Using Misconception-Based Worksheets to Encourage Meta-Cognitive Thinking in a Pre-Algebra Classroom

By

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Name: Joshua Glass
Title: Using Misconceptions to Encourage Meta-Cognitive Thinking in a Pre-Algebra Classroom
Research Question(s): How can a sequenced set of worksheets on misconceptions encourage students to reflect on their own and others' misconceptions improve student achievement in a high school Pre-Algebra classroom?

> Can using these misconception worksheets help close the wide achievement gap that exists between high-achieving and low-achieving students?
> Can worksheets that ask students to create common misconceptions on certain problems be used for students at multiple-skill levels to help keep all students engaged?
> Are there any differences between low-achieving and high-achieving students in their abilities to generate likely errors and produce explanations for common misconceptions?
> After learning a concept, will completing a misconceptions worksheet activity and discussing common mistakes help students eliminate these same mistakes in subsequent practice?
> Do students become better at identifying common misconceptions after completing the worksheet activities?

Research Activities:

This study took place in 2 Pre-Algebra classroom at an affluent southern California High school. The participants were a group of 51 students with diverse backgrounds, 24 of which came from homes where languages other then English are spoken, with Farsi being the most heavily represented language. All of the students had failed the course or one similar to it in the past. During the course of the intervention, students were given 3 worksheets that asked students to identify and correct common, misconception-based errors as well as to create potential errors on their own. Students worked in pairs to discuss problems as well as to identify errors their partner made. Data was collected on student opinions, student work, and student performance on an achievement test. Improvement in the ability to solve pre-algebra problems correctly and to avoid misconceptions was seen among a large percentange of the students, including a large improvement from the low-performing students. Students were able to eliminate the mistakes covered in the worksheets in subsequent assessment. Additionally, students started the intervention believing that they could learn from their mistakes and at the end of the intervention a larger percentage of students felt that their teacher valued their mistakes. Misconception-based worksheets can be used to help students solve pre-algebra problems correctly and to avoid common misconception-based errors. The use of misconception-based worksheets appears to benefit different ability groups in qualitatively different ways. Low-achieving students learn the mistakes to avoid while high-achieving students learn to think meta-cognitively, connecting different types of problems. Additionally, the intervention showed students that there is value in making mistakes and became more comfortable making mistakes in front of the teacher and others, thus increasing participation and discussion in the classroom.

Grade Levels: 9, 10, 11
Data Collection Methods: Curriculum assessment, Student work, Survey-Attitude
Curriculum Areas: Math-Pre-Algebra, Math-Remedial
Instructional Approaches: Collaboration/Teaming, Math-Conceptual Understanding, Math-Misconceptions

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Using Misconception-Based Worksheets to Encourage Meta-Cognitive Thinking in a Pre-Algebra Classroom

Throughout my first few months of teaching in a Pre-Algebra classroom it was apparent that my students lacked some needed math skills. Many of the topics had been seen multiple times by my students, yet even after multiple lessons the students made mistakes. Most importantly, at least half of the class was making the same mistake and these students were making these mistakes repeatedly. Additionally, these mistakes were often predictable and were almost always at least mentioned during the initial lesson. For example, students often considered 4-(-3) to be the same as 4-3, and when students were asked about this, they explained that a minus sign meant to subtract, not understanding the fact that subtracting a negative actually meant to add.

As I went through student work and talked to students while they were doing their work, I began to notice more and more that the mistakes being made were conceptual like the one above, and not just simple mistakes in calculations. One of the biggest mistakes I saw students make came when we learned to multiply fractions. Students were taught to just multiply straight across, so $\frac{2}{3} \bullet \frac{1}{6} = \frac{2 \bullet 1}{3 \bullet 6} = \frac{2}{18}$. However, during the lesson I noticed students were trying to find a common denominator, much like they would do if they were adding fractions. They then were multiplying across the numerator and keeping the denominator the same. This was a prime example of a misconception, because students made a mistake understanding how to do the problem even confusing it with another type of problem, instead of a mistake doing the calculations once they understood the problem. It was apparent to me that these mistakes were more common than simple calculation ones. In the example above a calculation mistake would have been: $\frac{2}{3} \bullet \frac{1}{6} = \frac{2 \bullet 1}{3 \bullet 6} = \frac{2}{15}$, since the student understood to multiply straight across, but ended up get $3 \bullet 6 = 15$ instead of 18 when they multiplied the denominator.

As I began to look into what research had been done about misconceptions in the math classroom, I found a lot of articles and studies that discovered students come into the classroom with many misconceptions. Schechter (2006) points out misconceptions students have at the college level, including problems that come from lower level-classes like the Pre-Algebra classroom, such as adding fractions and keeping track of positive and negative signs. Keazer (2004) observed students in Indiana who had mastered previous skills but were unable to apply this old concept to a new topic. This particular issue came up in my own class, with students who were able to multiply integers but made a mistake when they applied this concept to multiplying fractions.

Eggleton & Moldavan (2001) suggested that the best way to get students to address misconceptions was to have them work through a problem on their own, allowing the mistake to occur naturally instead of having the teacher give direct instruction on what mistakes to avoid. As I designed my intervention, I wanted to capitalize on this concept and encourage students to think about where mistakes came from. Because many mistakes students made were based on previously mastered skills (Keazer, 2004), I thought having students try to come up with an incorrect answer would force them to think about their previously mastered skills and think about how someone might mistakenly use these skills. In addition, students would be forced to think meta-cognitively, because they would be thinking about what someone who made a mistake might be thinking.

While I felt that addressing these misconceptions would be helpful to most of my students, I also realized that repeatedly going over the material may not have fit the needs of my students who were already successful in class. To address this, I decided to make my intervention open-ended enough to allow previously successful students the chance to really use their meta-cognitive thinking while my less successful students would be able to complete and learn from the worksheet Literature has shown that open-ended questions can be very beneficial in a classroom of students with different levels of ability, because open-ended questions allow strong students to keep engaged and working on a higher level of thinking, while weaker students still have time to get the basic part of the problem done and feel successful (Lawrence-Brown, 2004).

In addition, research has shown that meta-cognitive thinking in the classroom can lead to a greater understanding of the material (Veenman & Spaans, 2005; Kramarski & Mevarech, 2003). Veenman & Spaans (2005) found that, from a young age, students who thought in a meta-cognitive way were often more successful on novel tasks than their peers who did not. Additionally, Kramarski and Mevarech (2003) compared how eighth graders performed when taught a unit on graphing lines using (1) a meta-cognitive, collaborative method, (2) a meta-cognitive, non-collaborative method, (3) a non-meta-cognitive collaborative method, and (4) a non-meta-cognitive, non-collaborative method. They found that students performed best when given the meta-cognitive, collaborative treatment. Both of the above studies suggested that the use of meta-cognitive lessons could be very effective in improving younger students' abilities, which would seem not to fit in with the ages of the students in my study. However, since the research was focused on a Pre-Algebra classroom, where many students were developmentally behind their peers cognitively, specifically with their math skills, the research suggested that teaching meta-cognition in the classroom should have a positive result.

After looking at the research and my own students' work, I looked for a way to help address student misconceptions. Specifically, if students directly addressed these misconceptions, and used meta-cognitive thinking, I anticipated seeing an improvement in student achievement. My major research question was: *How can a sequenced set of worksheets on misconceptions encourage students to reflect on their own and others' misconceptions and improve student achievement in a high school Pre-Algebra classroom?*

This major research question suggested additional lines of inquiry that surround the misconceptions students have. One major issue had to do with students who did not have as many misconceptions as the lower performing students. While I anticipated the skills of the lowest performing students would improve, I also wanted the highest performing students to stay engaged and actually improve their own skills. The issue of closing this gap while still keeping all students engaged led me to my first sub-question: *How can using these misconception worksheets help close the wide achievement gap that exists between high achieving and low achieving students?*

In the same sense, I wanted to see if studying the misconceptions worksheets would keep all of my students engaged, which was addressed by my second sub-question: *Can worksheets that ask students to create common misconceptions on certain problems be used for students at multiple-skill levels to help keep all students engaged?* While I looked at the differences between high- and low-achieving students, it was beneficial for future planning to see if there were major differences in the way each type of student attacked these misconception problems. In fact I was interested in seeing *if there were any differences between low-achieving and high-achieving students in their abilities to generate likely errors and produce explanations for common misconceptions.*

Having students work on problems that show mistakes may have become problematic because students may have remembered the mistake instead of the correct way to do the problem. This was a potentially dangerous issue in my study, teaching students incorrect concepts, and I wanted to make sure that even if scores improved, students were actually avoiding the mistakes they learned about. This led to my fourth sub-question: *After learning a concept, will completing a misconceptions worksheet activity and discussing common mistakes help students eliminate these same mistakes in subsequent practice?*

Like any other mathematical lesson, the ultimate goal of the misconception worksheets was to get students to be autonomous, self-directed learners, able to perform the task on their own. While my intervention encouraged students to identify mistakes on their own, eventually they had to do the process independently. A good way to see if students did this was to see if they actually got better at the process of identifying and creating mistakes as the intervention went on. This led to my fifth sub-question: *Do students become better at identifying common misconceptions after completing the worksheet activities?*

Finally, after looking at how the intervention affected students' abilities, I wanted to investigate if the intervention changed student attitudes. Specifically, the intervention aimed to help students deal with wrong answers and learn from them, so I wanted to see how students felt about making mistakes before and after the intervention. This led to my last sub-question: *Will the worksheets affect the way students feel about making mistakes?*

Method

Participants

The research focused on two Pre-Algebra classrooms at Park City High School[1] located in an affluent area of Southern California. I had a total of 51 Pre-Algebra students broken into two different classes, one that met during 4th period and the other that met during 7th period. The first class, 4th period, met Monday, Tuesday, and Friday from 10:10-11:04 and Thursdays from 9:48-11:20 while 7th period met on Monday, Tuesday, and Friday from 1:44-2:36 and on Wednesdays from 1:44-3:15. Both classes were made up of mostly freshman students (see Table 1 for full breakdown), all of whom, along with the sophomores, had been in Pre-Algebra the year before and were recommended by their old teacher to take Pre-Algebra again. However, there were two juniors who were transfers from other schools where they had been exposed to Algebra 1 and were put in

Pre-Algebra by a counselor who had given them a standardized test. All of these students needed to pass Pre-Algebra so they can get to Algebra and graduate from high school.

My class was full of a diverse group of English Learners, including students who were still designated English Learners and some who were designated as proficient but came from households where English was not spoken. The distribution of home languages in both classes was similar, and both classes had students from many language

Table 1

Grade and Home Language Breakdown of Target Pre-Algebra Classes

	4th Period (n=24)	7th Period (n=27)
Grade		
Juniors	0	2
Sophomores	4	4
Freshmen	20	21
Home Language		
Russian	1	0
Farsi	7	4
Hebrew	1	1
French	2	1
Korean	1	1
Spanish	0	3
Polish	0	1
Japanese	0	1

backgrounds (Table 1).

The focus classrooms were similar in breakdown to the school as a whole, although there were noticeably more minority students in the intervention classrooms. Park City High is a fairly large school with a population of 2,362 students studying in its

[1] All names are pseudonyms

75 year-old walls. Although the demographics may appear to show that Park City High does not have a very diverse population (Table 2), the large Persian population that counts under the "White" category skews that data.

Table 2

Ethnic Breakdown of Park City High School and of Target Pre-Algebra Classrooms

Ethnicity	% in Park City High School	% in period 4	% in period 7
Asian/ Pacific Islander	19%	8%	7%
Hispanic	4%	8%	15%
Black/ Non-Hispanic	5%	21%	22%

Intervention

My intervention aimed at using meta-cognition, the way students understand the way they think, to help improve student understanding of mathematical concepts and potential misconceptions. This was done through the use of worksheets that showed common errors and asked students to create problems that show common errors. The first of these worksheets was introduced as students prepared to take a test. At this point, students were given a worksheet that showed problems, similar to those that they had studied for the upcoming test, completely worked out with a common mistake made.

This first worksheet covered subtracting and adding decimals, adding fractions, one-step solving for a variable problems involving fractions and decimals, one-step solving inequalities, and writing a decimal as a fraction. The worksheet problems were chosen for one of two reasons, either students had been observed making mistakes on similar problems or the problem was easily confused with a problem students had previously attempted. The errors were created by taking either real student mistakes or by solving the problem in a way that would have been used on a similar problem, but did not fit the current model. Students were asked to identify the error, find the correct answer, identify what someone may have been thinking if they made that error, and discuss their findings with a partner or a group (Figure 1 and Appendix B). In the example from Figure 1, it was my belief that students would see that the correct solution would have been to add the two numbers instead of subtract. I expected students to answer the third part, *why might someone make this mistake?* by commenting on how a subtraction symbol could make someone think of subtraction, instead of understanding the idea of subtracting a negative.

Identifying Mistakes

In the following worksheet you will be asked to identify what mistake the person doing these problems made. Identify ALL the errors you can find and explain why someone may have made that mistake. We will do example 1 together:

1. Evaluate the following:

3.53-(-2.12)

My solution:

$$\begin{array}{r} 3.53 \\ \underline{-\ 2.12} \\ 1.41 \end{array}$$

Identify the mistake made:

What should the solution be?

Why might someone make this mistake?

Figure 1. Sample misconception problem taken from Intervention Worksheet #1

During the completion of this worksheet and subsequent worksheets, I walked around the room assisting students. My main role was as facilitator, that is I did not give the answer, but I helped lead students to the answers. My main response to students' questions was to have them look at what the "mistake is" and to explain what was being done in their own words. I also asked students how they would solve the problem correctly, with the idea that comparing what they would do to what someone incorrectly did would help their thought process. Finally, I reminded students that the solution on paper is incorrect, so if the solution given was what they got, they needed to go back through their notes and homework to see exactly what they were doing wrong. For those students who still had no idea what to do after reading the book, I went through a quick lesson on how to do the problem correctly, which led them to see the error. Additionally, since these worksheets were new to students and a new way of thinking for students, I spent some time explaining exactly what the instructions were asking.

Ten days after this initial worksheet and after covering new material, students were asked to complete a similar worksheet, this one covering writing fractions as a decimal, multiplying fractions, multiplying mixed numbers, and dividing fractions. However, the last problem on this worksheet asked students to create their own mistake for a fraction multiplication problem. Students were asked to identify what mistake they made and why they thought a fellow student might make that mistake. After this worksheet, students returned from their three week long winter break and did the third worksheet, one where they identified the mistakes made on four problems and then they created the mistake on four problems. Additionally, once they had created the mistake on four problems, they traded papers with another student, who attempted to identify what mistake was made and why someone would make that mistake. This last worksheet covered multiplying decimals, finding the median, scientific notation, geometric sequences, one-step solving equations, writing fractions as decimals, multiplying fractions, and dividing fractions.

Data Collection Procedures

To investigate whether or not my intervention worksheets improved student achievement on the concepts covered, I gave a pre-and post-test that covered all of the material covered by the last two intervention sheets. I focused on material from the last two intervention worksheets because I considered the first worksheet to be a phase-in for students to start to understand what was being asked of them, not as much as a learning source. This 25 multiple-choice questions test was taken from the textbook test resources (Appendix C). Because the worksheets were based on student responses and predicted responses from questions from the book, the book test was a reliable way to assess the learning that had taken place. These pre-and post-tests, which covered topics including scientific notation, multiplying and dividing fractions, and multiplying and dividing decimals, helped me analyze whether or not students repeated the mistakes that were purposely given on the worksheets, or if going through the analysis helped them avoid it later. Additionally, the test gave me an opportunity to see if the achievement gap was closed, that is did my intervention help those most in need, the low achievers, more then the high achievers?

I used the written student work on the worksheets to see if students were using meta-cognition and staying engaged as they went through the worksheets. By comparing the way students responded to the worksheets at the beginning of the intervention to the way they responded at the end of the intervention I was able to see if student abilities improved over the course of the intervention. Additionally, looking at the detail of the written responses, or lack of detail, allowed me to see if students were engaged.

While looking at how student scores improved was interesting, it was also important to see if student abilities to analyze and create misconceptions improved through the intervention. If students were able to improve their ability to create misconceptions, then it would have shown that the intervention improved meta-cognitive ability in at least one context. To analyze whether or not this occurred, I looked at the types of responses students gave at the beginning of the intervention and compared them to the responses given at the end of the intervention.

My intervention also aimed to see how student attitudes changed through the process, specifically how students viewed making mistakes and going over mistakes made. To analyze this, students were given a survey before and after the intervention and I noted any changes that happened over time (Appendix A). If students saw value in the worksheets they were doing, there should have been an improvement in how they saw their own mistakes. Additionally, I anticipated an increase in the number of students who thought that looking at their mistakes would help them learn.

Results

In order to determine the effects of my intervention I investigated my research questions in a variety of ways. To determine if student achievement improved through the course of my intervention I used a pre-and post-intervention assessment and compared the results (Appendix C). The pre-test was given before students saw any intervention worksheet and before students had been introduced to the topics being covered. However, every student had seen every topic covered by the assessment in a previous class. This assessment was given in class and students, who were allowed to use a calculator if they brought one, were given 56 minutes to work individually on the 25 multiple-choice questions. Students were scored as correct or incorrect on each question, and the total number of correct responses is reported in the table below (Table 3).

Table 3

Pre-and Post-Intervention Assessment results

Class Period	Average Score Pre-Intervention Test (***n***=25)	Average Score Post-Intervention Test (***n***=25)	Average Difference in Score (***n***=25)
Period 4	9.86	17.54	+7.68
Period 7	11.38	14.67	+3.29
Combined	10.65	16.04	+5.39

One month later, students had received direct instruction on most of the topics (question 14 and 15 had been covered previously) and had been given intervention worksheets covering most of the topics. Students did not see their results on the original assessment and none of the material was reviewed. The assessment was then re-administered in the exact same situation, and graded in the exact same manner. The results of the assessment tests can be seen in Table 3. The major focus of the analysis was the change in score over the course of the intervention.

The results showed that there was an improvement in each class as well as in the intervention group as a whole. The overall average student improvement was 5.39 questions out of 25, or the equivalent of raising their score by 20% of the test questions.

While the results varied as to who improved the most, there was at least some improvement in nearly every student. Only four students, all in period seven, showed a decrease in correct answers while three students, two in period seven and one in period four, saw their scores stay the same. Alternatively, there were ten students whose score improved by more than ten questions. Adrian, a low-achieving, male freshman, showed the largest improvement, getting 17 more questions right on the post-intervention.

I also investigated whether or not the intervention helped close the wide achievement gap that existed in my pre-algebra classroom by analyzing the differences in improvement between the high-achieving and the low-achieving subgroups. To group students, I looked at overall test grades from the first semester in both of my classes. Students who averaged a 65% or below on all of our assessments were grouped into the low-achieving level while students who scored an 85% or better average on all tests or quizzes were grouped in the high-achieving level. Out of the 51 students in both classes, there were 12 students that fit the criteria of low-achieving and 18 students who fit the criteria of high achieving. As predicted, the low-achieving students showed a larger increase than their high-achieving peers, a percent increase of 65.17% and 57.78% respectively (Table 4). The results, tested for significance by running an ANOVA test with a null hypothesis of no difference in change between the two groups and the alternative hypothesis being there was a difference in score change between high-achieving and low-achieving students, were significant, $F(1, 29) = 5.11$, $p < .05$.

Table 4

Pre- and Post-Intervention Achievement Group Comparison

Grouping	Average Score Pre-Intervention Test	Average Score Post-Intervention Test	Average % Increase
Low Achieving (n=12)	7.42	12.25	65.17%
High Achieving (n=18)	13.17	19.00	57.78%

To investigate whether or not students were engaged in the material and able to comprehend the intervention, I went through each problem from the intervention worksheets and analyzed student responses. Student responses were counted as "correct" or "incorrect" as well as marked for "meta-cognitive" responses. Meta-cognitive responses were comments that indicated the student was thinking about the thought process behind the problem. Figure 2 compares two sample student responses to problem two taken from the second worksheet.

Solve the following equation: $\frac{3}{8} \bullet \frac{1}{4} = y$

Incorrect solution: *First, find a common denominator* $\frac{3}{8} \bullet \frac{1}{4}(\frac{2}{2})$

Then multiply across the top $= \frac{3}{8} \bullet \frac{2}{8} = \frac{3 \bullet 2}{8}$

Then simplify $= \frac{6}{8} = \frac{3}{4}$

What mistake did this student make?

Why might someone have made this mistake?

What should the correct answer be?

Bob	Joe
What mistake did this student make?	What mistake did this student make?
The student kept the common denominator instead of multiplying across the bottom	*The student didn't multiply the top and the bottom*
Why might someone have made this mistake?	Why might someone have made this mistake?
They confused it with an adding problem where you do keep the common denominator	*They forgot the rule*
What should the correct answer be?	What should the correct answer be?
3/32	*3/32*

Figure 2. Two sample student responses to question two from worksheet two. Bob's response was marked as correct and meta-cognitive. Joe's response was correct but not meta-cognitive.

More meta-cognitive comments would have indicated students were actively thinking as they went through the intervention, thus students were staying engaged. I considered any comment that compared the problem to a similar type of problem or any comment that indicated why a student would make a mistake as a meta-cognitive thought. This is very different then the majority of the comments, which indicated what the mistake was but failed to indicate why someone would make that mistake, other then "not paying attention."

Through my investigation I found that most students were able to identify the mistake made but few students were able to use their meta-cognitive skills to express why someone might make this mistake. Table 5 shows that the majority of students got the correct response, with a large increase from the first worksheet to the last two worksheets. However, the number of meta-cognitive responses was small considering that there were 51 students who responded. Also, the number of meta-cognitive responses did not grow between the first and second worksheet.

Table 5

Student responses to the Intervention Worksheets

Worksheet #	% of Correct Responses (n = 51)	% of Incorrect Responses (n = 51)	Number of meta-cognitive responses per problem (n = 51)
1	60.63	39.37	8.80
2	80.38	19.62	7.75
3	75.75	24.25	9.25

In addition to assessing how the class as a whole handled my intervention, I investigated how high performing students differed from low performing students in their abilities to create errors on the worksheet. Unfortunately, the short length of time students were in class limited the amount of time students had to create errors on their worksheets. Since the error creation section of the intervention was at the end of each worksheet many students did not have time to finish this section before they had to leave class. However, for those that did complete this part of the intervention, there were interesting results.

The first time students had to come up with their own errors was on a fraction multiplication problem at the end of the second worksheet. The 30 students who had time to answer the question generated seven different types of errors. The second time that students created their own mistakes there were four questions and students generated 4, 5, 8, and 5 different types of errors respectively, meaning there appeared to be no improvement in the creativity in student answers. This variety, or lack of variety, in answers was important because the more types of responses students gave meant that more students were thinking on their own. Based on the different types of responses from high- and low-achieving students it is impossible to discern any type of pattern between the sub-groups. The answer types that were most popular were given by high performing students, low performing students, and average students.

To determine whether or not students were able to later avoid errors they saw during the intervention, I carefully analyzed the pre-and post-intervention assessment to see what kind of mistakes students made. I selected seven problems (#8, 10, 12, 13, 14, 20, and 25 from Appendix C) to analyze, each of which had a potential wrong answer that was similar to a misconception that had been on a worksheet from the intervention. These seven were important because they allowed me to not only see whether or not students were getting the question right or wrong, but it allowed me to see if students were marking the misconception choice as their wrong answer. Over the course of the intervention multiplying decimals was covered once, scientific notation was covered once, dividing decimals was covered once, dividing and multiplying mixed numbers was covered twice, and one step equations was covered three times.

Looking at Table 6 it is clear that there was a marked improvement throughout the classes from the pre-to post-intervention. In order to see if there was a significant change in the proportion of students who marked the misconception between the pre-intervention assessment and the post-intervention assessment a one-tail, two-proportion z-test comparing the proportion of students who marked the misconception before the intervention to the proportion of students who marked the misconception after the intervention was run. The results were shown to be significant for questions #8, 13, 14, and 25, $p < .05$ (with test statistics of 1.83, 2.23, 2.33, and 4.10 respectively). Because an alpha level of .05 would have predicted that about one of the seven questions would have shown a significant change by chance alone, it is extremely unlikely that the above result of four questions showing a significant change was due to random luck. For each question the null hypothesis was that there would be no change or an increase in the number of students who marked the misconception answer and the alternative hypothesis was that there would be a drop in the number of students who marked the misconception answer.

Table 6

Students who marked tempting "misconception" answer on assessments ordered by percent change from greatest change to lowest change

Question # and Misconception Category	% of students who made mistake on Pre-intervention Assessment (n=51)	% of students who made mistake on Post-intervention Assessment (n=51)	% change in students who made the mistake
#13- Multiplying Decimals	9/51 (17.65%)	2/51 (3.92%)	-77.78%
#25- Scientific Notation	29/51 (56.86%)	9/51 (17.65%)	-68.97%
#14- Dividing Decimals	17/51 (33.33%)	7/51 (13.72%)	-58.82%
#10- Multiplying	11/51 (21.57%)	5/51 (9.80%)	-54.55%

Decimals			
#12- Dividing mixed numbers	11/51 (21.57%)	6/51 (11.76%)	-45.45%
#8- Multiplying mixed numbers	24/51 (47.06%)	15/51 (29.41%)	-37.50%
#20- Solving one-step fraction equations	11/51 (21.57%)	8/51 (15.69%)	-27.27%

While the original intervention aimed to have students improve their ability to think about creating mistakes on their own, the time constraints on each class period often left students with little time to complete the worksheets. There was one opportunity to create an error on the second intervention worksheet and four opportunities to do it on the third intervention worksheet, but not all students had time to respond on the third worksheet so the results are very limited. However, there were a few important things to note comparing the second worksheet to the third worksheet. Firstly, on the second worksheet there were two types of created mistakes (given by 12% of the responding students) that had not been covered in any of the previous intervention problems. On the last problem of the third worksheet there were five types of created mistakes (given by 100% of responding students) that had not been covered in any of the previous intervention problems. This shows a growth in students who were able to think creatively in creating their own misconceptions that they had never seen before, which also signifies an improvement in student ability to come up with potential mistakes on their own.

To measure the effects of my intervention on student attitudes, I gave students a survey before the intervention (Appendix A) and the same survey after the intervention. The survey aimed to investigate how students viewed making mistakes. Specifically, I wanted to see if students felt they could learn from their own and others' mistakes. In addition, I felt that if my intervention was successful, student attitudes would become more accepting of making mistakes and more willing to learn from them.

This survey was given to every student in my class but, because of absences, two students from my 4^{th} period class and two students in my 7^{th} period class were removed from the sample (they were absent for the pre- or post-intervention survey). The survey was given to students after they had completed a test and they had as much time as they needed to complete it. Students were told that the survey was for a research project and that it would not affect their grades.

I analyzed the results from the surveys by calculating the average student response to each survey question. A result of four or above was considered an agreement, while a score of two and below was considered a disagreement. Anything in between two and four was considered a polarizing question that did not have a class consensus. The pre-intervention data set showed that the majority of students saw that making mistakes was a normal part of the learning process, and even students who received A's made mistakes. This was an important result because it meant that students were not frustrated by making mistakes, they knew it was a part of learning. Additionally, the majority of students noted that they could learn from their own and others' mistakes (question seven and question eight respectively). However, the average on these questions was 4.5 and 4 out of 5 respectively, which meant there was the opportunity for growth on this question. The expectation was that through the intervention, students would begin to see the value in analyzing errors and the average on the post-intervention survey would be closer to 5.

The post-intervention survey showed some changes in the results, but that change was only shown to be significant on two questions. Question two, *I believe my teacher gets upset when I make mistakes,* and question seven, *I believe that looking at mistakes will make me better at math*, were tested with a two tailed, paired sample *t*-test at an alpha level of .05. The null hypothesis, there will be no change in student responses to this question, and the alternative hypothesis, there will be a change in student responses on this question, were the same for both questions and in both cases the alternative hypothesis was accepted with a *p*-level of .04 (Table 7). It is unlikely that two questions showed significant improvement due to chance alone, since an alpha level of .05 would have predicted that no more than one of the eight questions would have shown a significant improvement due to chance.

Table 7

Pre-and Post-Intervention Survey Data

Item	**Pre-Intervention Average** (*n*=47)	**Post-Intervention Average** (n=47)	**Change in Scores**	**P-value**
(1- Strongly disagree, 2- somewhat disagree, 3- unsure, 4- somewhat				

unsure, 4- somewhat agree, 5- strongly agree)				
1- I believe that I learn from mistakes	4.10	4.30	+.20	.24
2- I believe my teacher gets upset when I make mistakes	2.28	1.79	-.49	.04
3- I believe that students who get A's in math never make mistakes	1.66	1.62	-.02	.86
4- I believe that I can learn mathematics	4.51	4.51	0	1
5- I believe that working with others will help me in my math class	3.83	3.77	-.06	.81
6- I believe that everyone makes mistakes	4.77	4.79	+.02	.89
7- I believe that looking at my mistakes will allow me to become better at math	4.50	4.08	-.38	.04
8- Looking at the mistakes others make will help me avoid the same mistakes in the future	4.00	3.60	-.40	.07

Discussion

My achievement data clearly shows that students were able to improve their skills through the duration of my intervention. In fact, only 7 of the 51 students failed to improve over the course of the intervention. While the direct instruction that took place in the classroom may account for some of the improvement, we can see improvement on topics specifically covered by the intervention worksheets. Additionally, the format of the assessment, multiple-choice, meant a lot of the misconceptions covered in the intervention would be given as potentially tempting answers. Multiple-choice is often challenging for students because the wrong answer can be especially tempting, which is even more true if students have misconceptions in their head. Figure 3 gives an example from the pre-and post-intervention assessment.

Solve $2\frac{1}{4} \div 1\frac{1}{4} = y$

A. $1\frac{4}{5}$ B. 2 C. $2\frac{13}{16}$ D. $\frac{5}{9}$

Figure 3. Problem #12 from the pre-and post-intervention assessment.

A student may confuse this problem with a fraction subtraction problem, where one can subtract the whole numbers and subtract the fractions and combine the results. If a student did this, their result would be $(2 \div 1) \bullet (\frac{1}{4} \div \frac{1}{4}) = 2$. A student who had this misconception would have put B, 2, as their answer choice for this problem but the correct answer was A, $1\frac{4}{5}$. Thus, because students showed improvement in this format and avoided misconceptions after the intervention, the intervention was successful at improving student achievement.

Student achievement data was also analyzed to see if the low-achieving students were able to close the wide gap between them and their high-achieving peers. The results show that there were strong gains by both groups, but the low-achieving students showed a larger percentage increase than their peers. This result was encouraging because it showed the intervention to be effective for both sub-groups and the students were able to avoid the “Matthew Effect”. Stanovich (1986) described the Matthew effect as strong performing students improving more than their low performing peers, despite both groups being given the same intervention or resources. This is a common finding in research and Stanovich attributed it to the bidirectional nature of learning. For my students to avoid this while still showing improvement shows the effectiveness of the error misconception worksheets.

There are a few possible explanations as to why the results happened in this manner. The intervention was successful with low-achieving students because it forced them to address their own misconceptions and see that some of the thought processes they had in their head were wrong. While the intervention did this for all students, it was especially important for the low-achievers because these are the students who often make the same mistake repeatedly. Additionally, the low-achieving students tested lower on the pre-intervention test than the high-achievers, meaning that they had a lot more room to grow than their peers. Four students from the low-achieving subgroup scored lower than would have been expected just by chance, so it is logical that they would all improve even with the briefest of interventions. While the fact that there was a small difference between the high-achievers and low-achievers improvement may seem to point to a lack of huge gains in the low-achievers, it actually can be explained as surprisingly high gains in the high-achieving subgroup.

Through my intervention it quickly became clear that there were two levels of meta-cognitive thought taking place. The first level, which almost all students were able to do, involved identifying an error; that is students were able to think about a problem being wrong and compare it with the correct solution. The second level, which much fewer students attained, involved finding a mistake and identifying where that mistake came from. Students at this level were able to state that a student made a mistake because they confused it with another type of problem, whereas students at the first level could only identify the mistake and come up with the correct response. The students who were thinking on the second level tended to be high-achievers, and they were able to learn by thinking deeply about where the mistakes came from and analyzing the similarities and differences in problems. This appeared to lead to gains on topics they previously had a slight understanding of, thus leading to higher scores despite scoring well on the pre-intervention assessment. Meanwhile, the low-achieving students were able to learn by actively addressing misconceptions they would make or seeing how they should and should not solve a problem, leading to gains for their subgroup.

The intervention clearly shows that these worksheets were able to keep students of all abilities engaged. Students were much more successful on the second and third intervention worksheet than they were on the first, showing that after a little practice they were able and willing to find mistakes. However, the number of second level meta-cognitive comments did not increase over the duration of the intervention, which would indicate that students were unable to grasp that intended aspect of the intervention. While this seems logical it is incorrect because of the way the meta-cognitive thoughts were clumped together, that is most of the meta-cognitive thoughts were given on the same question. For example, look at question #2 on worksheet #2:

Solve the following equation: $\frac{3}{8} \bullet \frac{1}{4} = y$

Incorrect solution: *First, find a common denominator* $\frac{3}{8} \bullet \frac{1}{4}(\frac{2}{2})$

Then multiply across the top $= \frac{3}{8} \bullet \frac{2}{8} = \frac{3 \bullet 2}{8}$

Then simplify $= \frac{6}{8} = \frac{3}{4}$

This question clearly showed a mistake that confused adding fractions and finding a common denominator with the process of multiplying fractions. While most students were able to identify the mistake and correct it, only the second level meta-cognitive students expressed that a student made this mistake because they confused it with an addition problem. In fact 18 students were able to state this because the memory of addition was fresh in their minds from the last chapter and the idea of a common denominator is unique to adding fractions. However, some of the problems were not as straight forward. For example, #3 (Figure 4) from the same worksheet asked students to multiply mixed numbers.

---

1. Solve the following equation: $2\frac{1}{6} \bullet 3\frac{2}{3} = x$

Incorrect solution:

$$(2 \bullet 3)(\frac{1}{6} \bullet \frac{2}{3})$$
$$= 6\frac{2}{18} = 6\frac{1}{9}$$

What mistake did this student make?

Why might someone have made this mistake?

What should the correct answer be?

Figure 4. Problem #3 from the second intervention worksheet.

The mistake made was doing it in parts, multiplying the whole number and combining this answer with the multiplied fractions. While this is exactly how one could do it in addition, it is not the required way to do an addition problem. Unlike the common denominator mistake, this misconception would only be held by some students, meaning few students would even be prepared to mention this misconception. Thus, many students were unaware of how these problems were related, so they just responded that someone who made this mistake is “dumb” or “didn’t pay attention in class”.

Student comments such as this point out a major difference between the first level of meta-cognition some students showed and the second level of meta-cognition others were able to attain. To the first level students, many of the mistakes given were not an option because they do not remember making that mistake or they thought the problem was so easy no one could miss it. The second level students were able to think about the math problem in general, and by comparing the mistake to previously learned material they were able to see that not only was the mistake possible, but it was quite logical. This difference between first level and second level students is incredibly interesting and future research could focus on these differences and how they affect student learning. Additionally, research could investigate any method that would help the first level students start to think like second level students, greatly improving the achievement of the first level students.

The differences between the types of problems and the types of responses each problem elicited was prevalent throughout the intervention. While some problems quickly triggered memories and allowed students to relate to past problems, others seemed abstract or the mistake was too obvious and did not force students to think beyond the basic level. If this intervention were to be reproduced I believe that the instructor should give hints about the similarities between problems, such as sub-questions asking students to think of similar problems or similar types of situations. Additionally, I would spend more time going over a difficult example with the class, emphasizing a discussion on where mistakes come from. This concept could really improve the intervention, since so many students are confident in their perceived abilities and would benefit from learning why some of their concepts are incorrect.

My data shows that there was no noticeable difference in the way high-achieving students and low-achieving students handled the mistake creation part of the intervention. While this is counter to what I originally thought would happen, there were some interesting trends that are important to note. Firstly, student errors tended to be extreme errors that someone would not easily make. For example, if the original problem was multiplying fractions many student errors (9 of the 15 responses) had the wrong answer as an addition problem. There are very few high school students who would actually confuse a multiplication symbol with an addition symbol, which is very different from a student confusing the ways to solve an addition problem with the ways to solve a multiplication problem. This happened because most of my students lacked the second level of meta-cognitive ability required to link the current problem to similarities in problems they had seen before. Additionally, students may have just been trying to get the assignment finished and rushed by putting the first error they could think of, no matter how unrealistic it was.

While this result was somewhat disappointing, there were also some encouraging results. For example many students used mistakes they had seen on previous intervention worksheets when creating their own errors. Students who did this showed that they were able to remember common errors as well as the similarities between problems and apply it in a new situation. This is extremely valuable because students were able to remember the incorrect way to solve a problem, meaning they would be able to avoid this mistake in later problems. Future research may focus on whether or not students can successfully remember the misconceptions that were made (not just the right way to solve a problem) which they could use as a “thing to avoid” learning strategy.

I also think that working in partners was very beneficial for students in this exercise. Partners gave students the opportunity to discuss wrong answers with another person without feeling stupid for bringing up a wrong answer. This also meant that students who got it wrong were actually helping the partnership, because the wrong answer was what the teacher was looking for. As with all collaborative work, students who worked in partners had the benefit of two minds working together as well as the opportunity for a strong student to strengthen his or her own knowledge by teaching the material while the weaker student benefited from a one-on-one lesson. Finally, both students were forced to learn from one person’s mistakes, which would lead both students to avoid making that same mistake in the future.

The data clearly shows that students were able to later avoid making the misconception errors they saw during the intervention. This result is the most important of the study because it shows that students were able to learn from their errors and improve on specific topics. This ties directly into one of my sub-questions, *would students be able to avoid the mistakes on future assessments?* and puts to rest one of the biggest worries going into the intervention, that students would remember the misconception problems and think of them as the right way to do a problem. The results show that most of the class was able to improve in their ability to avoid the misconceptions covered, which implies that even if students are unable to reach the second level of meta-cognitive thought the intervention tried to evoke, facing misconceptions head-on can still be valuable. This idea, while researched before, should be further expanded upon in future research and could prove vital to those teachers who are tired of students making the same mistake repeatedly.

One example of students avoiding mistakes is Adrian. Adrian showed the largest improvement between the pre-and post- intervention, from 3 correct to 20 correct, and the questions he showed improvement on are interesting. Looking at question 13, "Solve (9.6)(0.4)=w; (A)10, (B)3.44, (C)3.84, (D)38.4," on the pre-test one can see Adrian chose option D, which is an incorrect answer because the decimal is in the wrong place. On the second intervention worksheet, the first problem asked students to identify the mistake in a decimal multiplication problem similar to the one found on the assessment. Adrian correctly identified that the decimal was in the wrong place on this worksheet and he was able to carry this information onto the post-intervention assessment.

Over the course of the intervention students were able to improve their ability to think of potential mistakes people could make, showing that the intervention did improve meta-cognition. Because the last intervention worksheet had students come up with more types of previously unseen mistakes, with students creating five different types of mistakes compared to students creating two types of mistakes on the second worksheet, we know that more students thought independently at the end of the intervention than did at the beginning. However, this claim must be considered cautiously due to the extremely small number of students who responded to the last question on the third intervention. Additionally, looking at the problems closer shows that the result may be partially due to their placement.

The last problem on the second worksheet deals with multiplying fractions and it is placed directly after an incorrectly solved problem dealing with the division of fractions. It is not surprising then that 13 students had their answer involve “flipping” the second fraction much like one would do in a division problem. This could have obviously influenced student opinions and would explain why there were many less response types to this problem. Although it is interesting to note that students still had to apply this concept to a new problem, showing that they did in fact learn the concept. Alternatively, the last problem on the third worksheet dealt with division of fractions, which had not been covered on that worksheet. So, students who tried this problem had no real reference and were forced to use their meta-cognitive abilities, leading to many different types of answers.

Knowing that many students ended up modeling their own mistake after one that the instructor gave them in an earlier problem would cause me to change the intervention if I were to do it again in the future. Rather then give students a model incorrect problem and then allowing them to copy it when they create their own incorrect answers, I would have the creative problems be completely different from the problems I modeled for them. This would force more exploration and creativity, which is one of the major benefits this intervention offers. Additionally, doing this allows an instructor to see what students believe about misconceptions rather than what they saw the worksheet show as a misconception. A prime example of this is on the last problem of the third worksheet where students came up with the misconception of dividing fractions by dividing straight across the numerator and straight across the denominator, making sure to just divide the bigger number by the smaller one (e.g., $\frac{2}{8} \div \frac{10}{4} = \frac{10 \div 2}{8 \div 4} = \frac{5}{2}$). This is a misconception that I would not have thought of on my own, so forcing students to think creatively allowed me to see a mistake students could potentially make.

The current data also shows that my intervention did not have a strong effect on most of the questions from my student attitude survey. Most surprisingly, the post-intervention survey data actually showed a significant drop in question seven, meaning more students disagreed with the statements, "I believe that looking at my mistakes will allow me to become better at math." This drop may signify student displeasure with the intervention worksheets, or it may represent students not seeing the value in the worksheets. While the drop may seem disheartening it is also somewhat insignificant because student scores improved. So while students may not have enjoyed what they were doing or actually felt it was helping them, the worksheets worked.

The other large and significant drop in score came on question two, “I believe my teacher gets upset when I make mistakes.” Student attitude shifted more towards disagreeing with the above statement, which is a positive intended result of the intervention. This result shows that through the brief intervention, many students started to see how much their instructor valued mistakes and learning from mistakes. In fact, with more time this concept would have become a major part of the classroom thinking and lead to more volunteering and taking chances in the classroom.

This intervention has addressed a very interesting and common problem, the acknowledgement and use of misconceptions in the math classroom, in a unique way. Because all math teachers see mistakes repeated over and over again, it is vital for research to find an effective way to address these problems. Additionally, it can be very valuable for a teacher to use meta-cognitive thinking to help improve student comprehension. While this study aimed to combine these two concepts into one style of teaching and reinforcing concepts, it was fairly limited in its scope and timeline. With more time in the classroom, it is my belief that students can begin to use these worksheets as a stepping stone to higher-level thinking and deep conceptual understanding of all topics. Despite the negative responses some students gave in the post-intervention survey, the assessment results showed that a series of worksheets dealing with misconceptions can be a very effective tool.

Since the results showed that student achievement did improve through the use of this intervention, I believe this concept should be applied to higher-level math courses, where students have been exposed to meta-cognitive thought and have a deeper understanding of mathematics. This deep understanding is vital to the meta-cognitive aspect of my intervention because I believe my students were unable to relate previously learned math concepts to current ones, which made it hard for them to think about why certain mistakes were made, such as seeing why someone would confuse multiplying fractions and adding fractions. The true value of this intervention in the Pre-Algebra classroom may be in how it allowed students to see potential mistakes and compare them to correct answers, rather than the intended result of having students understand why someone may have a misconception about a topic similar to a previously learned one. For example, with the problem in Figure 5, I had intended for students to notice that the person solving the problem had confused the idea of common denominators with a problem on multiplying fractions. Unfortunately, a lot of students were unable to reach this level of thinking, but they were able to avoid this mistake in the future, meaning they had learned to avoid one potential pitfall, even if they did not understand why someone had made it.

Solve the following equation: $\frac{3}{8} \bullet \frac{1}{4} = y$

Incorrect solution:	*First, find a common denominator*	$\frac{3}{8} \bullet \frac{1}{4}(\frac{2}{2})$
	Then multiply across the top	$= \frac{3}{8} \bullet \frac{2}{8} = \frac{3 \bullet 2}{8}$
	Then simplify	$= \frac{6}{8} = \frac{3}{4}$

Figure 5. Sample problem from intervention worksheet #2.

I think these worksheets can be a great resource in all classrooms, but especially important in a course like Algebra 2, where functions are explored and related to each other. Specifically, students need to be able to see how similar functions are subtly different and these worksheets point that out while challenging students to discover it on their own. In addition, in Algebra 2 students must look at equations and state what type of graph they would produce; these worksheets can challenge students to not only correctly identify the graph, but to identify why this graph has an equation that is similar to the equation of another graph. With misconceptions more likely in a Pre-Algebra classroom I thought these worksheets would be extremely beneficial, but the higher level of thinking required often took away from the lessons being learned. Students were often caught up not understanding why someone would make a mistake, instead of trying to find a problem similar to it that may have caused the mistake.

If I were to use this type of sheet again in a lower level class, I would walk through more examples of using meta-cognitive thinking. For example, I would spend time talking about how easy it is to confuse adding fractions and multiplying fractions, hopefully getting students to think about things like this when they go through the intervention on their own. Additionally, I would consider listing potential mistakes one could make, hopefully igniting the meta-cognitive thoughts inside students' minds.

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Appendix A

Pre-and Post-Intervention Survey

Baseline Survey

Please answer the following questions honestly. Your grade and Mr. Glass's opinion of you will not be affected whatsoever.

For the following questions, please circle one number

1. I believe that I learn from mistakes.

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

2. I believe my teacher gets upset when I make mistakes.

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

3. I believe that students who get A's in math never make mistakes.

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

4. I believe that I can learn mathematics.

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

5. I believe that working with others will help me in my math class.

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

6. I believe that everyone makes mistakes

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

7. I believe that looking at my mistakes will allow me to become better at math

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

8. Looking at the mistakes others make will help me avoid the same mistake in the future

1	2	3	4	5
Strongly disagree	Somewhat disagree	Unsure	Somewhat agree	Strongly agree

Appendix B

Intervention Worksheets #1

Name____________
Date_______Period______

Identifying Mistakes

In the following worksheet you will be asked to identify what mistake the person doing these problems made. Identify ALL the errors you can find and explain why someone may have made that mistake. We will do example 1 together:

2. Evaluate the following:

3.53-(-2.12)

My solution:

$$\begin{array}{r} 3.53 \\ \underline{-\ 2.12} \\ 1.41 \end{array}$$

Identify the mistake made:

What should the solution be?

Why might someone make this mistake?

3. Evaluate the following:

$$\frac{2}{3}+\frac{1}{2}$$

My solution: $\frac{2}{3}+\frac{1}{2}=\frac{3}{5}$

Identify the mistake made:

What should the solution be?

Why might someone make this mistake?

4. Solve the following equation for the missing variable.

x+3.25=2.52

My solution:

$$
\begin{array}{ll}
x+3.25 & =2.52 \\
\quad +3.25 & \ +3.25 \\
x & =5.77
\end{array}
$$

Identify my mistake:

What should the solution be?

Why might someone make this mistake?

5. Solve the following equation for the missing variable.

$y-(-\frac{1}{6})=\frac{1}{4}$

My solution:

$$
\begin{array}{ll}
y-(-\frac{1}{6}) & =\frac{1}{4} \\
\quad -\frac{1}{6} & \ -\frac{1}{6} \\
y & =\frac{1}{4}(\frac{3}{3})-\frac{1}{6}(\frac{2}{2}) \\
y & =\frac{3}{12}-\frac{2}{12} \\
y & =\frac{1}{12}
\end{array}
$$

Identify the mistake made:

What should the solution be?

Why might someone make this mistake?

6. Solve each inequality.

$b + 3.21 \leq 2.55$

My solution:

$$\begin{array}{ll} b + 3.21 & \leq 2.55 \\ \underline{\quad -3.21} & \underline{\ \ -3.21} \\ b & =1.34 \end{array}$$

Identify the mistake or mistakes made:

What should the solution be?

Why might someone make this mistakes or multiple mistakes?

7. Express the following as a fraction in simplest form:

$.\overline{4}$

My solution: $\frac{1}{4}$

Identify the mistake made:

What should the solution be?

Why might someone make this mistake?

Intervention Worksheet #2

Name____________

Date_________Period_____

Chapter 6 Errors Worksheet

After looking through our work over the last week, Mr. Glass has identified some mistakes students are making. These problems were taken from student work or from discussions with students.

2. Write the following as a decimal: $2\frac{5}{6}$

Incorrect Solution:

First, write as an improper fraction, then divide $2\frac{5}{6} = \frac{25}{6} = 6\overline{)25.00}$

What mistake did this student make?

Why might someone have made this mistake?

What should the correct answer be?

3. Solve the following equation: $\frac{3}{8} \bullet \frac{1}{4} = y$

Incorrect solution: *First, find a common denominator* $\frac{3}{8} \bullet \frac{1}{4}(\frac{2}{2})$

Then multiply across the top $= \frac{3}{8} \bullet \frac{2}{8} = \frac{3 \bullet 2}{8}$

Then simplify $= \frac{6}{8} = \frac{3}{4}$

What mistake did this student make?

Why might someone have made this mistake?

What should the correct answer be?

4. Solve the following equation: $2\frac{1}{6} \bullet 3\frac{2}{3} = x$

Incorrect solution:

$$(2 \bullet 3)(\frac{1}{6} \bullet \frac{2}{3})$$
$$= 6\frac{2}{18} = 6\frac{1}{9}$$

What mistake did this student make?

Why might someone have made this mistake?

What should the correct answer be?

5. Solve the following equation: $t = \frac{9}{10} \div \frac{3}{5}$

Incorrect solution:

$$\frac{9 \div 3}{10 \div 5} = \frac{3}{2} = 1\frac{1}{2}$$

What mistake did this student make?

Why might someone have made this mistake?

What should the correct answer be?

6. Solve the following equation: $g = \frac{3}{7} \div \frac{2}{5}$

Incorrect solution:

$$\frac{3}{7} \div \frac{2}{5} = \frac{7}{3} \bullet \frac{2}{5}$$
$$= \frac{7 \bullet 2}{3 \bullet 5} = \frac{14}{15}$$

What mistake did this student make?

Why might someone have made this mistake?

What should the correct answer be?

7. Solve for the following equation: $b = \frac{3}{5} \bullet \frac{12}{30}$

Solve this problem incorrectly, show a mistake you think a classmate may make:

What is the error you did in the problem above?

What would the correct solution be?

Intervention Worksheet #3

Chapter 6 Mistakes Worksheet

The mistakes on these worksheets were found on real student work. Your task is to identify what mistake was made. The words in *italics* are describing what the student doing the problem was thinking. When you are asked to say why a student would make the mistake describe what they may have been thinking, do not just say they were stupid or they forgot!

1. Solve the following equation: t=(3.4)(2.65)

Incorrect solution:

Set up as a multiplication problem and do the multiplication, ignore the decimals until the end.

$$\begin{array}{r} 2.65 \\ \underline{\times\ 3.4} \\ 1060 \\ \underline{7950} \\ 9010 \end{array}$$

Then move the decimal over. You move the decimal over the number of decimal places in the number with more decimals, in this case we move it over 2 times because 2.65 has 2 numbers behind the decimal!

t=90.10

What mistake was made?

What would the correct answer be?

Why might someone make this mistake?

2. Find the median of the following data set: 3, 9, 6, 10, 3, 8, 4

Incorrect solution:
Just find the number in the middle of the set by counting in from each side.
4 in from the left and then 4 in from the right, the number in the middle is 10!
10

What mistake was made?

What would the correct answer be?

Why might someone make this mistake?

3. Write the following number in scientific notation: .0000034

Incorrect solution:
Move the decimal over until it is behind all of the numbers 0000034

Count how many times you moved the decimal over and since we started with a decimal we will make our final answer a negative exponent.

$$34 \times 10^{-7}$$

What mistake was made?

What would the correct answer be?

Why might someone make this mistake?

4. If the first term in a Geometric sequence is 2 and the common ratio is -3, find the 3rd term in the sequence.

Incorrect solution:

Since it is Geometric, you just add the common ratio to find the next term

2+(-3)= Term 2	Term 2+(-3)=Term 3
2+(-3)=-1	-1+(-3)=-4

So the 3rd term would be -4!

What mistake was made?

What would the correct answer be?

Why might someone make this mistake?

Now that you have experience identifying mistakes, it is your turn to try and create mistakes your classmates may make. Once you have created them you will switch with a partner and see if they can catch yours. AVOID arithmetic mistakes (errors in adding, subtracting, multiplying or dividing) and use conceptual mistakes!

1. Solve for y; $\frac{y}{3.2} = 1.4$

Incorrect solution:

Identify the mistake made?

What should the answer have been?

Why did your partner include this mistake?

2. Write $1\frac{2}{5}$ as a decimal

Incorrect solution:

Identify the mistake made?

What should the answer have been?

Why did your partner include this mistake?

3. Solve for x; $x = (1\frac{2}{3})(-2\frac{3}{4})$

Incorrect solution:

Identify the mistake made?

What should the answer have been?

Why did your partner include this mistake?

4. Solve for z; $z=\frac{3}{5}\div(-\frac{1}{10})$

Incorrect solution:

Identify the mistake made?

What should the answer have been?

Why did your partner include this mistake?

Appendix C

Pre-and Post-Intervention Test with Answers

NAME ______________________ DATE ________

CHAPTER 6 Test, Form 1B

1. Write $\frac{2}{3}$ as a decimal. If necessary, use bar notation to show a repeating decimal.
   A. -0.6 B. 0.6 C. $-0.\overline{6}$ D. $0.\overline{6}$ 1. D

2. Express $\frac{5}{16}$ as a decimal.
   A. 0.3125 B. 0.312 C. 5.16 D. 3 2. A

3. Replace ● with <, >, or = to make $\frac{2}{3}$ ● $\frac{3}{5}$ a true sentence.
   A. = B. < C. > D. none of these 3. C

4. Estimate the product $\frac{1}{5}(16)$.
   A. 2 B. 3 C. 5 D. 80 4. B

5. Estimate the quotient $32 \div 3.1$.
   A. 96 B. 8 C. 12 D. 10 5. D

6. Evaluate ab, if $a = 2\frac{1}{2}$ and $b = \frac{2}{3}$.
   A. $3\frac{3}{4}$ B. $1\frac{2}{3}$ C. 2 D. $1\frac{5}{6}$ 6. B

7. Evaluate $7\frac{x}{y}$, if $x = 7.12$, and $y = 3.2$.
   A. 4.413 B. 3.14 C. 15.575 D. 16.33 7. C

Choose the correct product or quotient.

$\frac{16}{5} \cdot \frac{15}{4} = 12$

8. $3\frac{1}{5} \cdot 3\frac{3}{4} = y$.
   A. 12 B. 10 C. $9\frac{4}{20}$ D. $9\frac{1}{5}$ 8. A

9. $-\frac{2}{3} \div \left(-\frac{3}{2}\right)$
   A. 1 B. -1 C. $-\frac{4}{9}$ D. $\frac{4}{9}$ 9. D

10. (3.6)(-1.1)
    A. -39.6 B. 4.7 C. -3.66 D. -3.96 10. D

11. $36.48 \div 8.0$
    A. 4.26 B. 4.06 C. 5.16 D. 4.56 11. D

12. Solve $2\frac{1}{4} \div 1\frac{1}{4} = y$.
    A. $1\frac{4}{5}$ B. 2 C. $2\frac{13}{16}$ D. $\frac{5}{9}$ 12. A

13. Solve $(9.6)(0.4) = w$.
    A. 10 B. 3.44 C. 3.84 D. 38.4 13. C

NAME ________________ DATE ________

Chapter 6 Test, Form 1B (continued)

14. Choose the quotient for $t = 17.92 \div 5.6$ in which the decimal is correctly placed.
A. 0.032 B. 0.32 C. 3.2 D. 32.0
14. C

15. Name the property of multiplication shown by $(3.6)(4.8) = (4.8)(3.6)$.
A. Identity B. Associative
C. Commutative D. Inverse
15. C

16. Name the property of multiplication shown by $(3.6)\frac{1}{3.6} = 1$.
A. Identity B. Associative
C. Commutative D. Inverse
16. D

17. Find the median of 62, 63, 63, 64, 65, 70, and 71.
A. 62 B. 63 C. 64 D. 65
17. C

18. Find the mean of 18, 20, 22, 25, and 30.
A. 12 B. 22 C. 23 D. 115
18. C

19. Find the mode of 18, 20, 19, 18, 20, 18, 19, 19, 19, 20, and 21.
A. 18 B. 19 C. 20 D. 21
19. B

20. Solve $\frac{2}{3}y = \frac{3}{7}$.
A. $\frac{9}{14}$ B. $\frac{2}{7}$ C. $\frac{6}{7}$ D. $\frac{6}{14}$
20. A

21. Solve $\frac{x}{-3.2} < 12.8$.
A. $x < -40.96$ B. $x > -40.96$
C. $x < -4$ D. $x > -4$
21. B

22. Find the next term of the geometric sequence 24, 12, 6, 3, · · ·.
A. 1.5 B. 1 C. 0.5 D. 0.25
22. A

23. Use the expression $a \cdot r^{n-1}$ to find the sixth term in the sequence 3, 6, 12, 24, · · ·.
A. 32 B. 48 C. 96 D. 192
23. C

24. Mary read that a red blood cell is about 7.5×10^{-4} centimeter long. Write this number in standard form.
A. 0.0075 B. 0.07500 C. 7,500 D. 0.00075
24. D

25. The speed of sound waves in the tissues of the human body averages about 154,000 centimeters per second. Write this number in scientific notation.
A. 15.4×10^5 B. 1.54×10^5
C. 0.154×10^5 D. 154×10^5
25. B