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CCSS Mathematics: Now What?

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California Mathematics Project, Co-Director

Plan

- Quick background
- Viable arguments
- Coherent Units
- Scope and Sequence
- IM

How are the CCSS different?

The CCSS are reverse engineered from an analysis of what students need to be **college and career ready**.

The design principals were **focus** and **coherence**. (No more mile-wide inch deep laundry lists of standards)

The CCSS in Mathematics have two sections:

CONTENT and PRACTICES

The Mathematical Content is what students should **know**.
The Mathematical Practices are what students should **do**.

Real life applications and **mathematical modeling** are essential.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CCSS Mathematical Practices

OVERARCHING HABITS OF MIND

1. Make sense of problems and persevere in solving them

6. Attend to precision

REASONING AND EXPLAINING

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

4. Model with mathematics

5. Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING

7. Look for and make use of structure

8. Look for and express regularity in repeated reasoning

Constructing viable arguments

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Constructing viable arguments

- *use* stated assumptions, definitions, and previously established results in constructing arguments.

- *make* conjectures

- *build* a logical progression of statements

- *analyze* situations by breaking them into cases

- *recognize* and use counterexamples

- *justify* their conclusions, *communicate* them to others, and *respond* to the arguments of others

- *distinguish* correct logic or reasoning from that which is flawed

Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Viable arguments are important beyond mathematics

21st Century Skills

Common Core Standards for English Language Arts

Career and College Readiness Anchor Standards for Writing

Text types and Purposes*
1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and distribution of Writing
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge
7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

Arguments to support claims

1. Write *arguments to support claims* in an analysis of substantive topics or texts, using *valid reasoning* and *relevant and sufficient evidence*.

2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

Practices for Next Generation Science Standards

1. Asking questions (for science) and defining problems (for engineering)
2. Developing and using models
3. Planning and carrying our investigations
4. Analyzing and interpreting data
5. Using mathematics and computational thinking
6. Constructing explanations (for science) and designing solutions (for engineering)
7. Engaging in argument from evidence
8. Obtaining, evaluating and communicating information

A note on definitions

argument (noun)

1.

a **:** a reason given in proof or rebuttal

b **:** discourse intended to persuade

2.

a **:** the act or process of arguing : argumentation

b **:** a coherent series of statements leading from a premise to a conclusion

c **:** quarrel, disagreement

viable (adjective)

a **:** capable of working, functioning <*viable* alternatives>

b **:** having a reasonable chance of succeeding <*viable* candidate>

The distinction between "evidence of understanding" and "viable argument"

Getting the correct answer or being right is NOT the same thing as explaining the reasoning, communicating, or convincing others!

Getting the correct answer or being right is not a bad thing. The important point is *to be career and college ready you need BOTH*, and mathematics classrooms have not always provided much opportunity or support of the latter.

Expectations and goals

In a mathematics class we sometimes have the expectation (or hope) that when asked a question most of the students should give the correct answer.

In a writing class the typical expectation when given a writing task is that most of the students will need to revise.

“Evidence of viable argument”

It may be helpful to interpret this as

“evidence of a student’s mathematical argument”

AND

specific places where this argument could be made more viable.

The expectation is that most arguments can be improved by revision.

Our goal is to look for evidence of the parts that are viable and identify those places where support could be given as feedback to the students.

Here are some categories we included:

Explains solution:

Logical sequence of steps:

Communicates precisely:

Responds to reasoning of others:

Makes mathematical sense:

Let’s try!

Take a few minutes solo to work on this task:

It is important in mathematics to give clear and logical explanations.

Please take a few minutes to write down your ideas:

1) What is the definition of an even number?

2) Explain why the sum of two even numbers is always even.

Key Feedback for Revision

- Terms are precise (look for vague words, pronouns)
- Statements are connected logically
- Assumptions and conclusions are clearly stated
- Diagrams are labeled.

2) Explain why the sum of two even numbers is always even.

because it hasent been proventhat

it's not

1) What is the definition of an even number?

An even number is a number that you usually divide with, and you can multiply 2 by something.

2) Explain why the sum of two even numbers is always even.

Two even numbers added are always even because when you add them, you're going to get a bigger even number.

2) Explain why the sum of two even numbers is always even.

Because an even number plus an even number makes the sum also divisible bu two therefore, its even.

2) Explain why the sum of two even numbers is always even.

Because when you go on a Date you go with another perso and theres two of You and if you go on a double date theres four of You but if there is a third wheel cause there date didn't show up that's just awkward and Odd... No pun intended

2) Explain why the sum of two even numbers is always even.

It is always even because thaire both even.

$$\begin{array}{r} 2 \\ +\,4 \\ \hline 6 \end{array}$$

2) Explain why the sum of two even numbers is always even.

The sum of two even numbers are always even because there are no odds.

Example $4 \cdot 4 = 16$

1) What is the definition of an even number?

I think the definition of an even number Is a number two time's like 1+1=2, two is an even number. If you put numbers on a paper and start at two and skip count you will find most of the even numbers.

2) Explain why the sum of two even numbers is always even.

Becaus an even plus an even equals even like 2+4=6, six is even and 12+14=26 twenty six is also even. Two even numbers ALWASe equal an even.

Arguments across the grades

Because if the numbers are both even, then they can't be odd if there are 2 numbers.

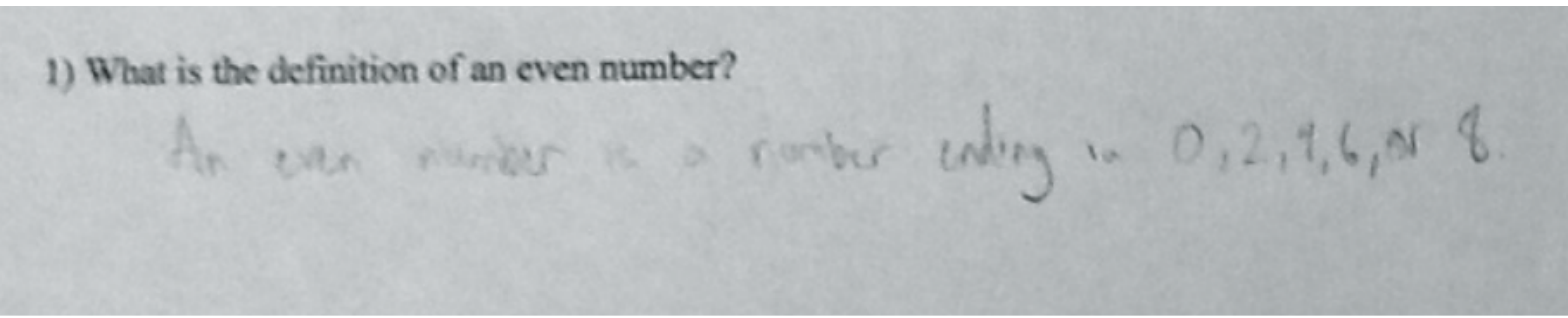
2+2=4 because it is always made up of 2's

If you add an even number plus an even number it always comes out even

When you add two even numbers the answer is always even because it equals an even number. For example, 2+2=4 and 4+4=8. It is always an even number.

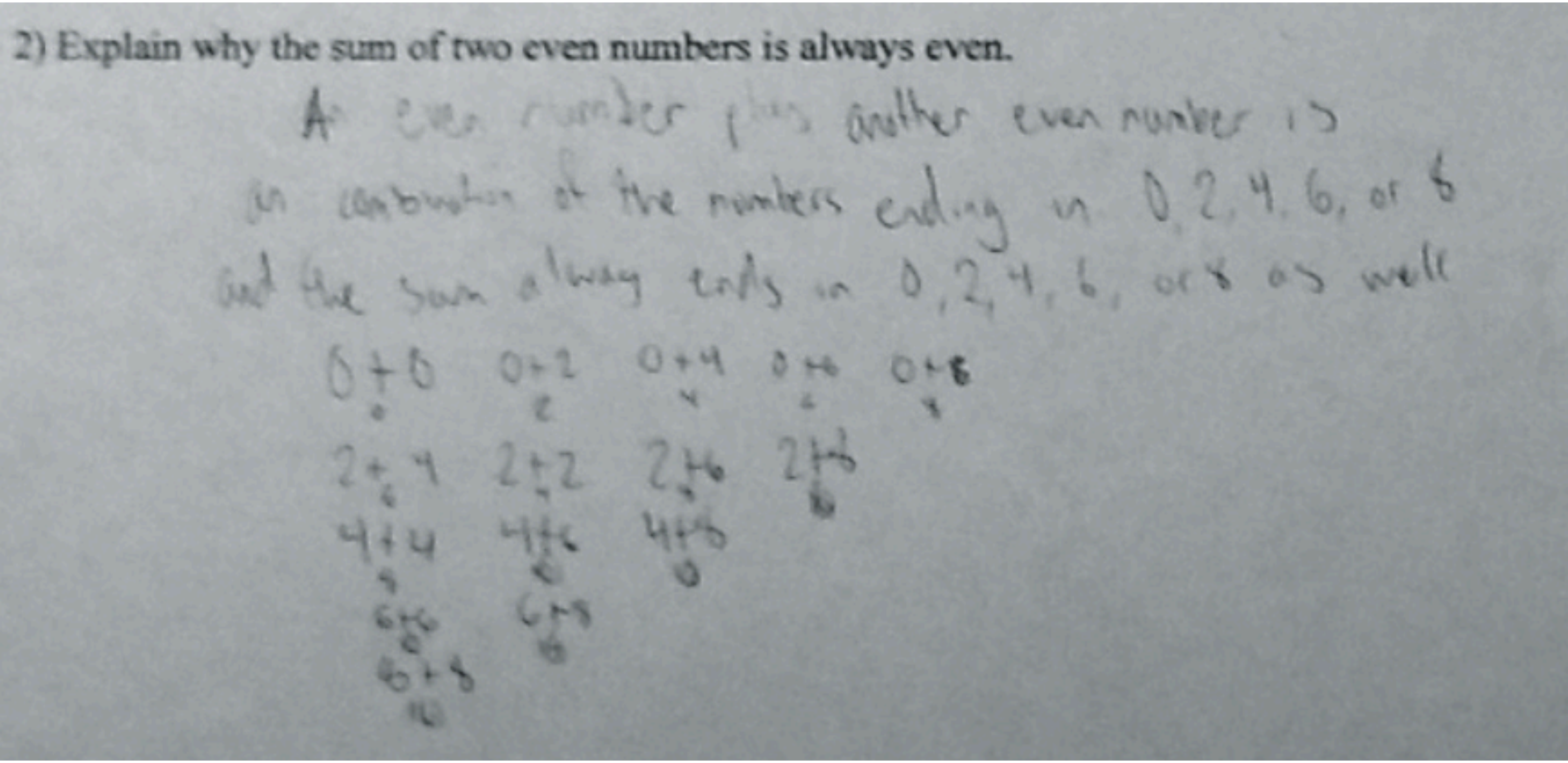
Because two even numbers cannot make an odd.

What *do* viable arguments look like?


1) What is the definition of an even number?

An even number is a number ending in 0, 2, 4, 6, or 8.

This is a viable argument based on the student's definition



2) Explain why the sum of two even numbers is always even.

An even number plus another even number is
an combination of the numbers ending in 0, 2, 4, 6, or 8
and the sum always ends in 0, 2, 4, 6, or 8 as well

0+0 0+2 0+4 0+6 0+8
0 2 4 6 8

2+4 2+2 2+6 2+8
6 4 8 10

4+4 4+6 4+8
8 10 12

6+6 6+8
12 14

8+8
16

A viable argument

Definition: An even number is a multiple of 2 which means it can be represented by $2k$ for some integer k.

Let **A** and **B** be any even numbers.
By the definition this means that $\mathbf{A=2}\boldsymbol{m}$ and $\mathbf{B=2}\boldsymbol{n}$ for some integers m and n.

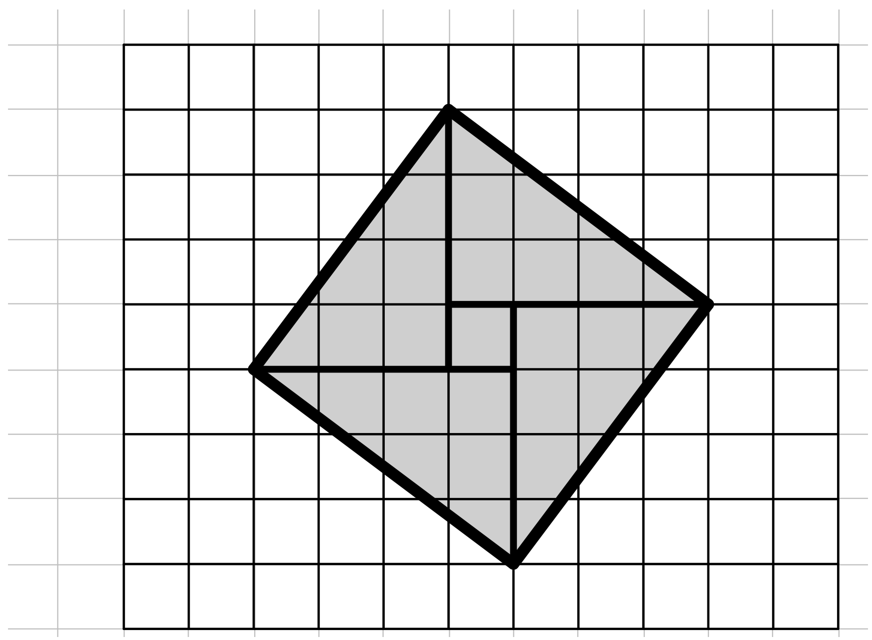
So, $\mathbf{A + B = 2}\boldsymbol{m} + \mathbf{2}\boldsymbol{n} = \mathbf{2(}\boldsymbol{m} + \boldsymbol{n}\mathbf{)}$ by the distributive property.

Therefore A + B is a multiple of two, hence an even number.

Thus the sum of any two even numbers is an even number.

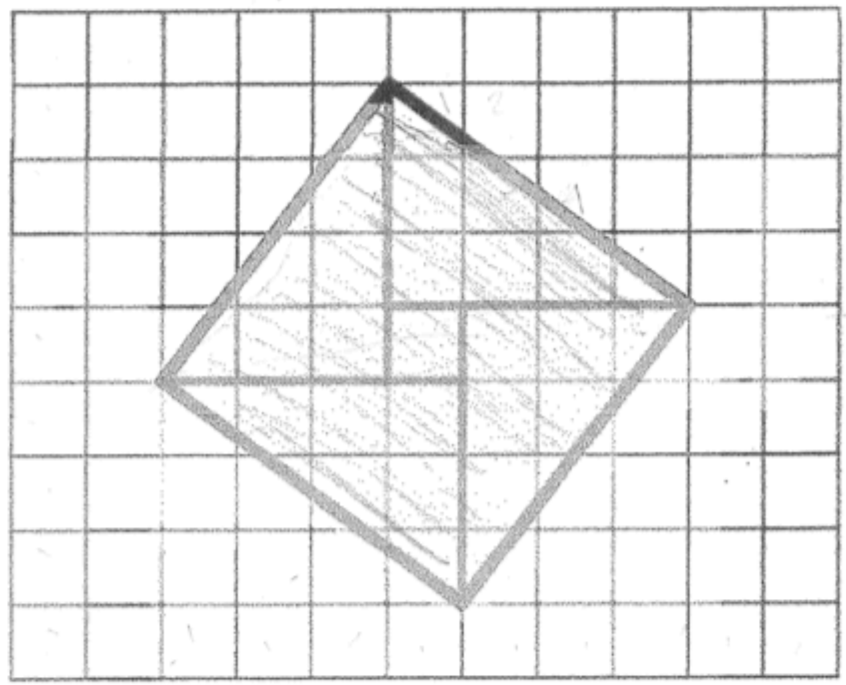
Area Prompt in Geometry

Explain why the shaded figure has an area of 25 square units:



$2\frac{3}{4}$

Explain why the shaded figure has an area of 25 square units:

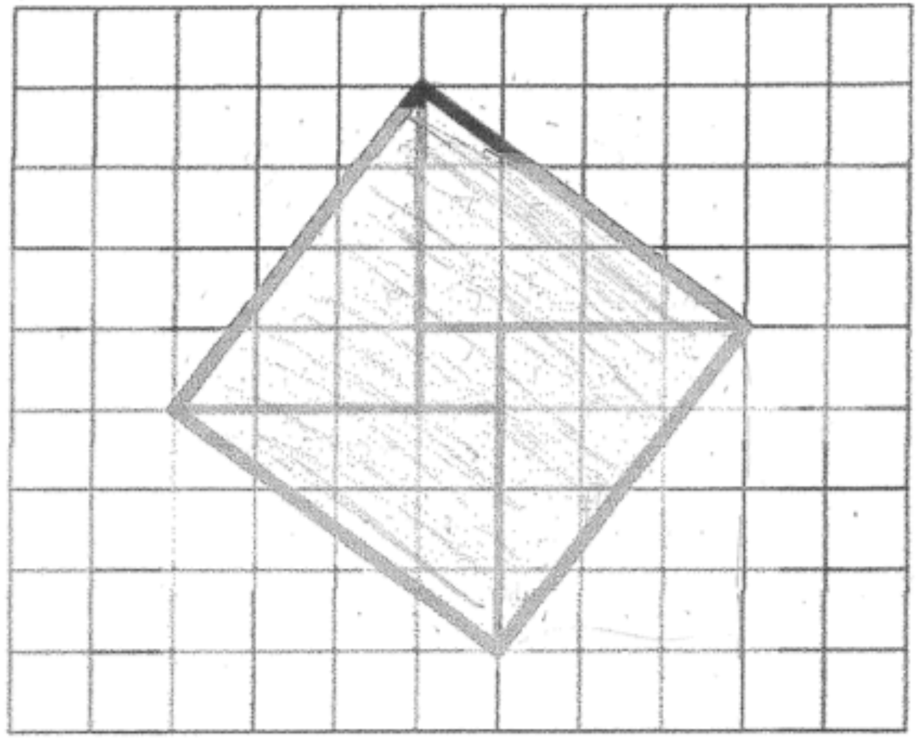


My explanation:

~~The total~~ number of squares is 49

One square is one centimetere.

Explain why the shaded figure has an area of 25 square units:

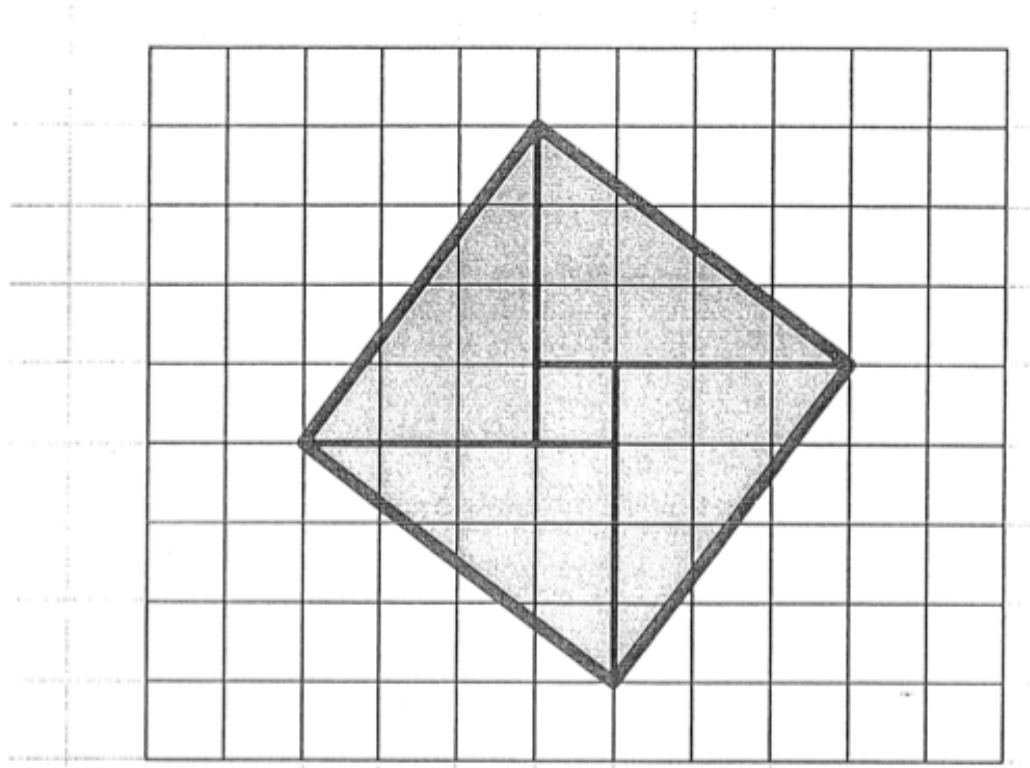


My explanation:

Area: L•W

5 . 5 = 25

Explain how you can find the area of the shaded figure below:



My explanation:

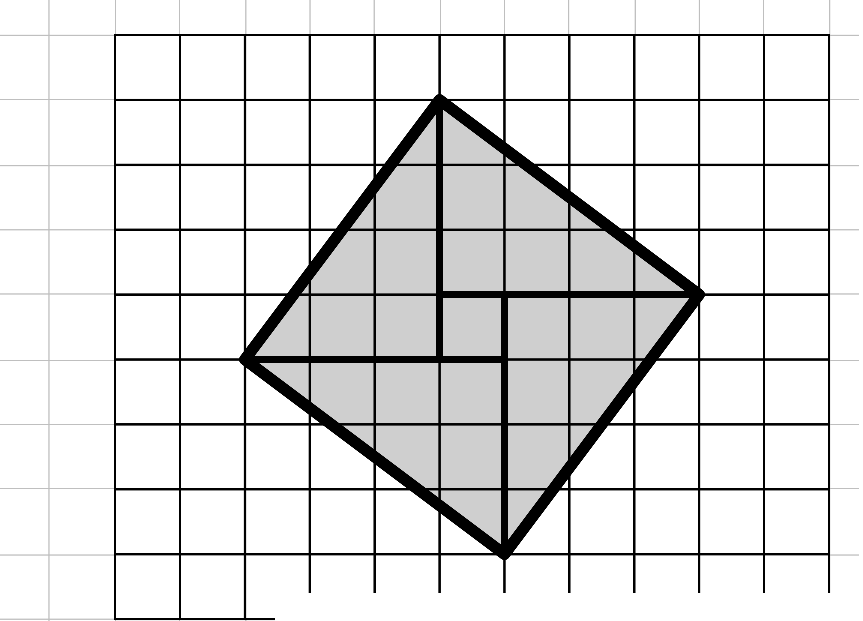
$SA = 6S^2$

$SA = 6 \cdot S^2$

$SA = 6 \cdot 8$

$SA = X$

you would do this problem



Describe a different way of showing why the figure has an area of 25 square units.

My second explanation:

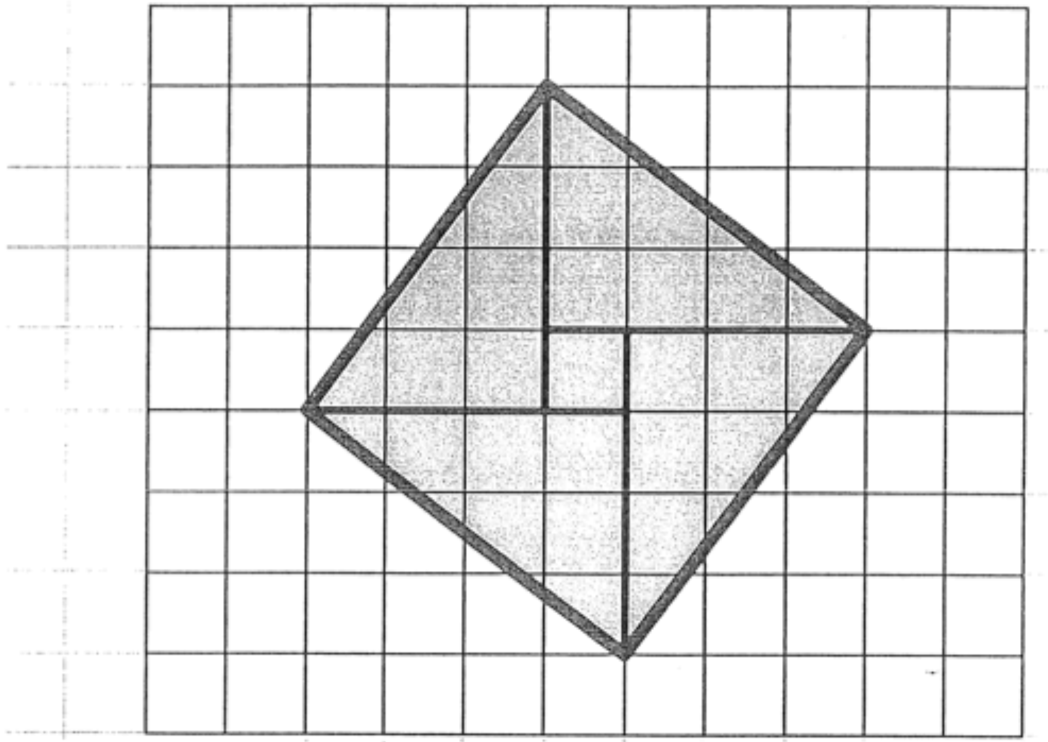
By counting all squares, there is a total of 14 complete squares. There are 22 half squares, & if divided by 2 & added to it to 14, it equals 25.

$14 + \frac{1}{2}x = 25$

$\frac{1}{2}x = 11$

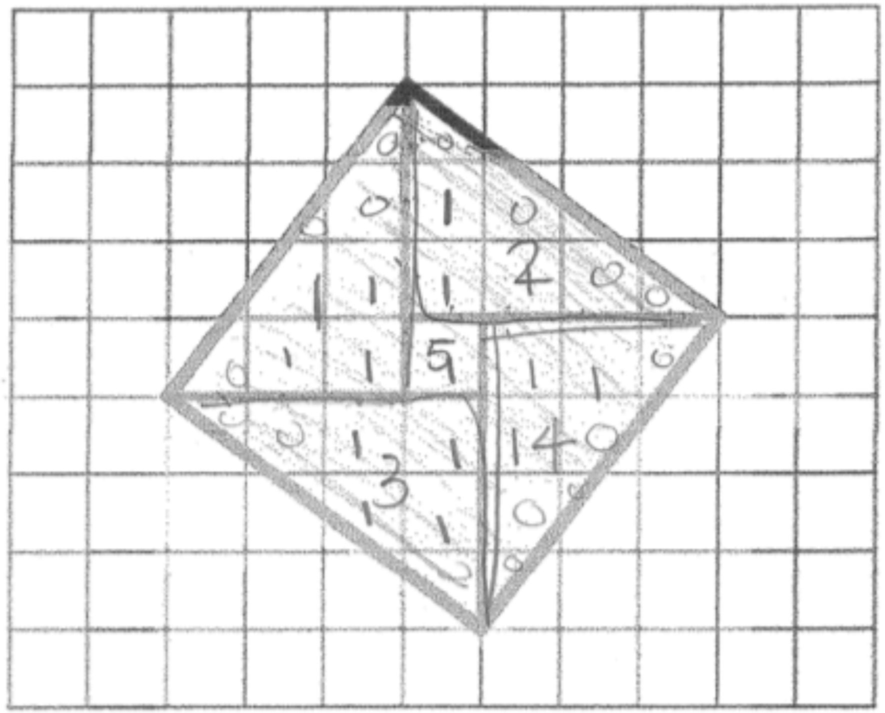
$x = 22$

Explain how you can find the area of the shaded figure below:



My explanation: count all the squares inside and divide by the out side

Explain why the shaded figure has an area of 25 square units:



My explanation:

Area of a triangle = $\frac{1}{2} lw$

$\Delta 1 = (3)(4)(\frac{1}{2}) = 6$

$\Delta 2 = (3)(4)(\frac{1}{2}) = 6$

$\Delta 3 = (3)(4)(\frac{1}{2}) = 6$

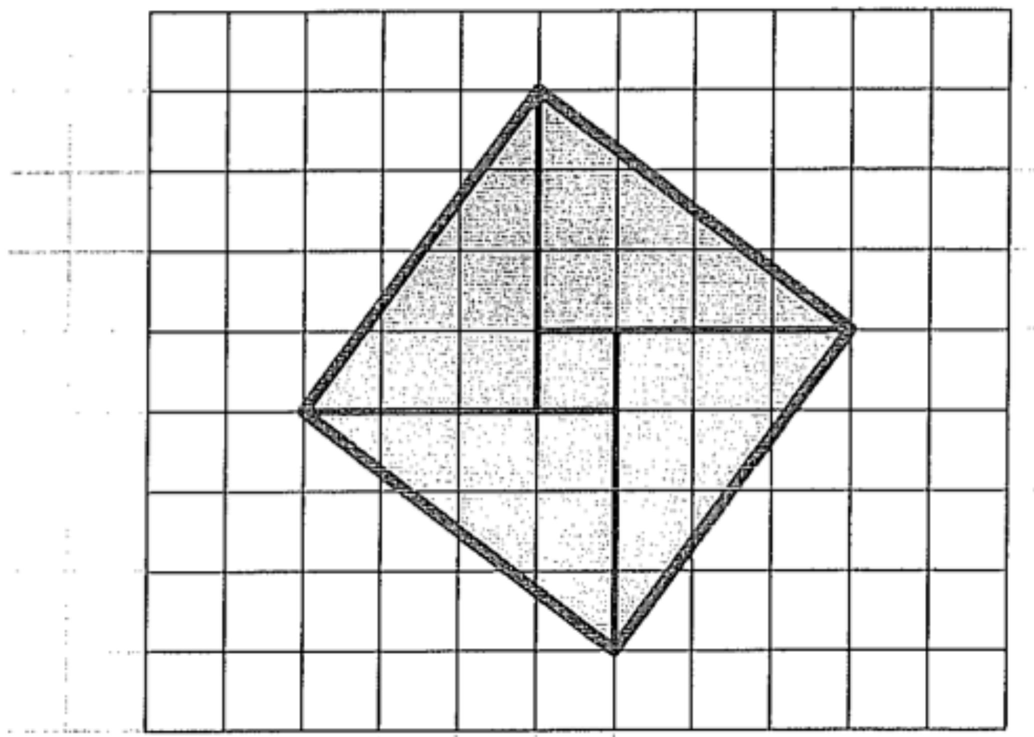
$\Delta 4 = (3)(4)(\frac{1}{2}) = 6$

$\square 5 = 1 \quad = 1$

$\boxed{25}$

If added, it is equal to 25 sq. units

Explain how you can find the area of the shaded figure below:



My explanation: Well first you would look for an easy object to find the area of. There are 4 triangle and an extra unit. the triangles sides are 3 + 4 so the other side must be 5. You do base times height divided by two. Then times 4 for all of the triangle then add 1 for the other unit.

Polygon Prompt

1. Write a precise mathematical definition for a **polygon**:

Breaking Polygon Definitions

Exchange papers with someone.

Draw an object that you do NOT think is a polygon, that nonetheless, satisfies their definition.

Breaking Polygon Definitions

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Draw an object that you do NOT think is a polygon, that nonetheless, satisfies their definition.

Example:

Polygon: A closed figure.

Breaking Polygon Definitions

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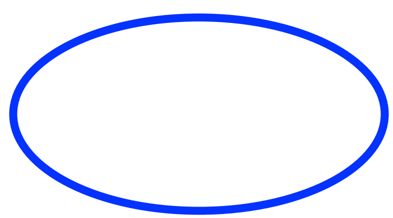


I don't this is should be a polygon, but it satisfies the definition

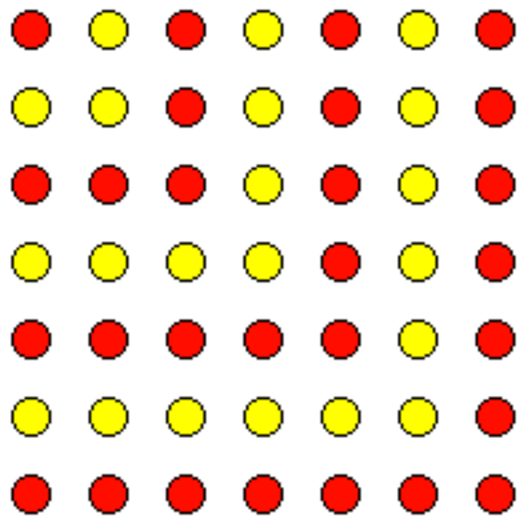
A viable argument without words?

Recall that many viable arguments can be made using diagrams or other representations.

Elementary students can construct arguments using **concrete referents such as objects, drawings, diagrams, and actions**. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

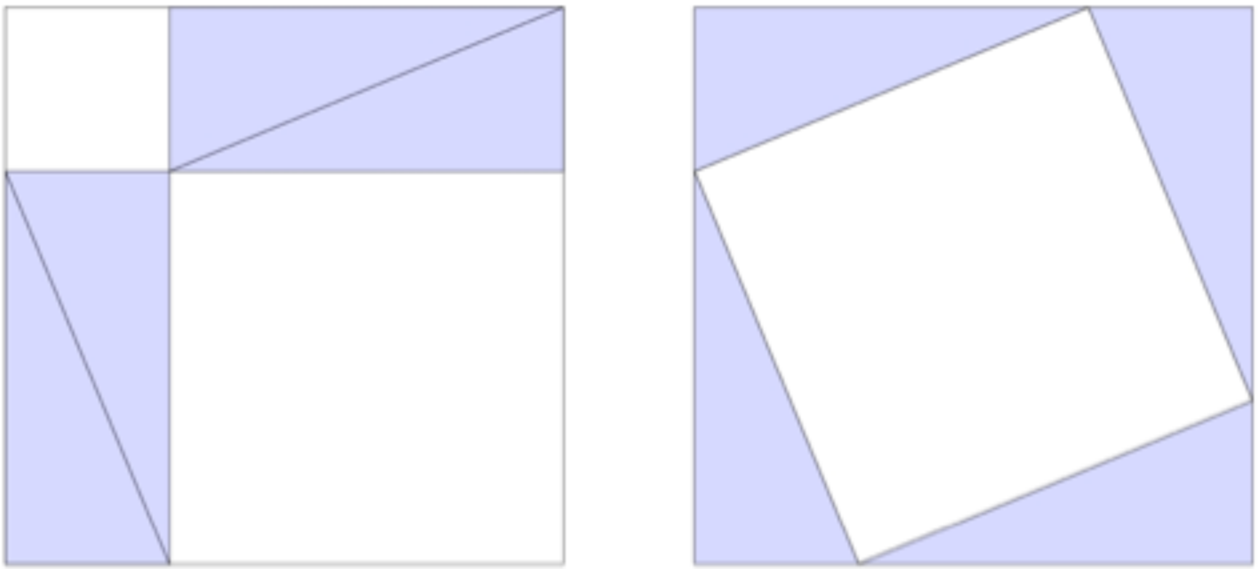
What is the sum of the first *n* odd numbers?

1+3+5+7+9…=?



$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

My favorite proof of the Pythagorean Theorem



Part 2

A long time ago…

(August 11, 2010) The California State Board of Education adopted new standards:

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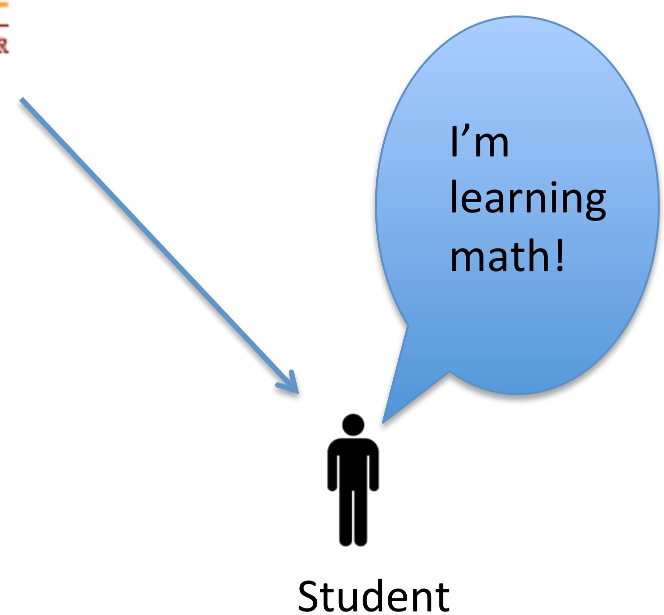
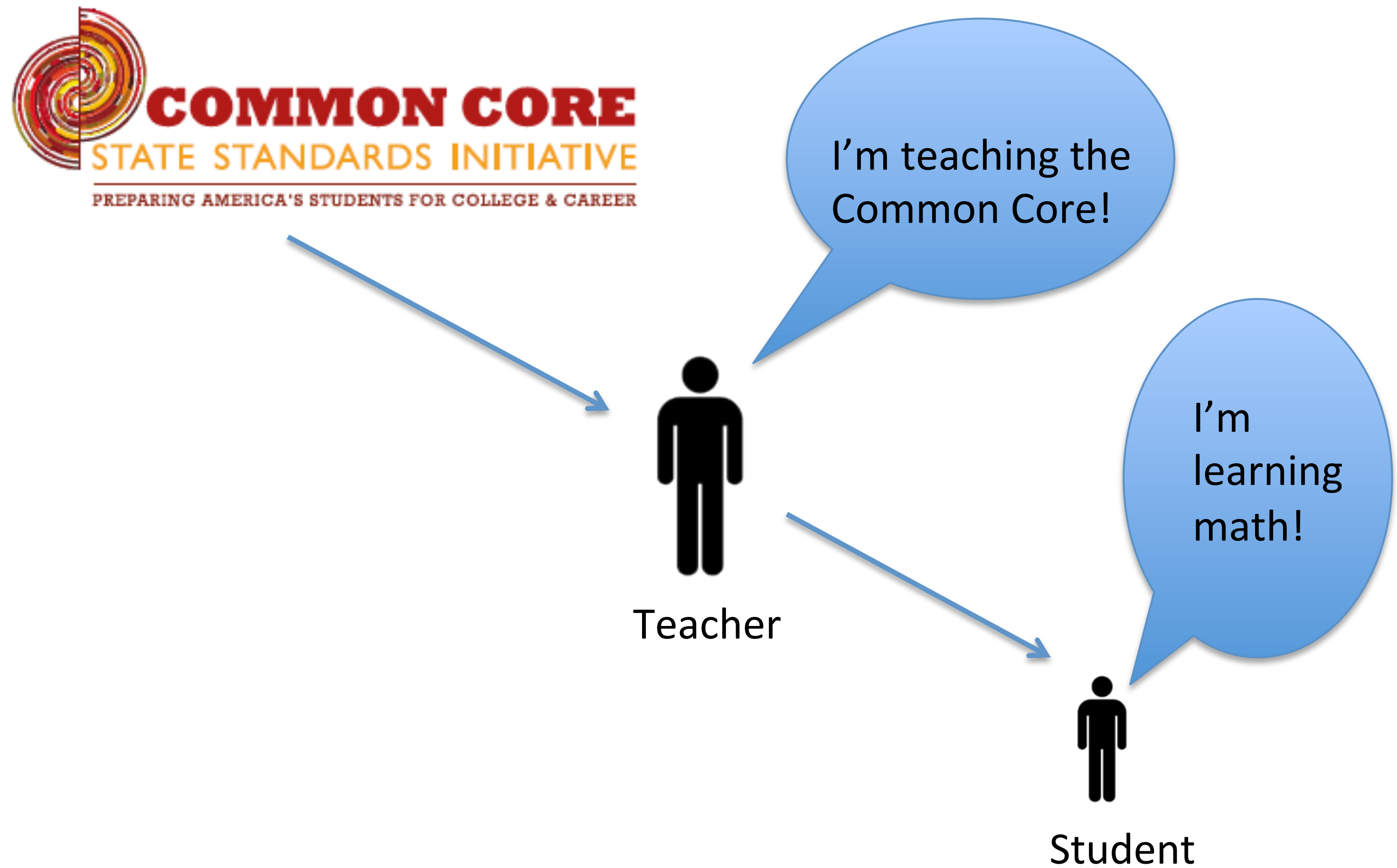
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

And everything was good.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER



Let’s try to map out a little more detail…



Meanwhile…

Let's take a look at the CCSS and see what I am supposed to teach

HS Teacher
(9^{th} grade)

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

OK, what am I supposed to teach?

HS Teacher
(9^{th} grade)

COMMON CORE STATE STANDARDS for MATHEMATICS

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The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus,

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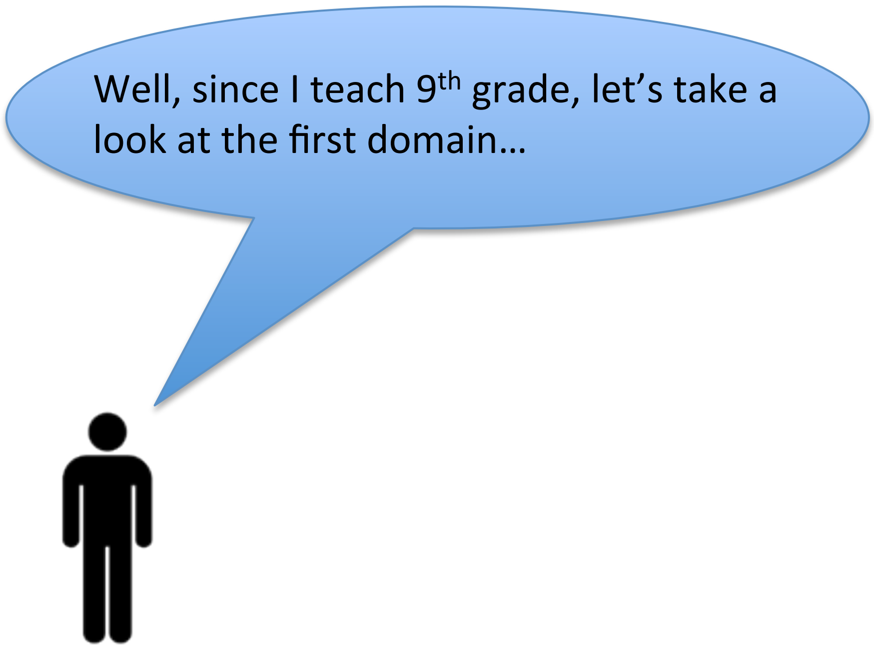


HS Teacher
(9th grade)

Number and Quantity Overvie

The Real Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

Quantities

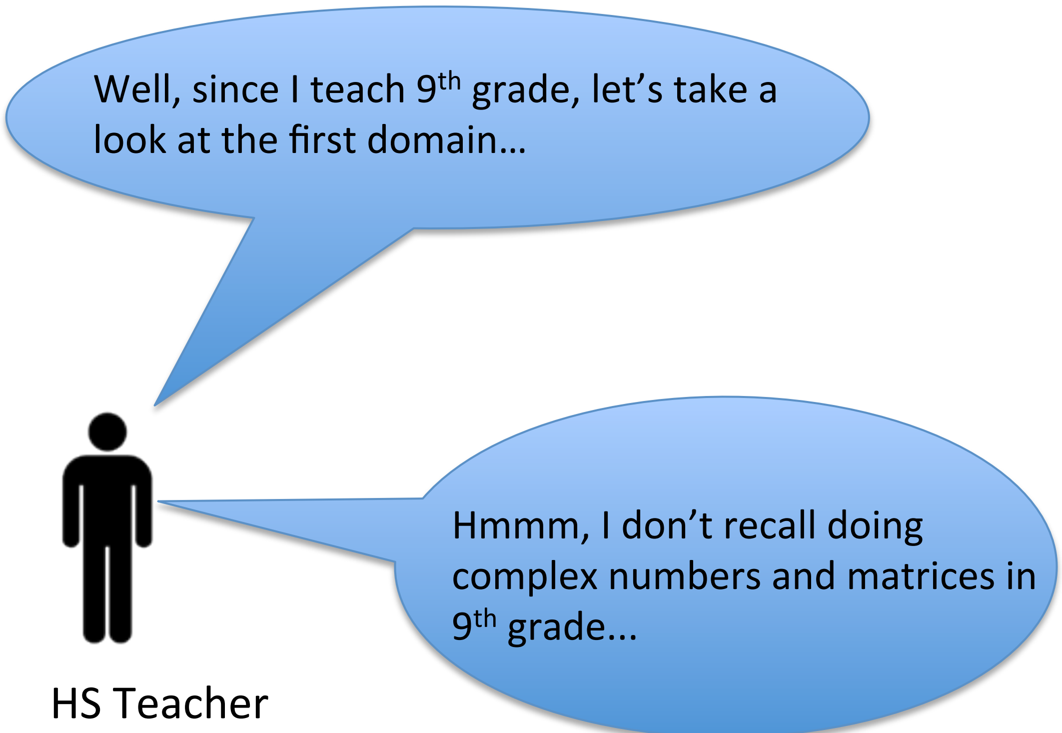
- Reason quantitatively and use units to solve problems

The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.



HS Teacher
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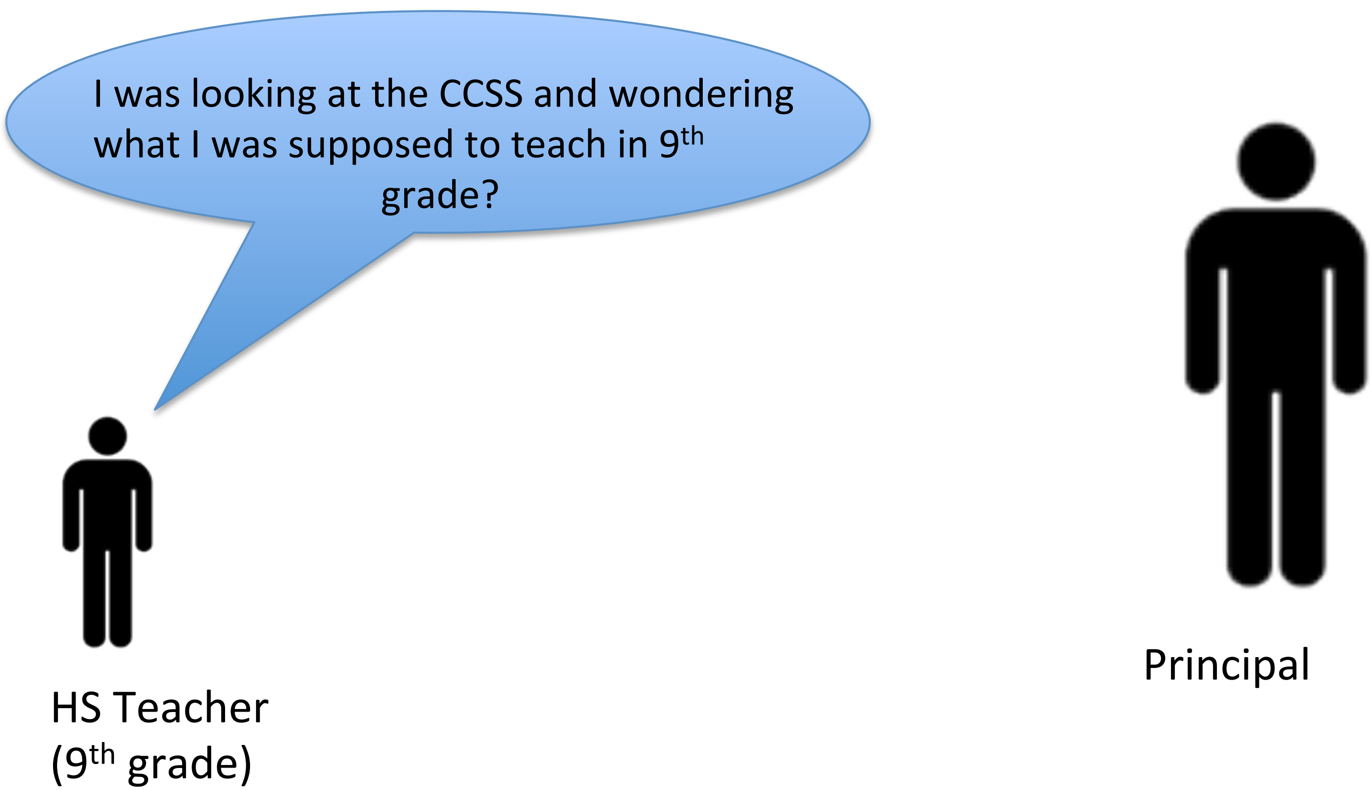
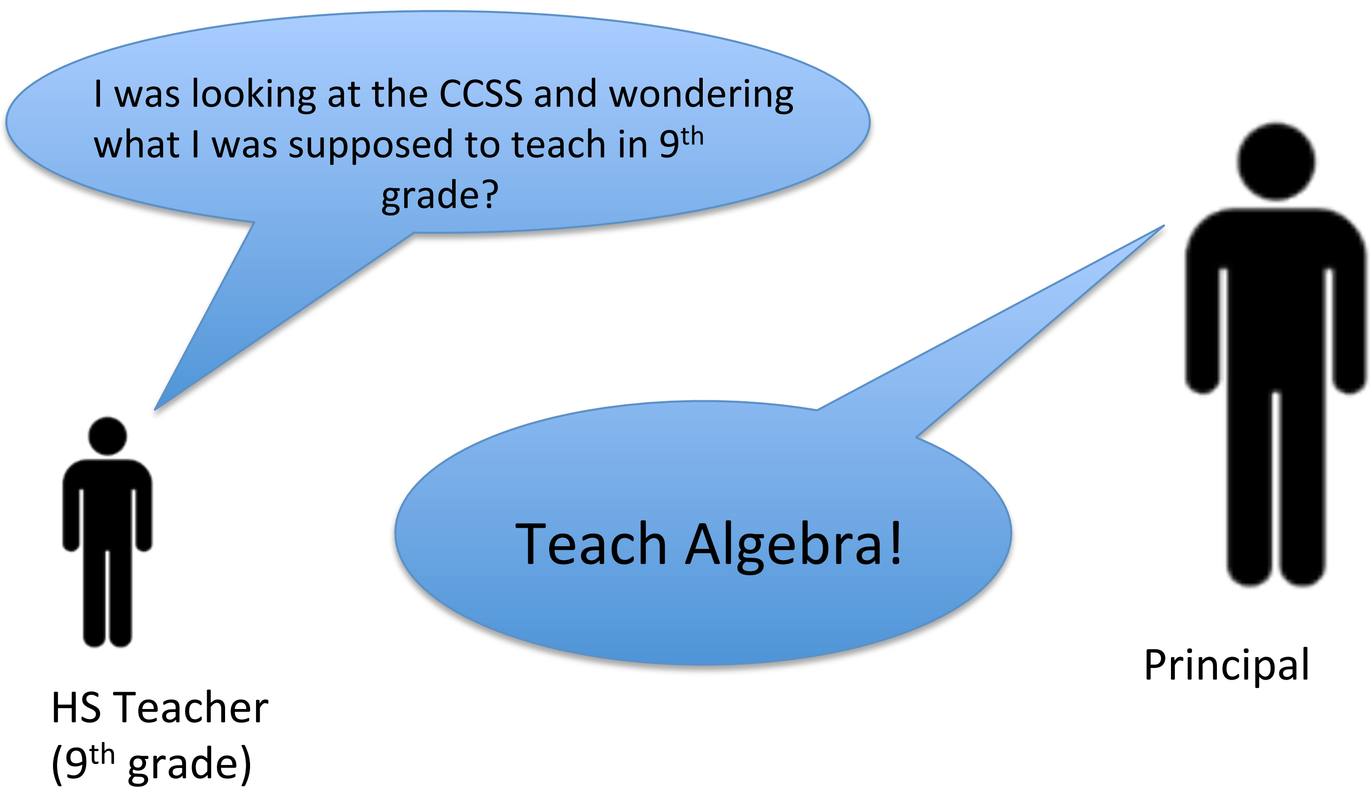
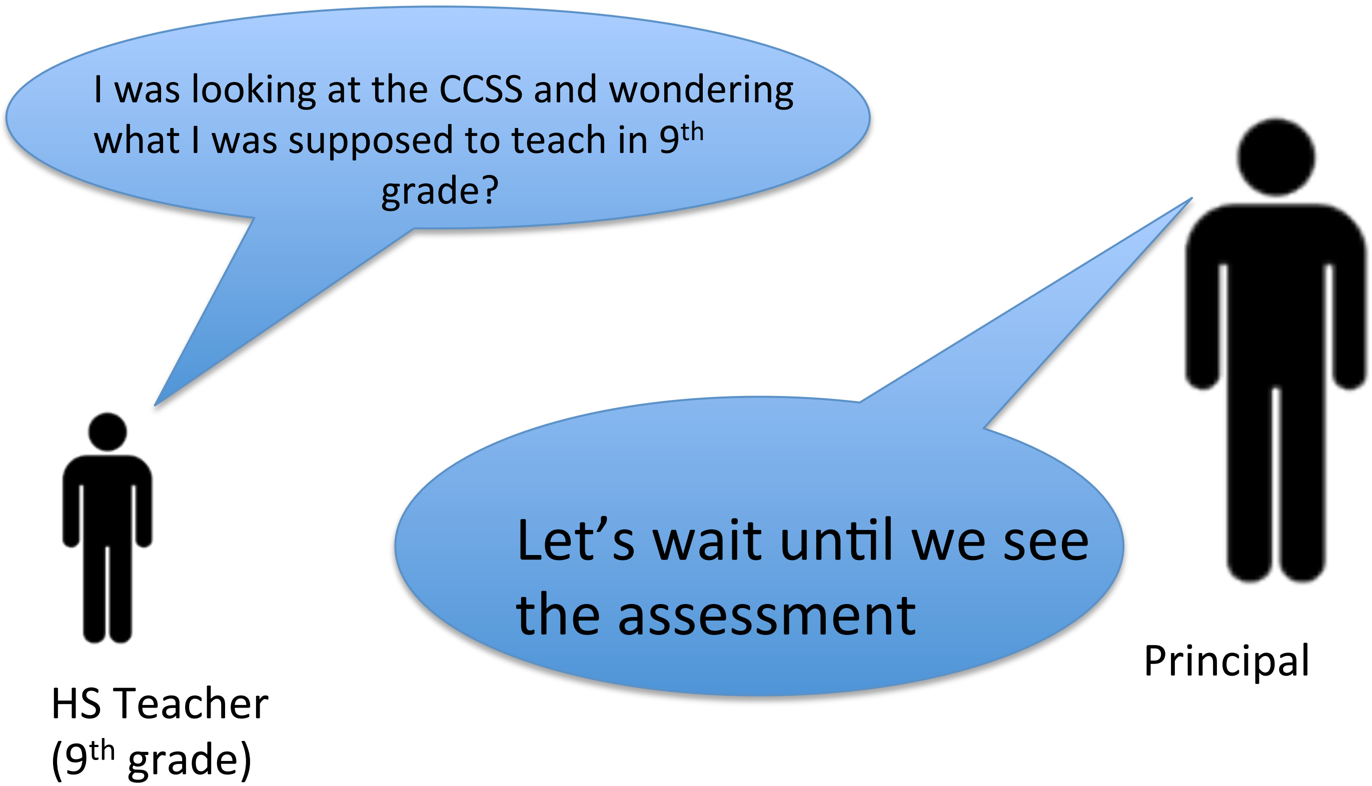
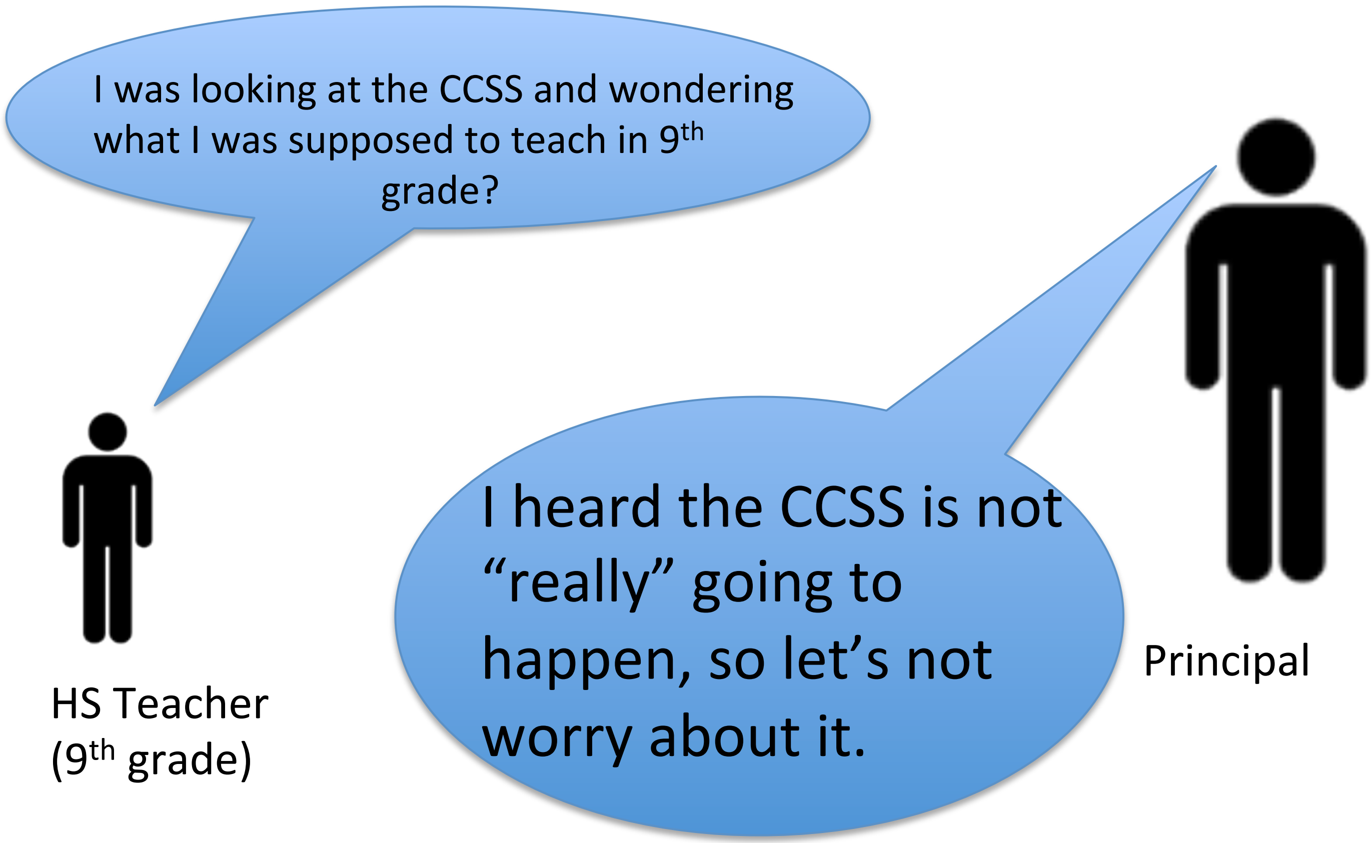
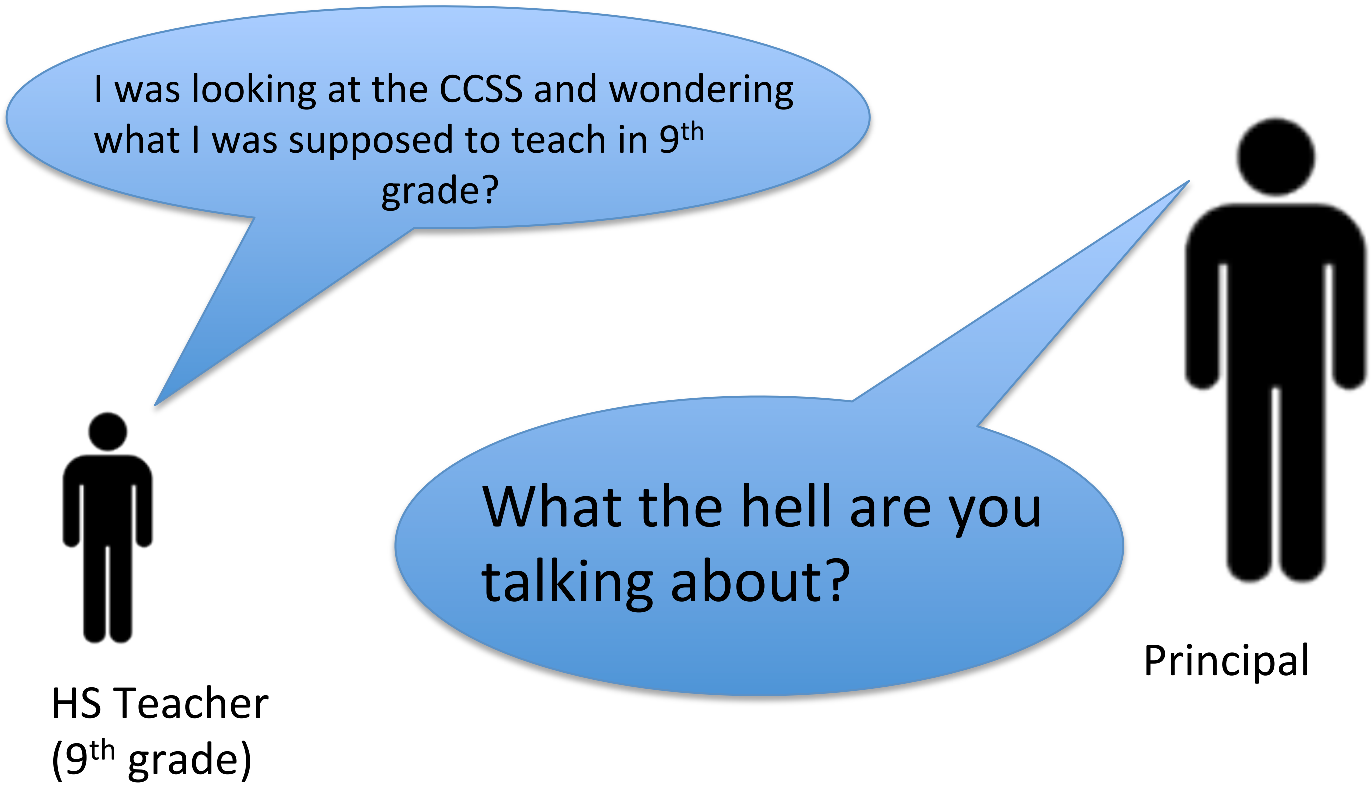

I was looking at the CCSS and wondering what I was supposed to teach in 9th grade?
HS Teacher (9th grade)
Principal

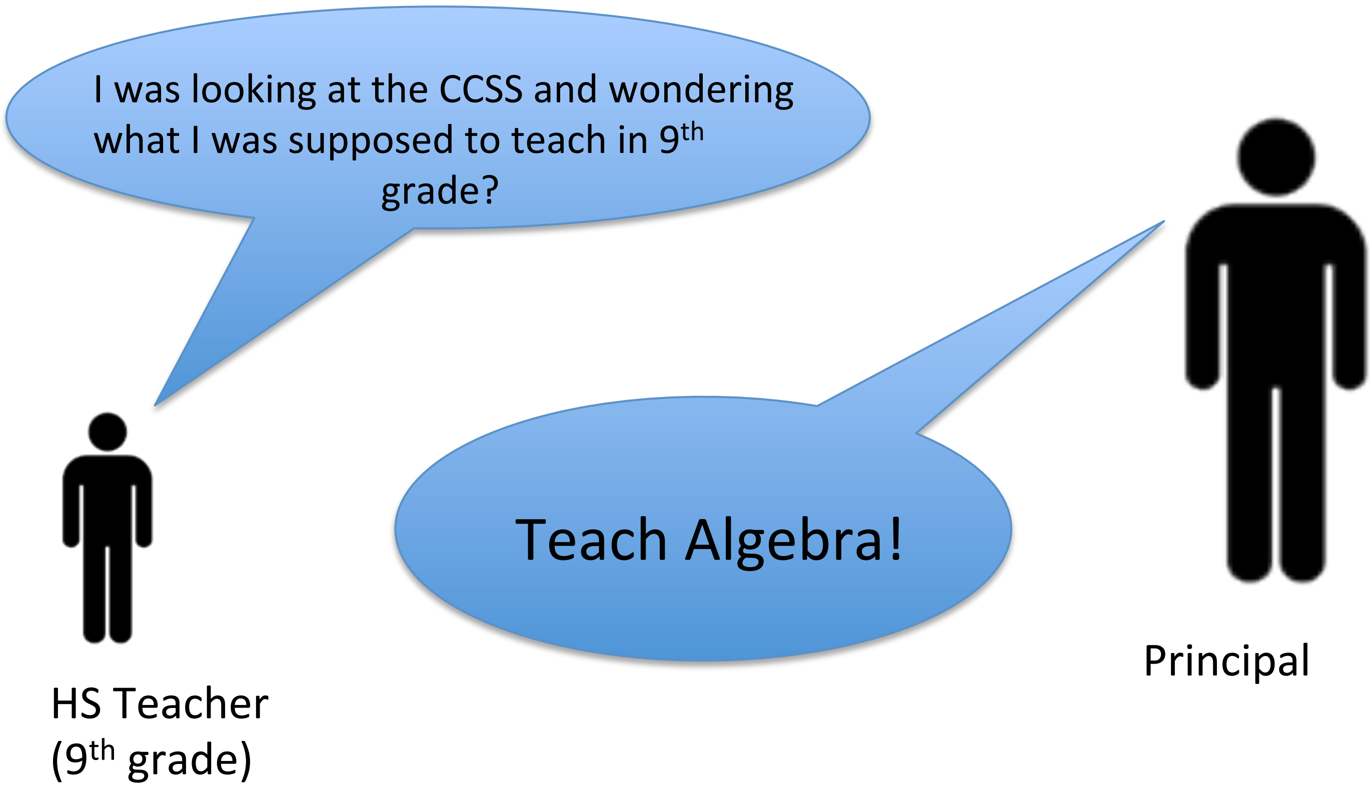


I was looking at the CCSS and wondering what I was supposed to teach in 9th grade?
Teach Algebra!
HS Teacher (9th grade)
Principal


I was looking at the CCSS and wondering what I was supposed to teach in 9th grade?
Let's wait until we see the assessment
HS Teacher (9th grade)
Principal


I was looking at the CCSS and wondering what I was supposed to teach in 9th grade?
I heard the CCSS is not "really" going to happen, so let's not worry about it.
HS Teacher (9th grade)
Principal


I was looking at the CCSS and wondering what I was supposed to teach in 9th grade?
What the hell are you talking about?
HS Teacher (9th grade)
Principal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Teach Algebra!
HS Teacher
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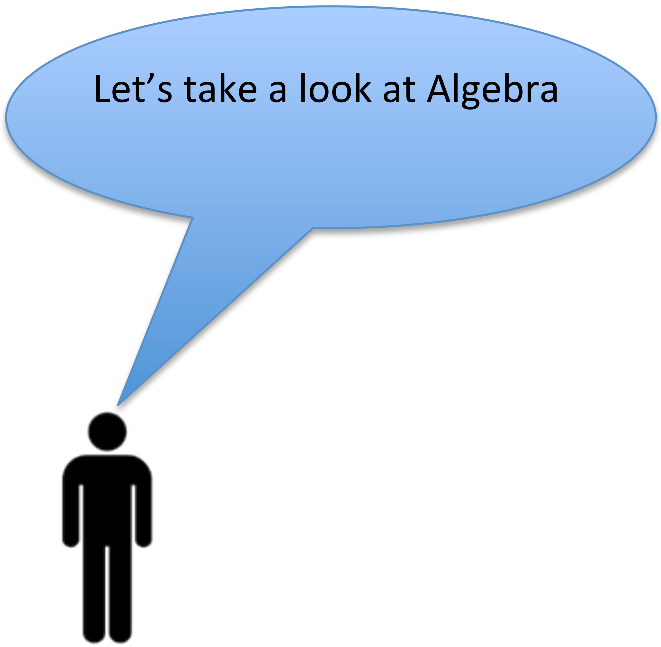

HS Teacher
(9th grade)

Algebra Overview

Seeing Structure in Expressions

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Algebra Overview

Seeing Structure in Expressions

- **Interpret the structure of expressions**

...olve

- **Perform arithmetic operations on polynomials**
- **Understand the relationship between zeros and**

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...reasoning

- **Solve equations and inequalities in one variable**
- **Solve systems of equations**
- **Represent and solve equations and inequalities graphically**

Arithmetic with Polynomials and Rational Expressions	A-APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

Seeing Structure in Expressions	A-SSE

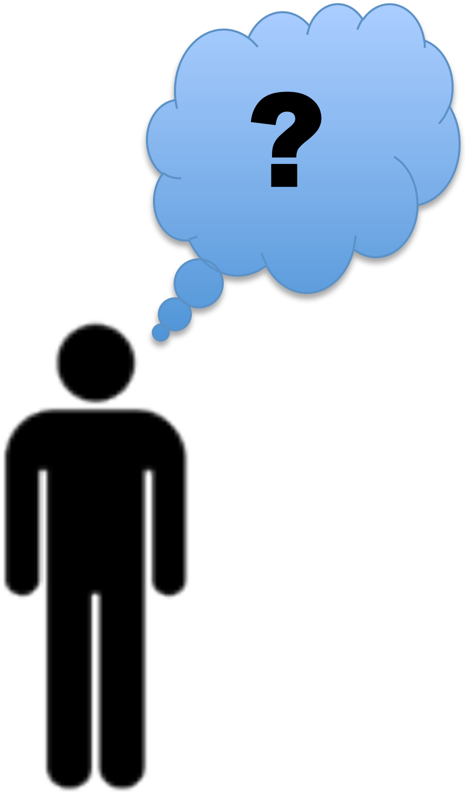
Interpret the structure of expressions

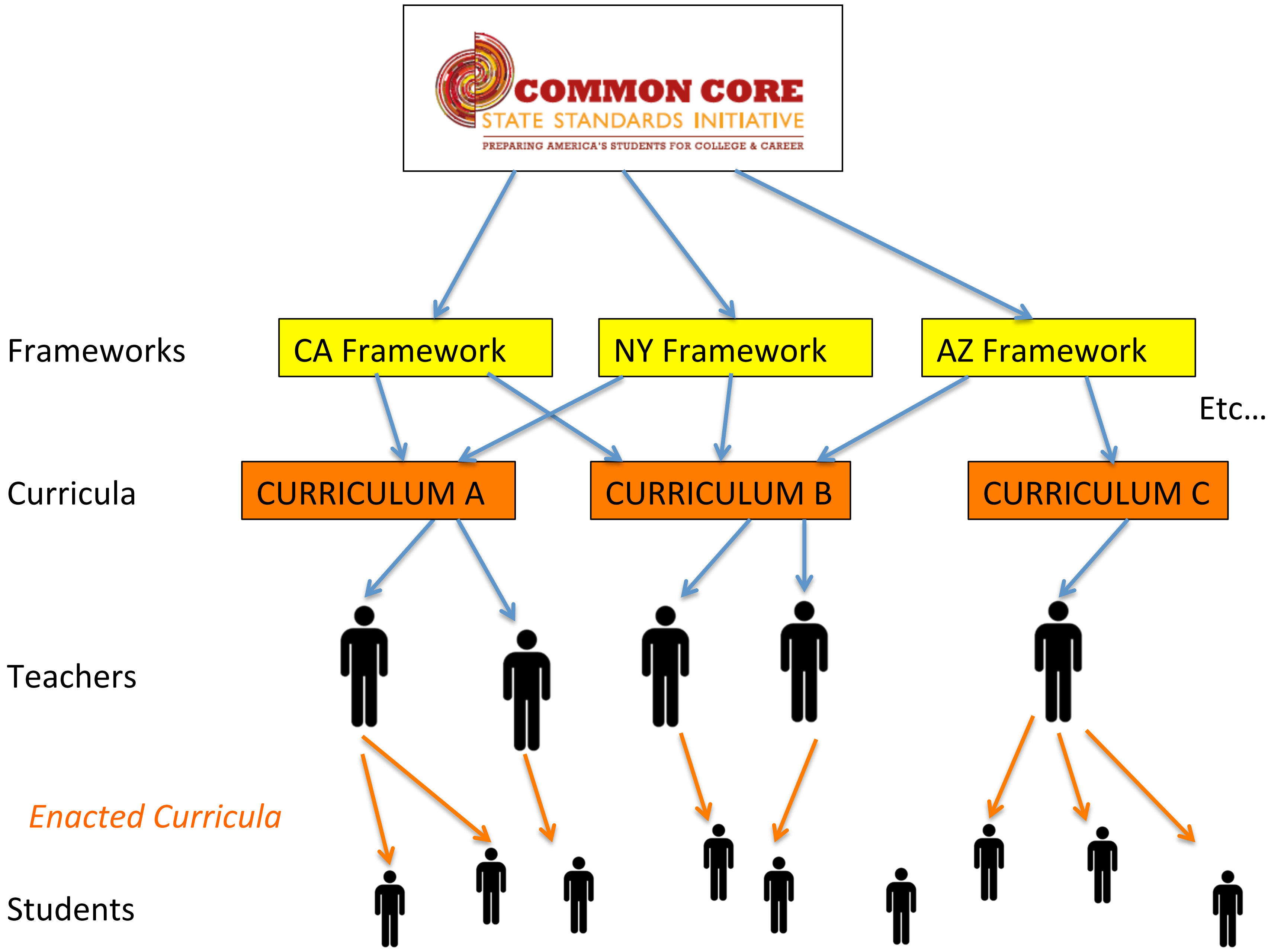
1. Interpret expressions that represent a quantity in terms of its context.*
   - **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
   - **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

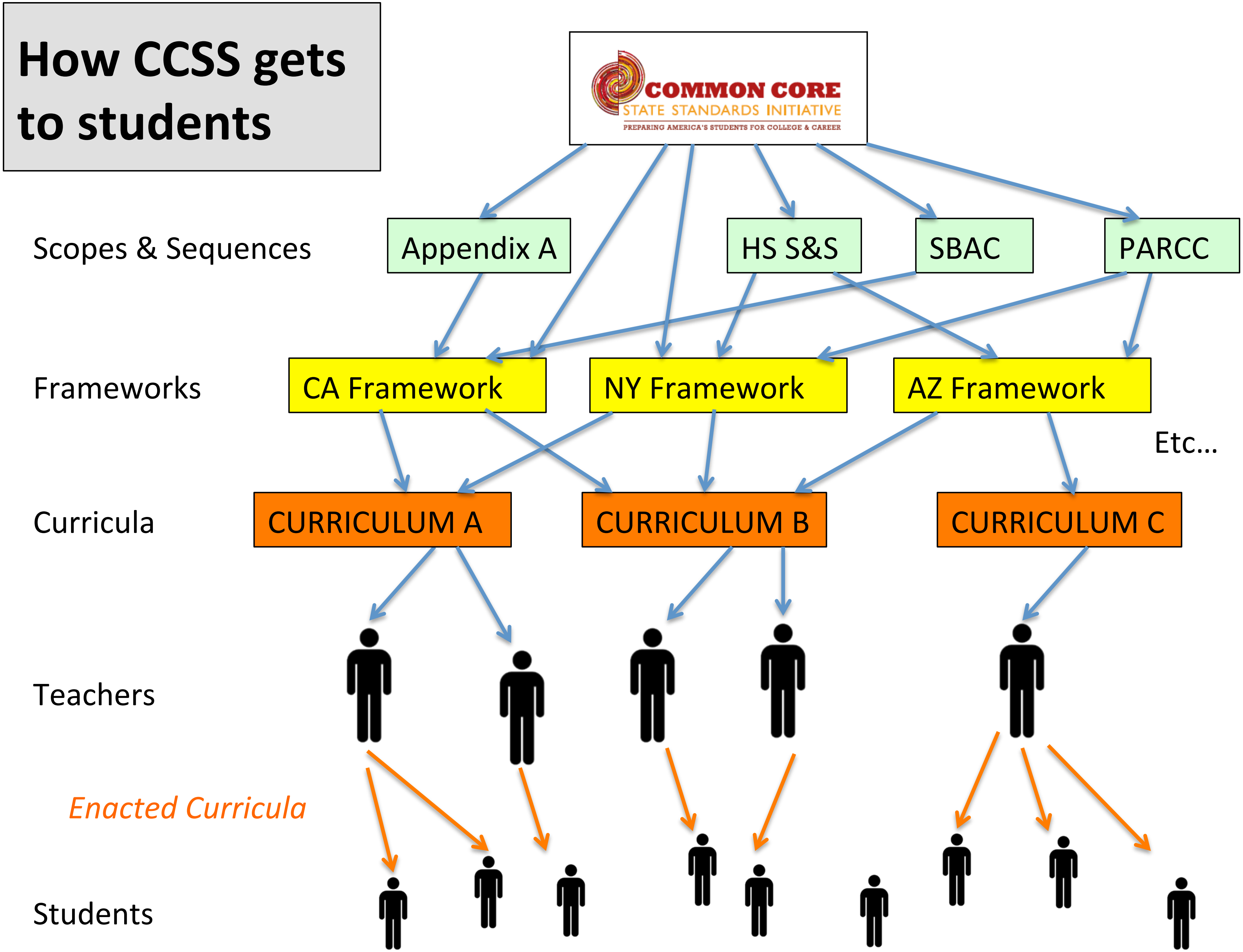
Now, which of these are Algebra One?

HS Teacher
(9th grade)

What is algebra?




COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER
Frameworks
CA Framework
NY Framework
AZ Framework
Etc…
Curricula
CURRICULUM A
CURRICULUM B
CURRICULUM C
Teachers
Enacted Curricula
Students


How CCSS gets to students
COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER
Scopes & Sequences
Appendix A
HS S&S
SBAC
PARCC
Frameworks
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Etc…
Curricula
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"Appendix A"

At the high school level, the standards are organized by conceptual category .... As states consider how to implement the high school standards, an important consideration is how the high school CCSS might be organized into courses that provide a strong foundation for post-secondary success. To address this need, Achieve (in partner- ship with the Common Core writing team) has convened a group of experts... to develop **Model Course Pathways** in Mathematics based on the Common Core State Standards.

The pathways and courses are models, not mandates. They illustrate possible approaches to organizing the content of the CCSS into coherent and rigorous courses that lead to college and career readiness. States and districts are not expected to adopt these courses as is; rather, they are encouraged to use these pathways and courses as a starting point for developing their own.

NB: I do not believe that there is a "correct" or "optimal" way to organize the standards into courses. I think that the enacted curriculum and curriculum matter more. However, I do think some sequences make more sense than others, and the chosen sequence has implications.

What is the HS S&S?

The Gates Foundation and the Pearson Foundation are funding a large scale project to create a system of courses to support the ELA and Mathematics CCSS. These will be a modular, electronic curriculum spanning all grade levels. A Santa Cruz based company, Learning In Motion, is working to write the lessons.

Phil Daro has suggested that it is not the lesson or activity, but rather the **unit** that is the "optimal grain-size for the learning of mathematics". Hence that was the starting point for our Scope and Sequence.

The design challenge was to

1. Start with organizing every standard into coherent units (*not start with* "courses", e.g. 9th grade, or Algebra 2, etc)
2. Structure the coherent units into three sequences Algebra, Geometry, Probability and Statistics
3. Design the units to be able to be organized into either traditional or integrated courses

Lead team: Patrick Callahan, Dick Stanley, David Foster, Phil Daro, Marge Cappo, Brad Findell

Coherent Mathematics Units

Definition of COHERENCE

1 : the quality or state of cohering: as

a : systematic or logical connection or consistency

b : integration of diverse elements, relationships, or values

The building blocks of mathematics instruction:

Problems, tasks, activities, projects

These are combined into **lessons**.

Sequences of lessons comprise a **unit**.

Units have many types of lessons that have different **purposes**

Some possible examples:

Sequence of problems or activities, purpose to develop specific concepts, designed to scaffold, outcome is a delicate (fragile) understanding

Purpose: attend to precision, pin down definitions, conventions, symbolism

INTRO LESSON | CONCEPT LESSON | CONCEPT LESSON | CONCEPT LESSON | GETTING PRECISE LESSON | GETTING GENERAL LESSON | FORMATIVE ASSEMENT LESSON

Purpose: Engage students, spark curiosity, "hook" and necessitate

Different students work on different things, goals of both moving from a fragile to robust understanding via a variety on problems

Purpose: use concepts across contexts, generalize via variables and parameters and different types of numbers, operations, functions, structures

Purpose: Revisit and organize the unit goals and outcomes

… CONCEPT LESSON | ROBUSTNESS AND DIFFEREN-TIATION LESSONS | ROBUSTNESS AND DIFFEREN-TIATION LESSONS … CLOSURE LESSON | SUMMATIVE ASSESSMENT LESSON

Designing for opportunities for SMPs happens at the unit level.

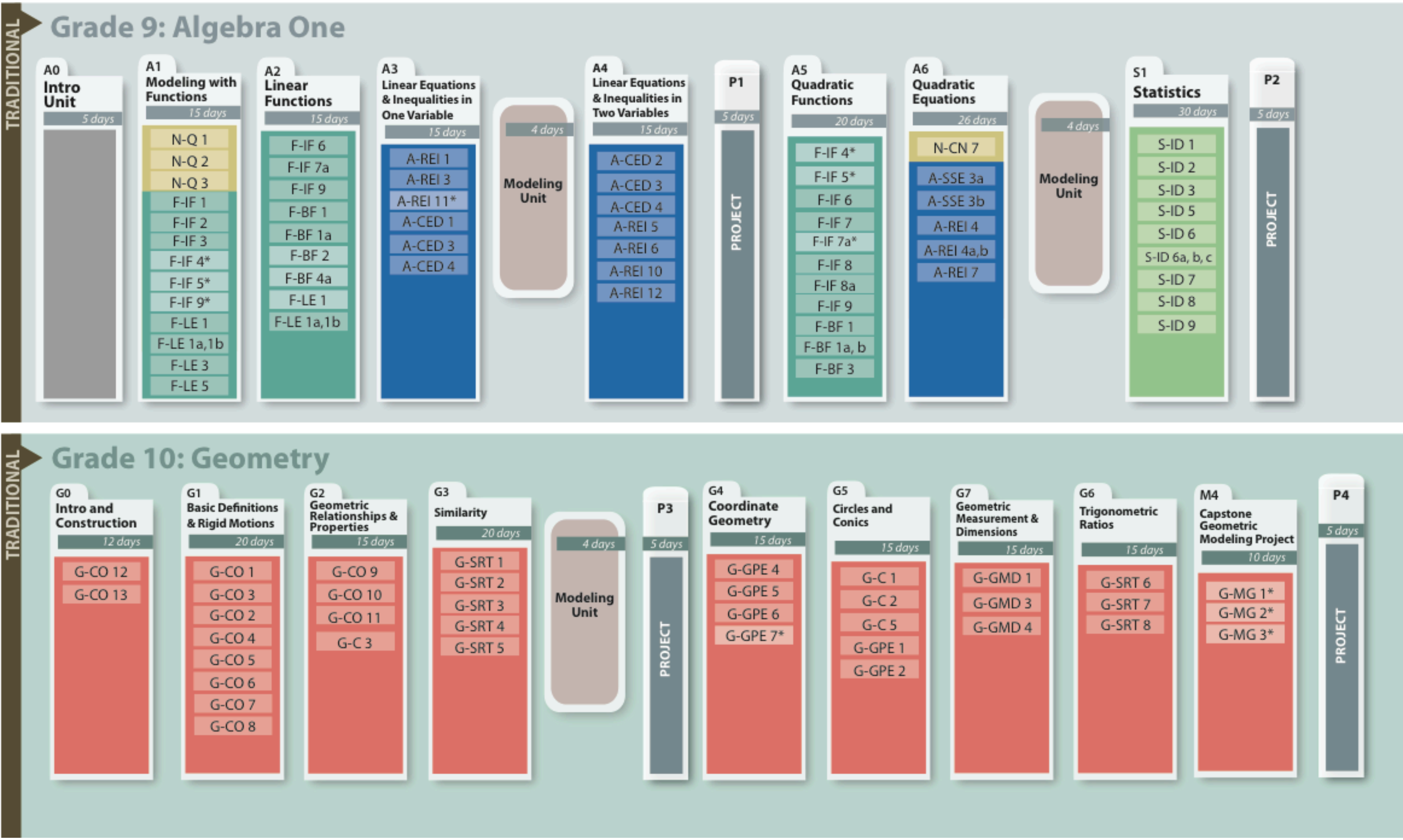
CCSS High School Units

High School Algebra Units:
A0 Introductory Unit
A1 Modeling with Functions
A2 Linear Functions
A3 Linear Equations and Ineq in One Var
A4 Linear Equations and Ineq in Two Var
A5 Quadratic Functions
A6 Quadratic Equations
A7 Exponential Functions
A8 Trigonometric Functions
A9 Functions
A10 Rational and Polynomial Expressions

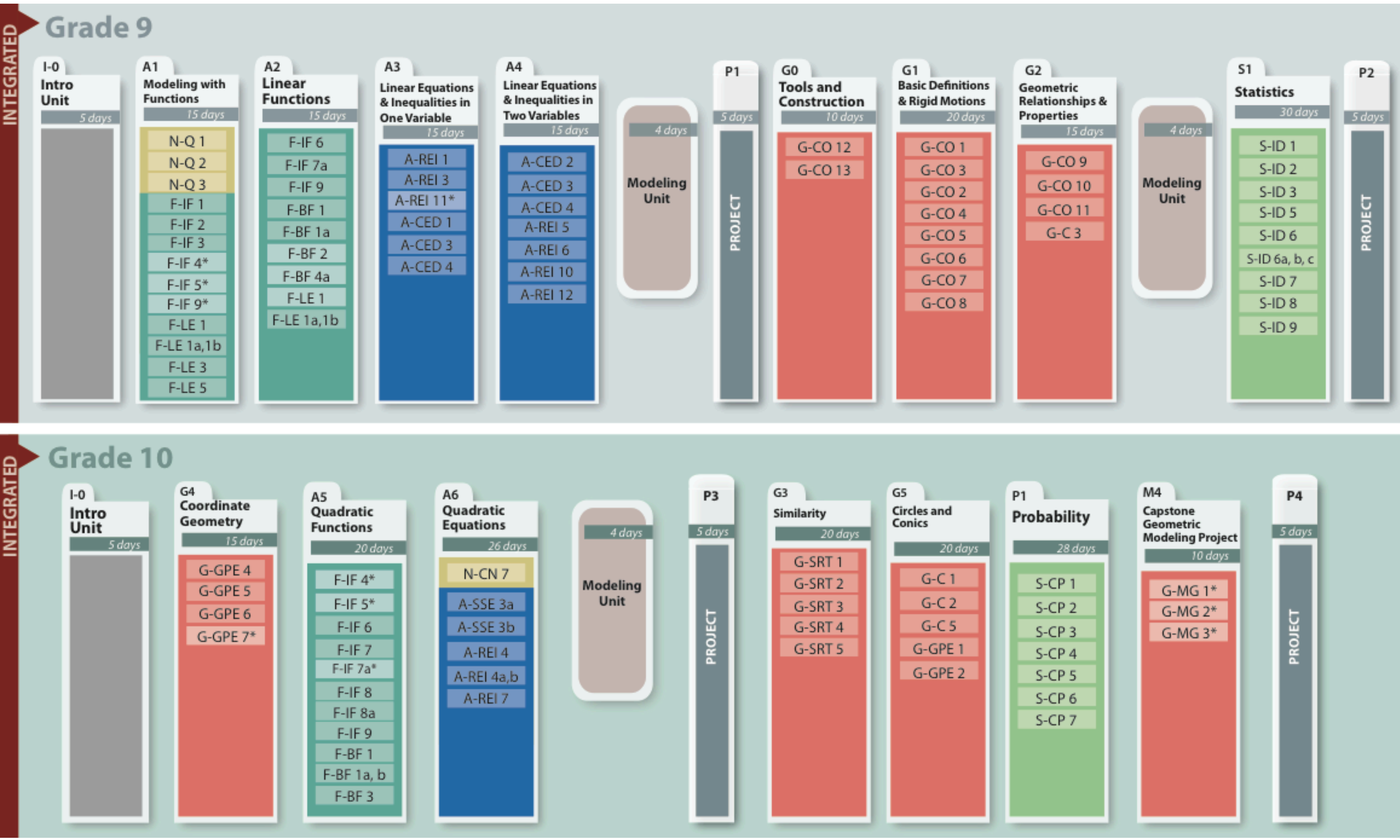
High School Geometry Units:
G0 Introduction and Construction
G1 Basic Definitions and Rigid Motions
G2 Geometric Relationships and Properties
G3 Similarity
G4 Coordinate Geometry
G5 Circle and Conics
G6 Trigonometric Ratios
G7 Geometric Measurement and Dimension
M4 Capstone Geometric Modeling Project

High School Prob & Stat Units:
P1 Probability
S1 Statistics
S2 Statistics (Random Process)

Traditional sequence



Integrated sequence



Tools for the Common Core Standards

News about tools that are being developed to support implementation of the Common Core State Standards

Arranging the high school standards into courses

Posted on March 16, 2012 by Bill McCallum

Here is a suggested arrangement of the high school standards into courses, developed with funding from the Bill and Melinda Gates Foundation and the Pearson Foundation, by a group of people including Patrick Callahan and Brad Findell. I haven't looked at it closely, but it seems to be a solid effort by people familiar with the standards, so I put it up for comment and discussion. There are five files: the first four are graphic displays of the arrangement of the standards into both traditional and integrated sequences, with the standards referred to by their codes. The fifth is a description of the arrangement with the text of the standards and commentary.

Algebra Sequence Design

High School Algebra Units:
A0 Introductory Unit
A1 Modeling with Functions
A2 Linear Functions
A3 Linear Equations and Ineq in One Var
A4 Linear Equations and Ineq in Two Var
A5 Quadratic Functions
A6 Quadratic Equations
A7 Exponential Functions
A8 Trigonometric Functions
A9 Functions
A10 Rational and Polynomial Expressions

We did not think in terms of Algebra 1 and Algebra 2.

We wove together the domains of Number and Quantity, Algebra, and Functions.

We repeat and revisit standards to create coherence.

We consider opportunities for the Standards of Mathematical Practice in the units.

A1 Modeling with Functions (15 days)

Description

This initial unit starts with a treatment of quantities as preparation for work with modeling. The work then shifts to a general look at functions with an emphasis on representation in graphs, and interpretation of graphs in terms of a context. More emphasis is placed on qualitative analyses than calculation and symbolic manipulation. Linear and non-linear examples are explored.

Quantities
A short treatment of the general notion of a "quantity" thought of as a number with a specific unit of measure. Includes unit analysis (dimensional analysis).

Examples of simple quantities with standard units of measure; the fundamental dimensions of quantities (length, time, weight, temperature, counts); division of quantities: quotient units; examples of quantities with quotient units: speed, flow rate, frequency, price, density, pressure; quotient units and "rates"; quotient units and unit conversion; unit analysis/dimensional analysis; multiplication of quantities: product units; area and volume as examples of quantities with product units; person-days and kilowatt hours as other examples of product units;

Functions
A general treatment of the function concept with minimal use of symbolic expressions, and instead emphasis on the idea of a function as a mapping represented in graphs or tables. The functions used in this unit, will be mostly linear and 'baby exponential'. In grade 11, student will thoroughly study exponential functions. But they will be introduced to them here so they can compare two different types of functions. Quadratics or piecewise functions can be used to illustrate the properties of functions.

Domain and range; functions defined by graphs and their interpretation; functions defined by tables and their interpretation; properties of particular functions (rate of change, zeros) and their meaning in an application; sums and differences of two functions; product of a function and a constant; vertical shifts and horizontal shifts; equality of two functions vs. values where two functions are equal; equations defined in terms of functions and their solution; functions defined by geometric conditions (projections); functions defined recursively; sequences.

This unit builds on 8.F 1, 8.F 2, 8.F 3
Functions: Define, evaluate, and compare functions.
and 8.F 4 and 8.F 5
Functions: Use functions to model relationships between quantities.

Interpreting Functions F-IF

Analyze functions using different representations.

F-IF 9 (page 70)

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Linear, Quadratic, and Exponential Models* F-LE

(Only linear, simple quadratic, and simple exponential functions.)

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE 1 (page 70)

Distinguish between situations that can be modeled with linear functions and with exponential functions.

F-LE 1a (page 70)

Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE 1b (page 70)

Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE 3 (page 71)

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model.

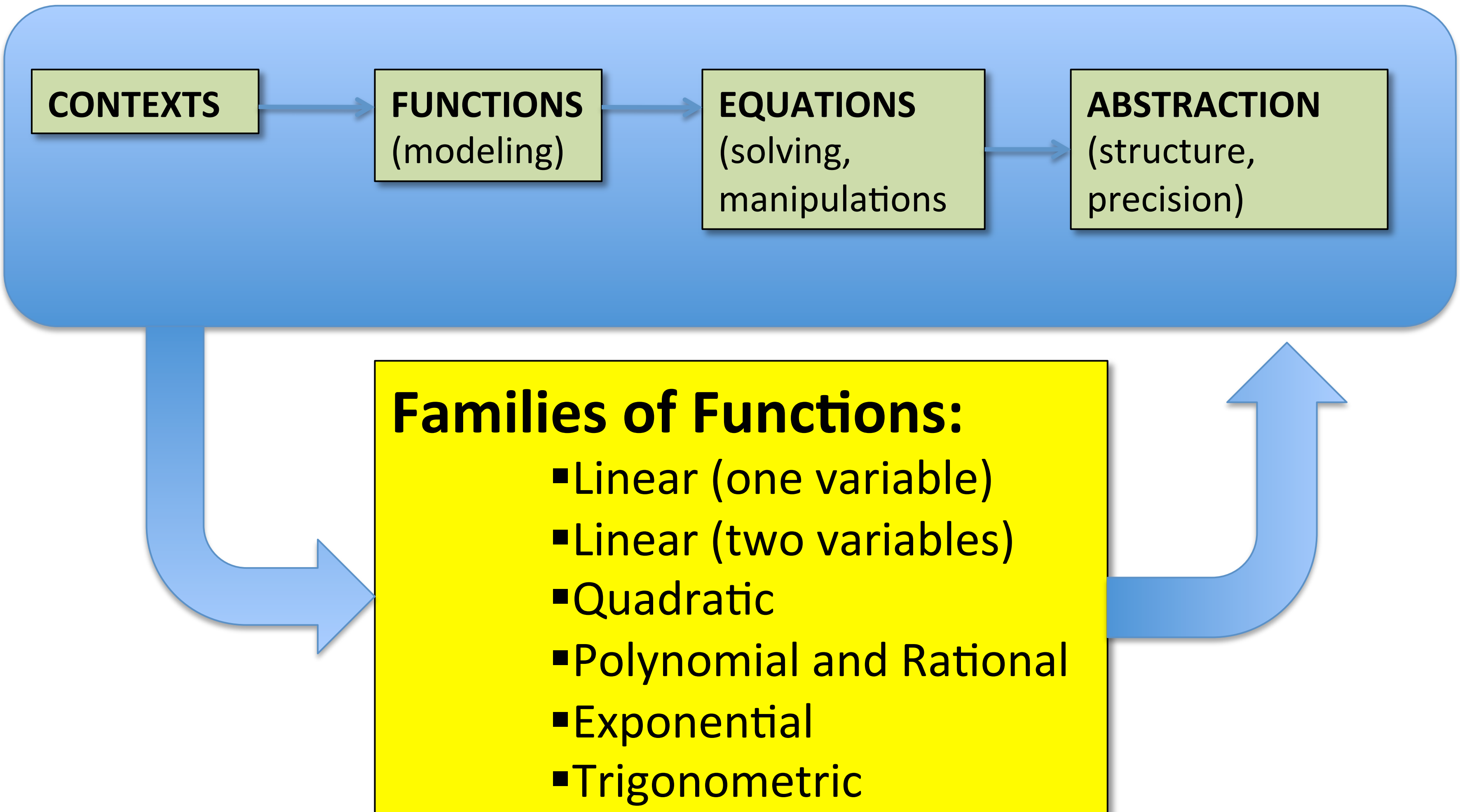
F-LE 5 (page 71)

Interpret the parameters in a linear or exponential function in terms of a context.

Algebra ≠ Bag of Tricks

To avoid the common experience of algebra of a “bag of tricks and procedures” we adopted a cycle of algebra structure based on a family of functions approach.

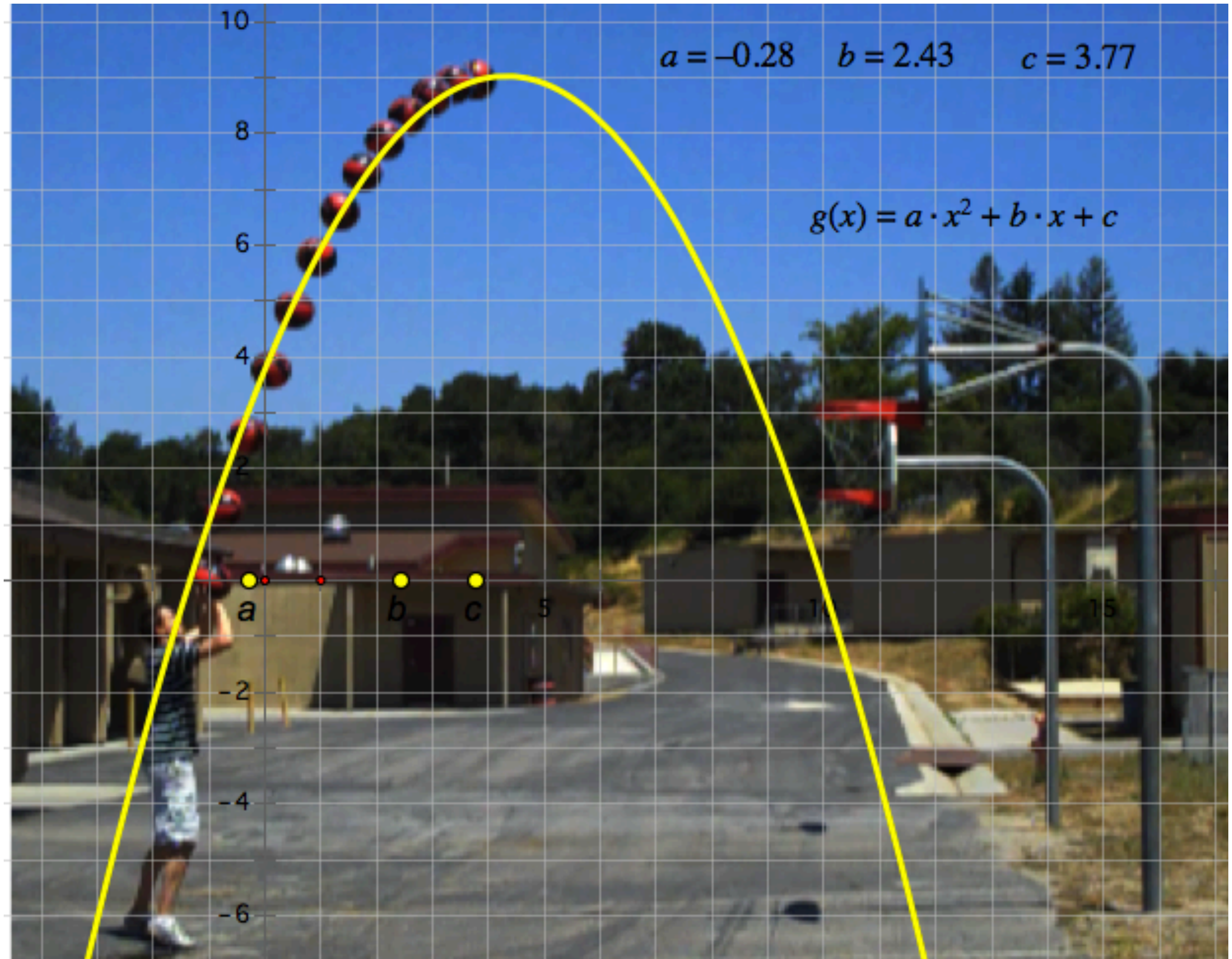
HS Algebra Families of Function Cycle



Context

From Dan Meyer’s blog

Model with functions



Equations

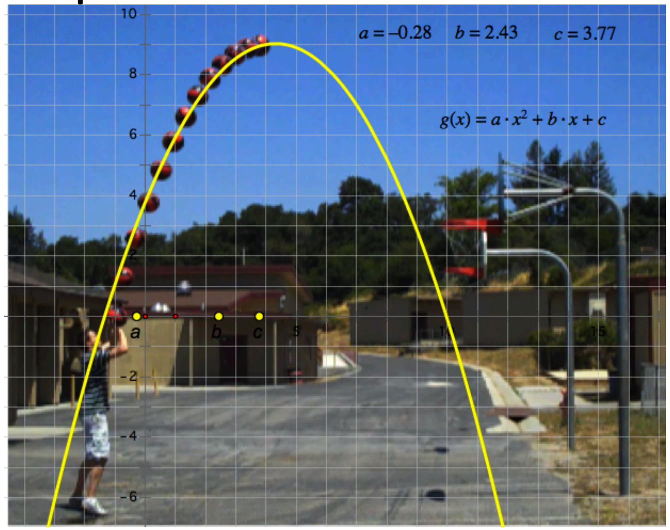
$$g(x) = -2.8x^2 + 2.43x + 3.77$$

$$0 = -2.8x^2 + 2.43x + 3.77$$

You can't "solve" a function. But functions can be analyzed and lead to equations, which can be solved.

What was the maximum height of the ball?
How close did the ball get to the hoop?

Symbolizing,
manipulating,
Equivalence…



Abstracting (structure, generalization)

Examples:

The maximum or minimum occurs at the midpoint of the roots.

The sign of the *a* coefficient determines whether the parabola is up or down (convexity)

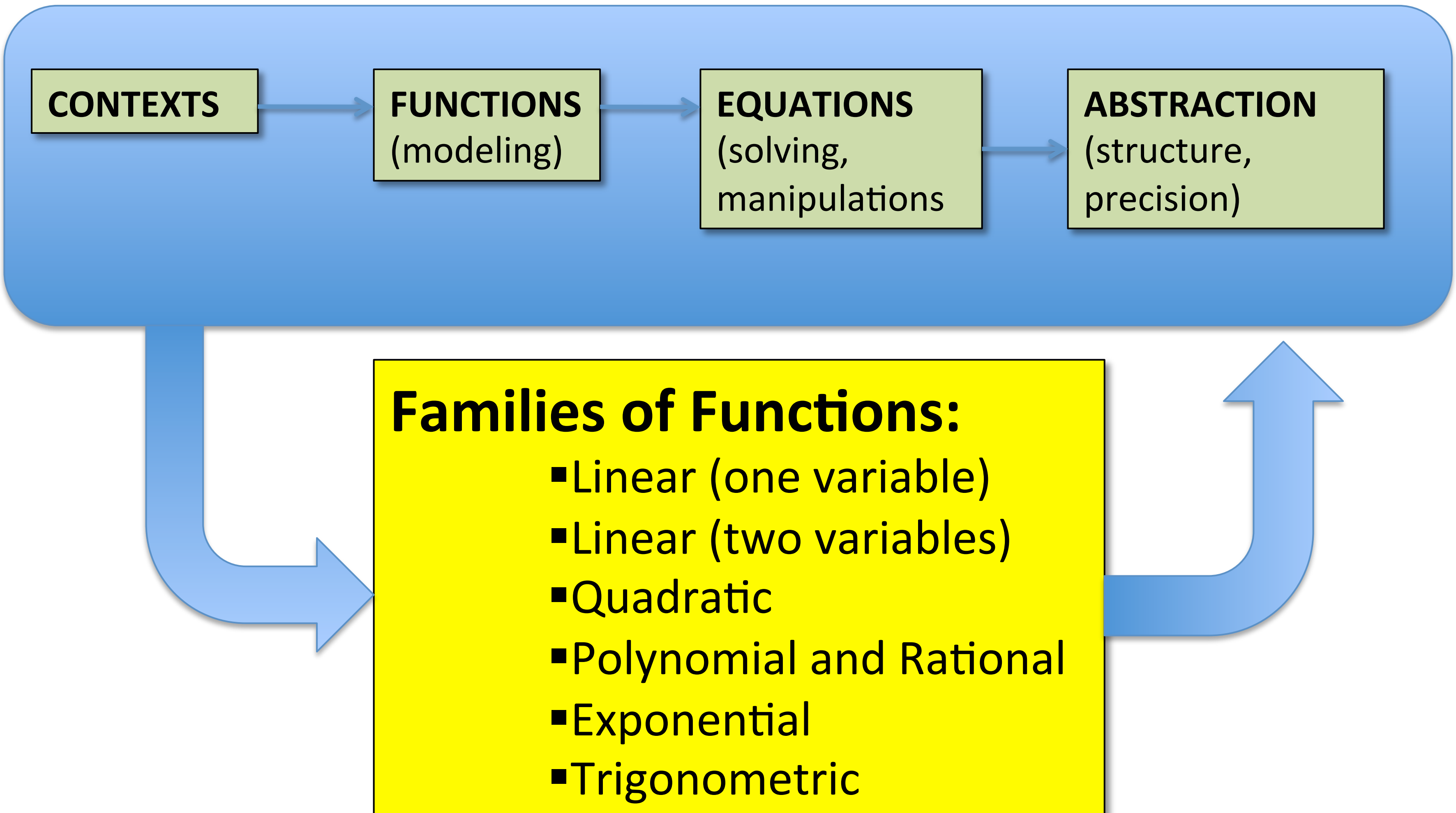
The *c* coefficient is the sum of the roots.

The roots can be determined in multiple ways: quadratic formula, factoring, completing the square, etc.

$$(x - p)(x - q) = x^2 - 2(p + q)x + pq$$

$$ax^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HS Algebra Families of Function Cycle



Catching-up Problem

Car A is traveling at 50 mph. Three hours later Car B starts to try to catch up. If Car B goes 75 mph how far must it travel to catch up with Car A?

What is the mathematical content of this problem?

What is the purpose?

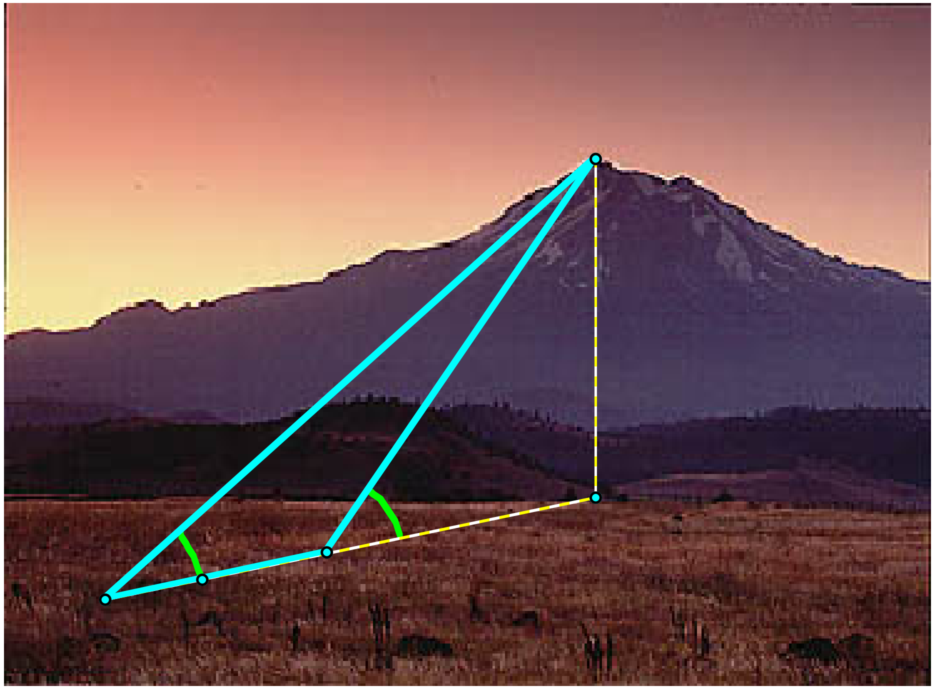
Mountain Problem

We take sightings on the mountain from a level plain. At one point we find the mountain top has an angle of elevation α, if we move directly toward the mountain a distance D we find that the angle of elevation is β.

What is the height of the mountain?

What is the content?

What is the purpose?

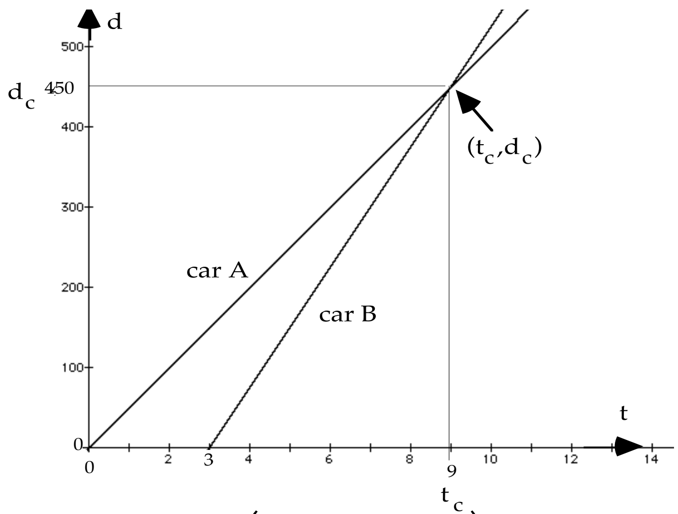


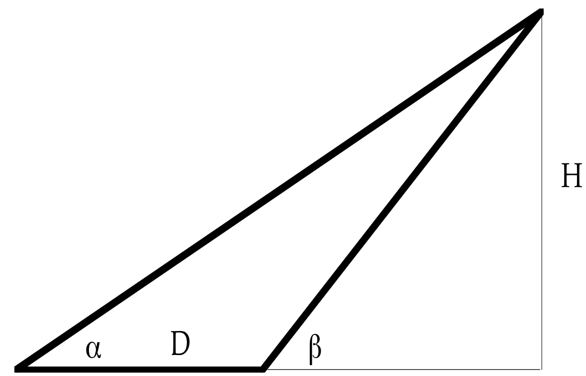
Typical views

Catching-up problem: algebra, graphing, distance = rate x time, setting up an equation, solving for unknown, linear functions

Mountain problem: geometry, trigonometry, angle relationships, determining unknown lengths and angles, definitions of trigonometric functions

Looking deeper





$$d_c = t_h \cdot \left(\frac{1}{\frac{1}{v_A} - \frac{1}{v_B}} \right)$$

$$H = D \cdot \left(\frac{1}{\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}} \right)$$

Isomorphism of problems, revealing structure through representations, finding connections and generalizations

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Illustrative Mathematics

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K-8 STANDARDS

HIGH SCHOOL STANDARDS

PRACTICE STANDARDS

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K-8 Standards

High School Standards

Practice Standards

Illustrative Mathematics provides guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the Common Core State Standards, and by publishing other tools that support implementation of the standards.

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Illustrative Mathematics

Content Standards: Kindergarten Through Grade Eight

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K-8 STANDARDS

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TERMS OF USE

K	1	2	3	4	5	6	7	8
Geometry								
Measurement and Data						Statistics and Probability		
Number and Operations in Base Ten						The Number System		
Operations and Algebraic Thinking						Expressions and Equations		
Counting and Cardinality			Number and Operations—Fractions			Ratios and Proportional Relationships		Functions

Show only illustrated standards (?)

Reveal standards automatically (?)

Expressions and Equations

Grade 6	6.EE	show all
Grade 7	7.EE	show all
Grade 8	8.EE	show all

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8.EE Coffee by the Pound

Direct link to this task

8.EE Coffee by the Pound

Alignment 1: 8.EE.5

This task is rated +1 (1 rating) • *Log in if you'd like to rate this task*

Lena paid $18.96 for 3 pounds of coffee.

a. What is the cost per pound for this coffee?

b. How many pounds of coffee could she buy for $1.00?

c. Draw a graph in the coordinate plane of the relationship between the number of pounds of coffee and the total cost.

d. In this situation, what is the meaning of the slope of the line you drew in part (c)?

Commentary:

A slight modification to this problem would make this appropriate for the 7th grade level (see 7.RP.2.d Coffee by the Pound). At the 8th grade level, the solver would be expected to identify the slope of the line with the unit rate.

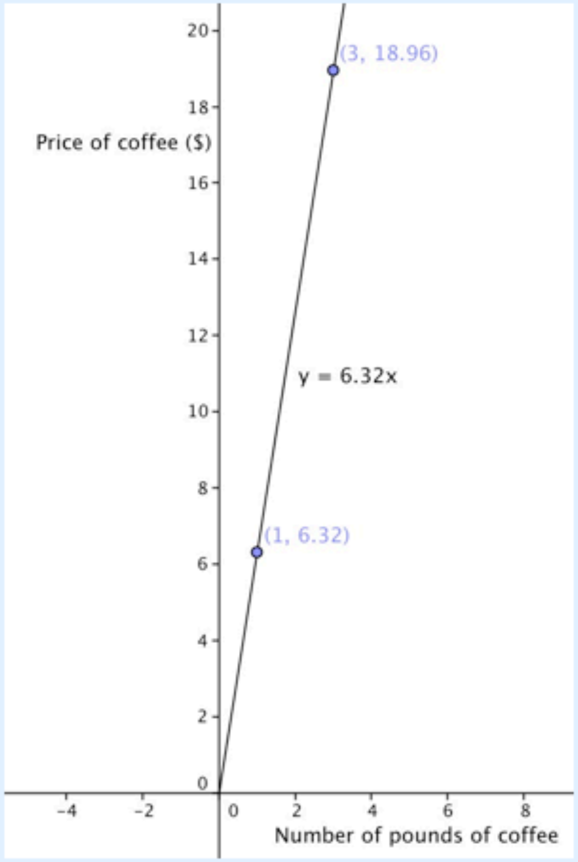
Solution: Two possible graphs

a. If you divide the cost for three pounds by three, you will get the cost per pound. Coffee costs \$6.32 per pound.

b. If you divide the number of pounds by the cost for three pounds, you will get the amount of coffee one can purchase for \$1.00. You can buy approximately 0.16 pounds of coffee for a dollar.

c. There are two possible graphs depending on what you choose *x* to represent and what you choose *y* to represent.

   If we let *x* indicate the number of pounds of coffee and let *y* indicate the total price, then the solver may produce a graph by drawing a line through the origin and the point (3, 18.96); see below.



   If we let *x* indicate the total price and let *y* indicate the number of pounds of coffee, then the solver may produce a graph by drawing a line through the origin and the point (18.96, 3).

d. With the choice for *x* and *y* we made, the slope is the cost per pound of coffee, which is \$6.32. If we had chosen the other order, the slope would have been the amount of coffee one could buy for a dollar, which is 0.16 pounds.

Illustrative Mathematics Task Bank

Illustrative Mathematics Task Bank

Goal is to illustrate every CCSS content standard with a collection of tasks.

Question: How do we decide if a task is “good enough” to be included in the Task Bank?

Answer: Every task is reviewed multiple times against a set of criteria by people with different expertise and experience.

There has to be at least one reviewer with classroom expertise and one reviewer with mathematics content expertise.

Coming up with criteria

What do you think is essential for “good” mathematics tasks?

Illustrative Mathematics Task Review Form

1. Task criteria

Indicate whether the task meets each of the following criteria.

YES	NO	
☐	☐	1. The task illustrates the specified standard, cluster, domain, or conceptual category.
☐	☐	2. The task's purpose is clearly stated in the commentary and is likely to be fulfilled.
☐	☐	3. The task has at least one appropriate solution.
☐	☐	4. The mathematics is correct.
☐	☐	5. Any diagrams or pictures have a clear mathematical or pedagogical purpose.
☐	☐	6. The context supports the purpose of the task.
☐	☐	7. The task write-up appropriately addresses units and numerical precision.
☐	☐	8. The language of the task is unambiguous and grade-appropriate.

Explain your selections above:

Purpose

<u>Purpose</u>

We will call the mathematical idea and/or habit of mind that a task is intending to develop or assess, along with its intended use, the *purpose of the task*.

The purpose of a task could be (but not restricted to):

- Introduce a new concept
- Engage students (i.e. get students interested in the concept)
- Review an old concept
- Establish connections between different concepts
- Assess students' understanding of a concept
- Elicit misconceptions
- Provide opportunity to contrast different approaches
- Provide opportunity to practice a specific approach or technique
- Necessitate attending to precision
- Provide opportunity to model mathematically

Sample 5th grade problem

Standard: 5-NS.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Purpose: Problem Solving

I wuv math

Solve the following problems:

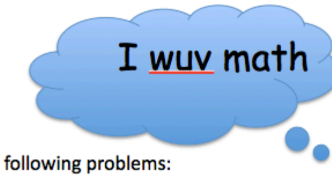
6 x 5 =

5/4 x 1 =

0 x 11 ½ =

Sample 5th grade problem

Standard: 5-NS.6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
Purpose: Problem Solving



Solve the following problems:

6 x 5 =

5/4 x 1 =

0 x 11 ½ =

Illustrative Mathematics Task

1. Task criteria

Indicate whether the task meets each of the following cri

YES	NO	
☐	☐	1. The task illustrates the specified standard, cluster, domain, or conceptual category.
☐	☐	2. The task's purpose is clearly stated in the commentary and is likely to be fulfilled.
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☐	☐	4. The mathematics is correct.
☐	☐	5. Any diagrams or pictures have a clear mathematical or pedagogical purpose.
☐	☐	6. The context supports the purpose of the task.
☐	☐	7. The task write-up appropriately addresses units and numerical precision.
☐	☐	8. The language of the task is unambiguous and grade-appropriate.

Explain your selections above:

Algebra and Functions

81. $x^3y^3 =$

A $9xy$

B $(xy)^6$

C $3xy$

D $xxxyyy$

M02879

82. What does x^5 equal when $x = -2$?

A -32

B -10

C $-\frac{1}{32}$

D 32

M12857

83. Which of the following is equivalent to $(6x - 2)(6x - 2)(6x + 2)$?

A $(6x - 2)^3$

B $(6x + 2)^3$

C $2(6x - 2)(6x + 2)$

D $(6x - 2)^2(6x + 2)$

M12845

85. Simplify the expression shown below.

$$(6a^4bc)(7ab^3c)$$

A $13a^4b^3c$

B $13a^5b^4c^2$

C $42a^4b^3c$

D $42a^5b^4c^2$

M02109

86. Which expression is equivalent to $7a^2b \bullet 7bc^2$?

A $14a^2b^2c^2$

B $49a^2bc^2$

C $49a^2b^2c^2$

D $343a^2b^2c^2$

M12872

87. Which expression is equal to $\sqrt{100a^2}$?

A $10a$

B $50a$

C $10a^2$

D $50a^2$

M20646

A quick example

Consider the following common problem from high school algebra:

Simplify $$\frac{1}{7-\sqrt{3}}$$

A quick example

The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.

Consider the following common problem from high school algebra:

Simplify $$\frac{1}{7-\sqrt{3}}$$

A typical solution might be to "rationalize the denominator by multiplying by the conjugate"

$$\frac{1}{7-\sqrt{3}} = \frac{1}{7-\sqrt{3}} \cdot \frac{7+\sqrt{3}}{7+\sqrt{3}} = \frac{7+\sqrt{3}}{46} = \frac{7}{46} + \frac{1}{46}\sqrt{3}$$

What is the mathematics?

Many questions come to mind:

Why is $\frac{7}{46} + \frac{1}{46}\sqrt{3}$ more simple than $\frac{1}{7-\sqrt{3}}$?

What is “rationalizing”?

What are conjugates? When and why do you use them?

How would you simplify $\frac{1}{7-\sqrt[3]{3}}$?

Coherence not required…

51. *Critical Thinking* A hot air balloon flies at a speed of $(n + 8)$ miles per hour. At this rate, how long will it take to fly $(n^2 + 5n - 24)$ miles?

Hot air balloons move with wind currents. Evaluate the expressions in Exercise 51 when $n = 27$ to find the speed and length of one journey.

Prentice Hall's *Algebra I: California Edition*:

Taken from Dan Meyer's blog, heading "psuedocontextsaturday"

So what can we do now?

Frameworks, assessments, CCSS aligned curricula (non-sticker versions) are at least a year or two away.
We are in a transition period with a great deal of uncertainty.

Thinking about how the standards can be arranged into coherent units is a great way to get a deeper understanding of the standards.

Working together to

- identify existing resources to build coherent units
- identify performance assessments for coherent units
- support the implementation of coherent units
- design and share lessons with different purposes.
- use the Illustrative Mathematics tasks
- submit, review, critique tasks in IM

These actions can all be done now to help pave the way for whatever happens.

patrick@illustrativemathematics.org

Conclusion:

Be coherent.
Have a purpose.