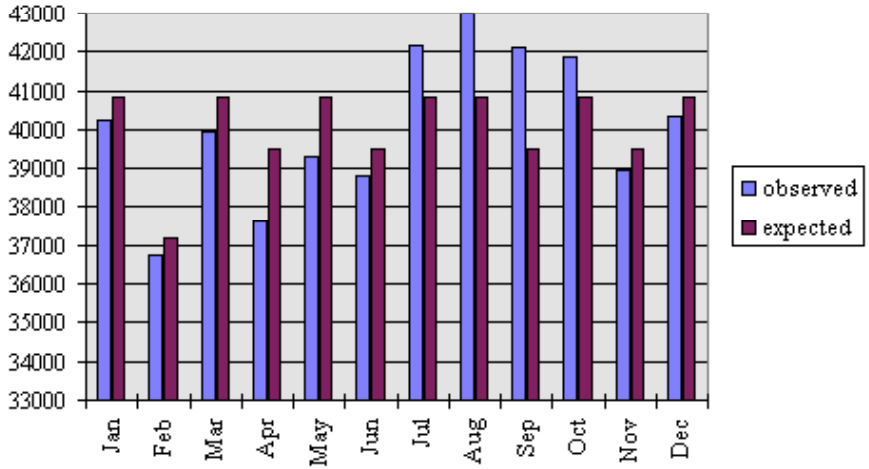
A1. If there are *N=25* people in a room, what is the probability that *two or more* people share a birthday?

$$p(N) = 1 - \left(\frac{365 \times 364 \times 363 \times \cdots \times (365 - N + 1)}{365 \times 365 \times 365 \times \cdots \times \quad 365} \right)$$

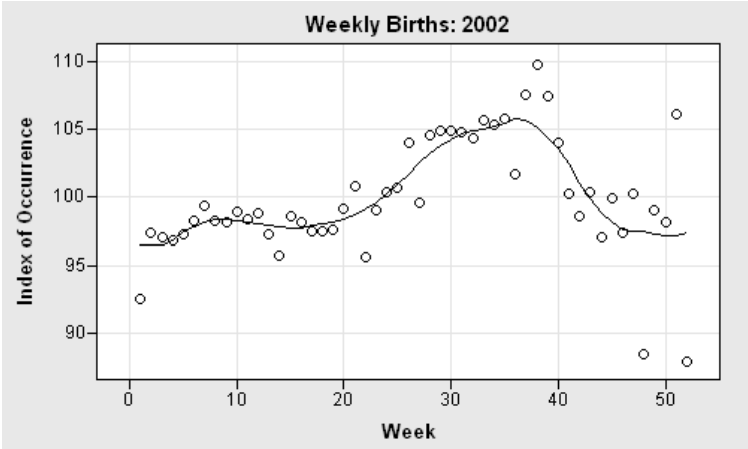
$$= 1 - \left(\frac{365 \times 364 \times 363 \times \cdots \times (365 - N + 1)}{365^N} \right)$$



distribution of birthdays by month/week



http://www.panix.com/~murphy/bday.html



http://blogs.sas.com/content/iml/2011/09/09/the-most-likely-birthday-in-the-us/

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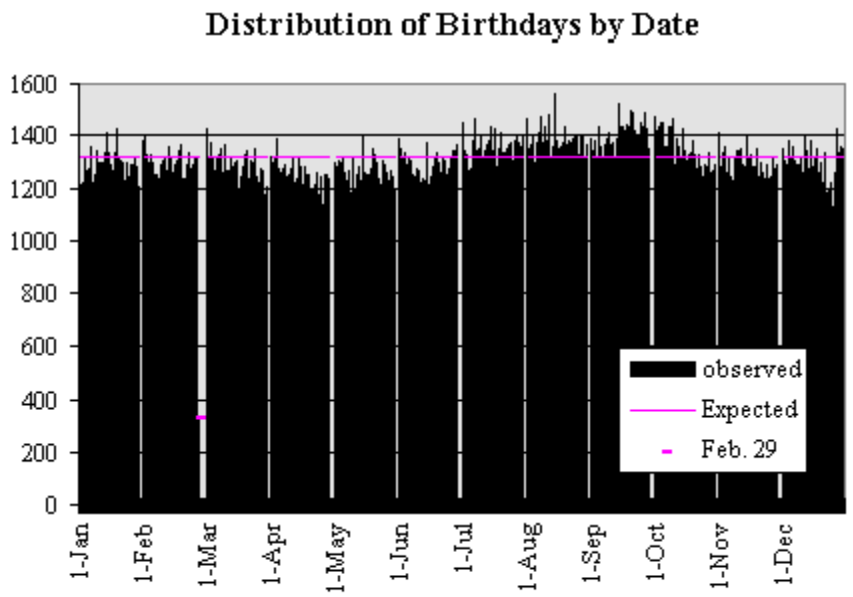
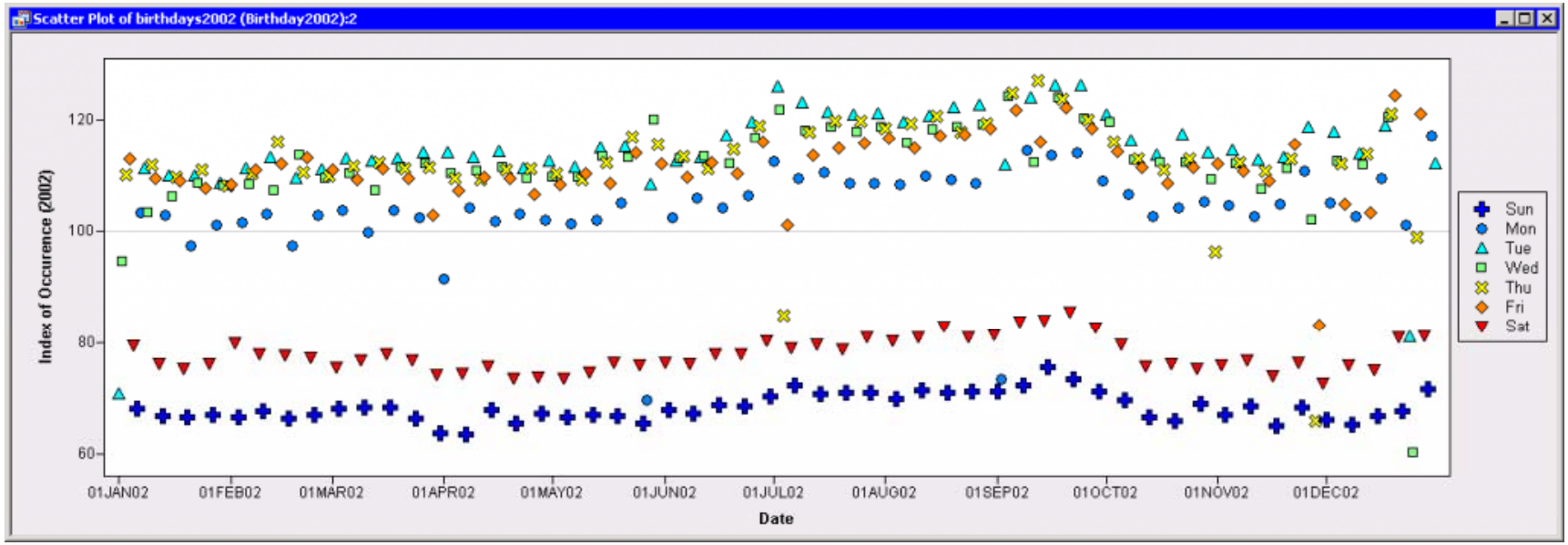


birthdates from 480,040 insurance policy applications made between 1981 through 1994 of a Life Insurance Company.
http://www.panix.com/~murphy/bday.html

The shapes and colors of the markers correspond to days of the week. Most US babies are born Tuesday through Friday. Compared with the other weekdays, Mondays have fewer births. The fewest babies are born on weekends and holidays.



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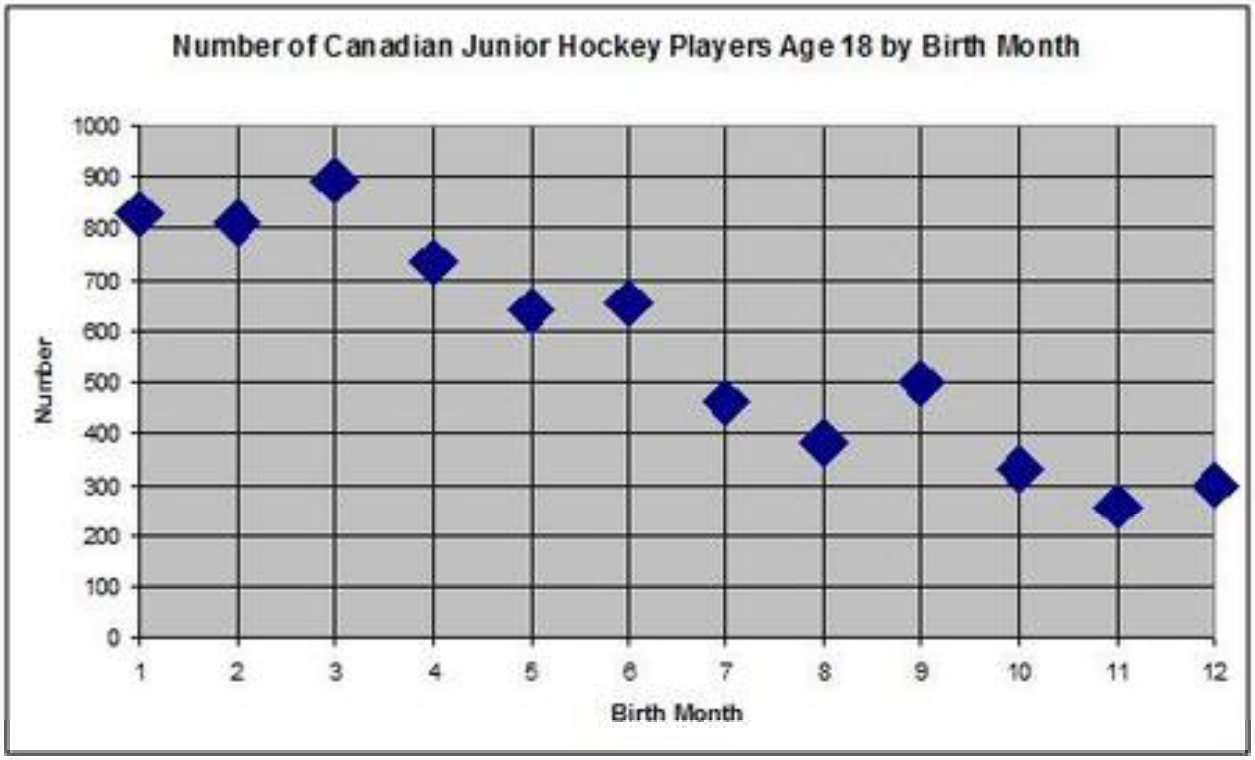
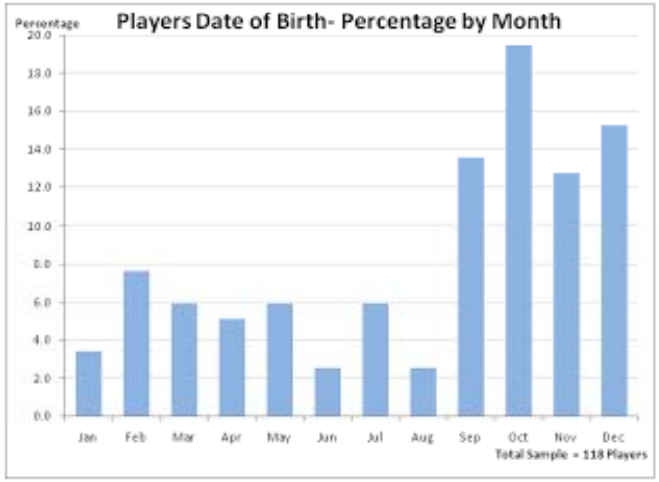
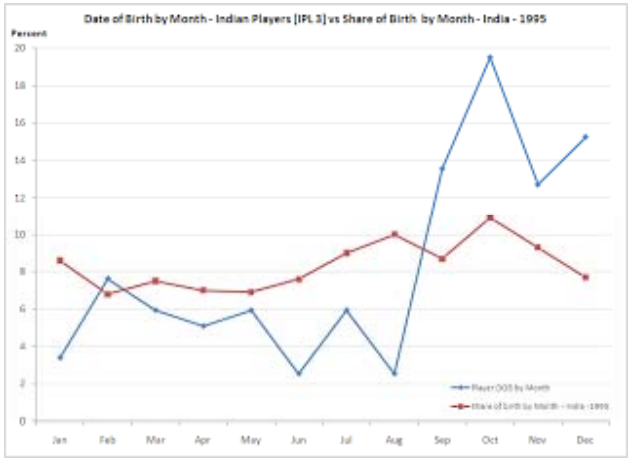


http://www.arcticicehockey.com/2010/8/3/1603630/inefficiencies-in-irrelevant

distribution of birthdays for those in the ‘elite class’ in Indian cricket



… compared to distribution of birthdays in Indian



http://www.visualquest.in/2010/04/mathew-effect-in-indian-cricket-and.html