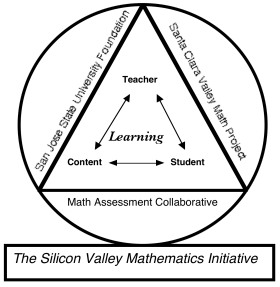
Common Core Standards: Math

How The CCSS Will Impact Teaching, Learning And Assessment In Mathematics

David Foster
Silicon Valley Mathematics Initiative
www.svmimac.org



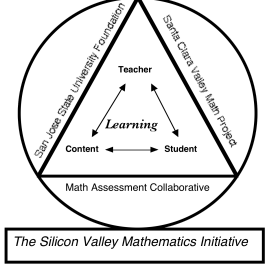
Optimism

"Optimism is an essential ingredient for innovation. How else can the individual welcome change over security, adventure over staying in safe places? A significant innovation has effects that reach much further than can be imagined at the time, and creates its own uses. It will not be held back by those who lack the imagination to exploit its use, but will be swept along by the creative members of our society for the good of all. Innovation cannot be mandated any more than a baseball coach can demand that the next batter hit a home run. He can, however, assemble a good team, encourage his players, and play the odds."

Robert N. Noyce

Silicon Valley Mathematics Initiative

83 Members - School Districts, Charter School Networks, and Schools



Albany USD
Alvord SD (Riverside County)
Antioch Unified SD
Aspire Charter School Network
Assumption School (San Leandro)
Bayshore SD
Belmont-Redwood Shores SD
Berryessa SD
Bolinas – Lagunitas SD
Brisbane SD
Buckeye SD
Cambrian SD
Castro Valley USD
Charter School of Morgan Hill
Chicago Public School
Creative Arts Charter (SF)
CSU San Bernardino
Cotati – Rohnert Park
Cupertino SD
Dade County Schools (GA)
Discovery Charter School
Dioceses of Santa Clara
Dublin USD
East Side UHSD
Edmonds Community College
Emery SD
Etiwanda SD (San Bernardino Co)
Gilroy (Brownell MS)
Fairfield-Suisun USD
Fremont USD
Forsyth County School (GA)
Hamilton County (Tn)
Hayward USD
Jefferson ESD
Jefferson HSD
Las Lomitas SD
La Honda-Pescadero Sd
Livermore USD
Los Altos SD
Los Gatos SD
Menlo Park SD
Monterey Peninsula USD
Moreland SD
Mountain SD
National Council of La Raza
New York City PS
New Visions for Public Schools
Oakland Unified SD
Pacifica SD
Pajaro Valley USD
Palo Alto USD
Pittsburgh USD
Portola Valley SD
Ravenswood City SD
Riverside COE
Redwood City Schools
Sacramento City USD
Salinas City Schools
San Carlos Charter Learning Center
San Francisco USD
SMFC (Park School)
San Jose Unified SD
San Leandro USD
San Ramon Valley USD
Santa Clara USD
Santa Cruz City Schools
Saint Michael's School (Poway)
Saint Patrick's School (San Jose)
Saratoga
Scotts Valley USD
SCCOE County Court Schools
Sequoia HSD
SMCOE County Court Schools
South Cook Service District
South San Francisco USD
Sumter County (GA)
The Nueva School
Union SD
University of Illinois, Chicago
Valley Christen (Dublin)
Valdosta City (GA)
Walnut Creek SD
Woodside SD

Supporting Teaching and Learning of Mathematics Since 1996

The state of mathematics education in America

After a decade of high-stakes accountability promising to equalize performances among students of all demographic classifications, what is happening to the "***Gap***"?

The Achievement Gap

NAEP 2009	US 4th Grade	US 8th Grade
All Students	240	283
Black	222	260
White	248	292
Hispanic	227	266
Asian	255	300
Low Income	228	266
Mid-High Income	250	293
English Learner	218	243
English Fluent	242	284

Source: U.S. Department of Education

National Achievement of Education Progress (NAEP 2009 Math) The Nation's Report Card

Approximately 10 scale points is equivalent to a grade level of learning

NY TIMES April 29, 2009

Persistent Racial Gap Seen in Students Test Scores

By SAM DILLON

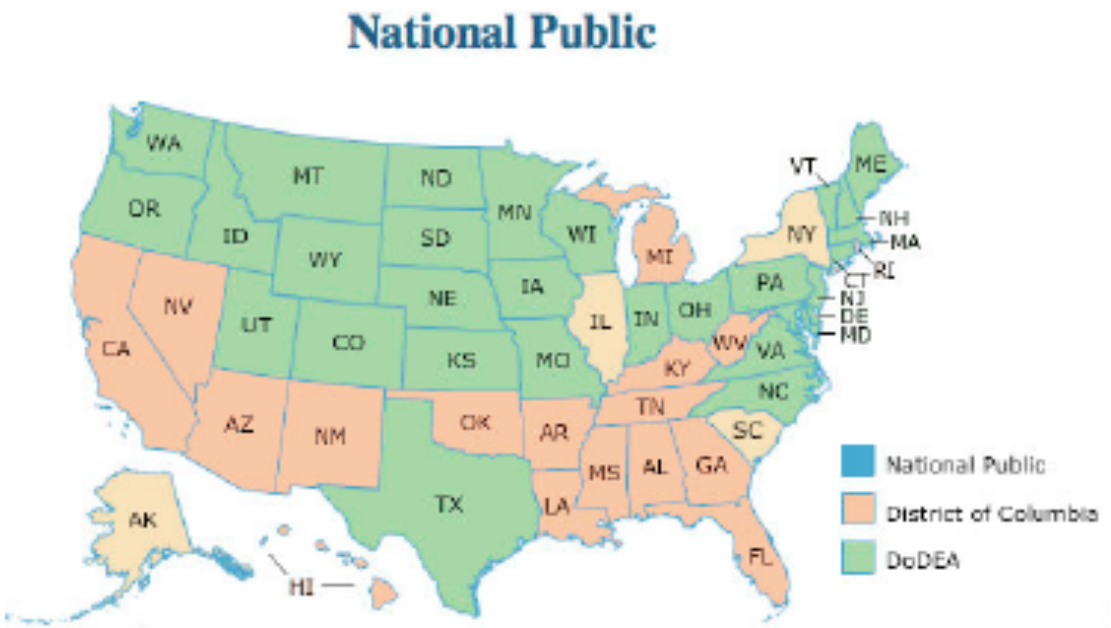
The achievement gap between white and minority students has not narrowed in recent years, despite the focus of the No Child Left Behind law on improving black and Hispanic scores, according to results of a federal test considered to be the nation's best measure of long-term trends in math and reading proficiency.

NAEP 2009 8th grade

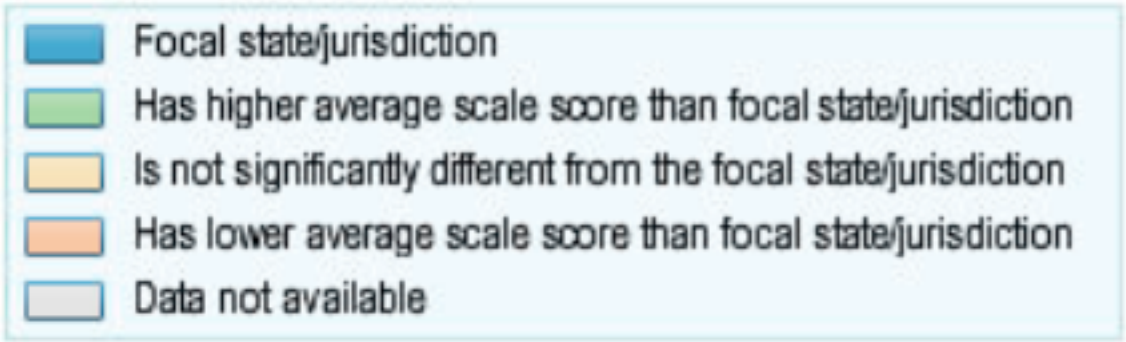
National Ave. 282

Massachusetts (1st) 299

California (49th) 270



Where you live and your background correlates to how you score on tests.



Common Core Standards:

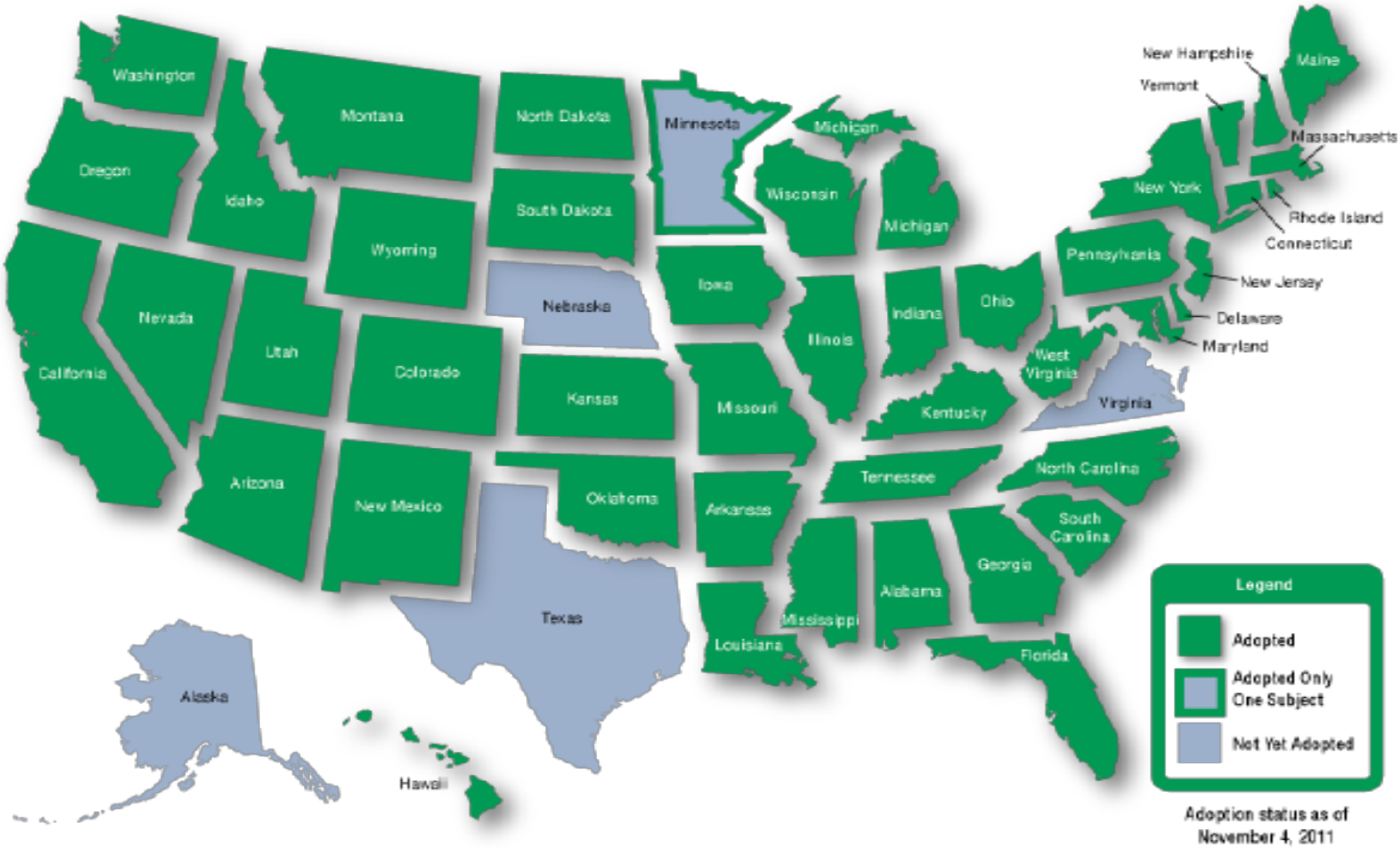
A New Direction linking Instruction and Assessment

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

46** States, DC, and US Virgin Islands



States have joined Assessment Consortia funded by RttT

Partnership for Assessment of Readiness of College and Careers (PARCC) is being managed by Achieve, Inc., a Washington-based non-profit. There are 23 states and DC in PARCC.

The SMARTER-Balanced Assessment Consortium is being managed by San Francisco-based WestEd and its senior program director, Stanley Rabinowitz. SMARTER-Balanced enlisted 31 states.

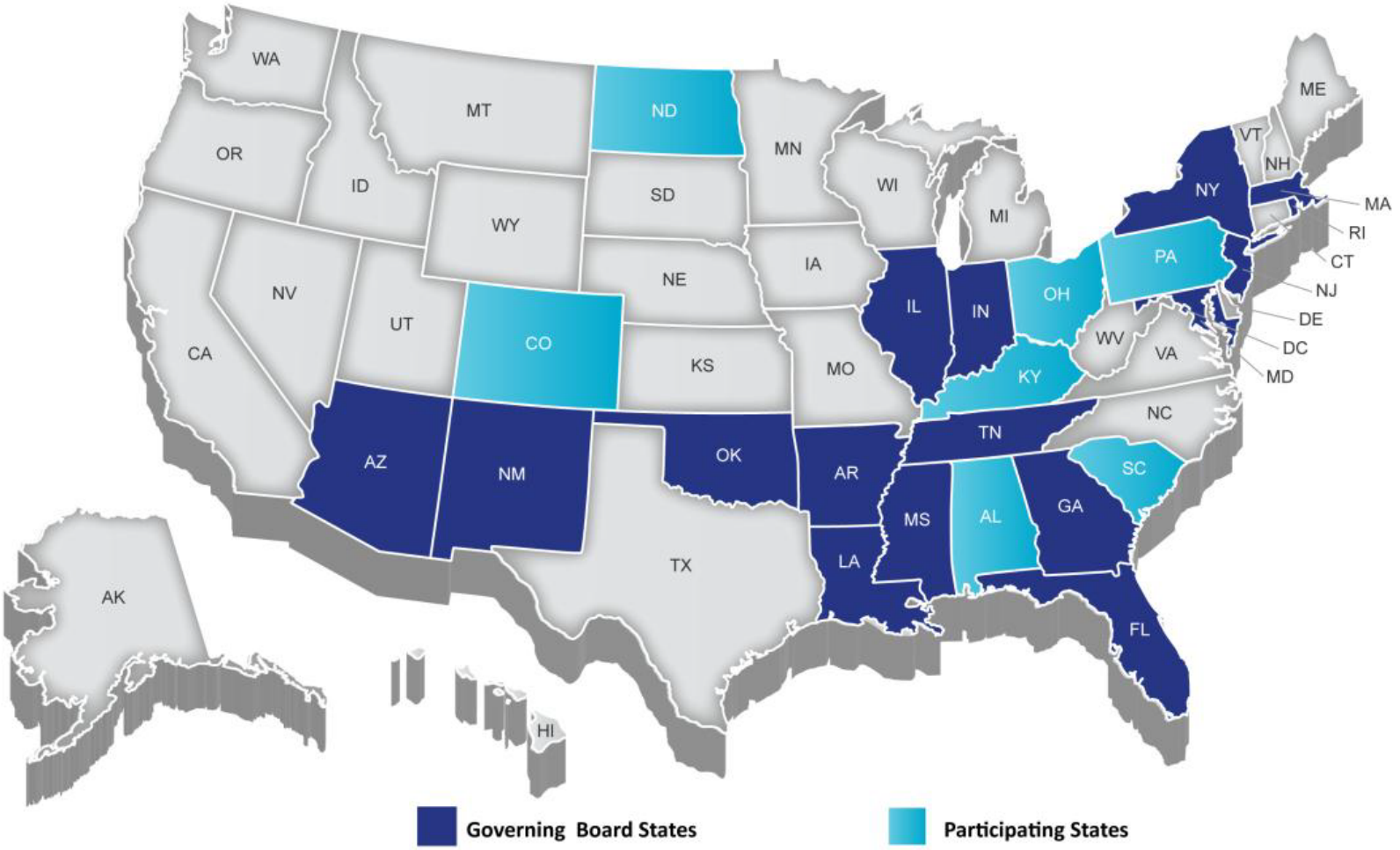
At this point, both consortia are targeting the first test administration by 2014-15. Both say they will integrate summative or end-of-the-year tests with interim and formative assessments that can guide instruction during the year. Both are promising to include performance-based tasks, such as conducting a science experiment and writing short answers to questions, that are intended to show deeper levels of learning and thinking than multiple choice questions supposedly can measure. Both indicate that technology will play a major role.

Goals of Assessment

"We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics."

Everybody Counts

PARCC States



PARCC States

Achieve

SMARTER
Balanced Assessment Consortium

SMARTER

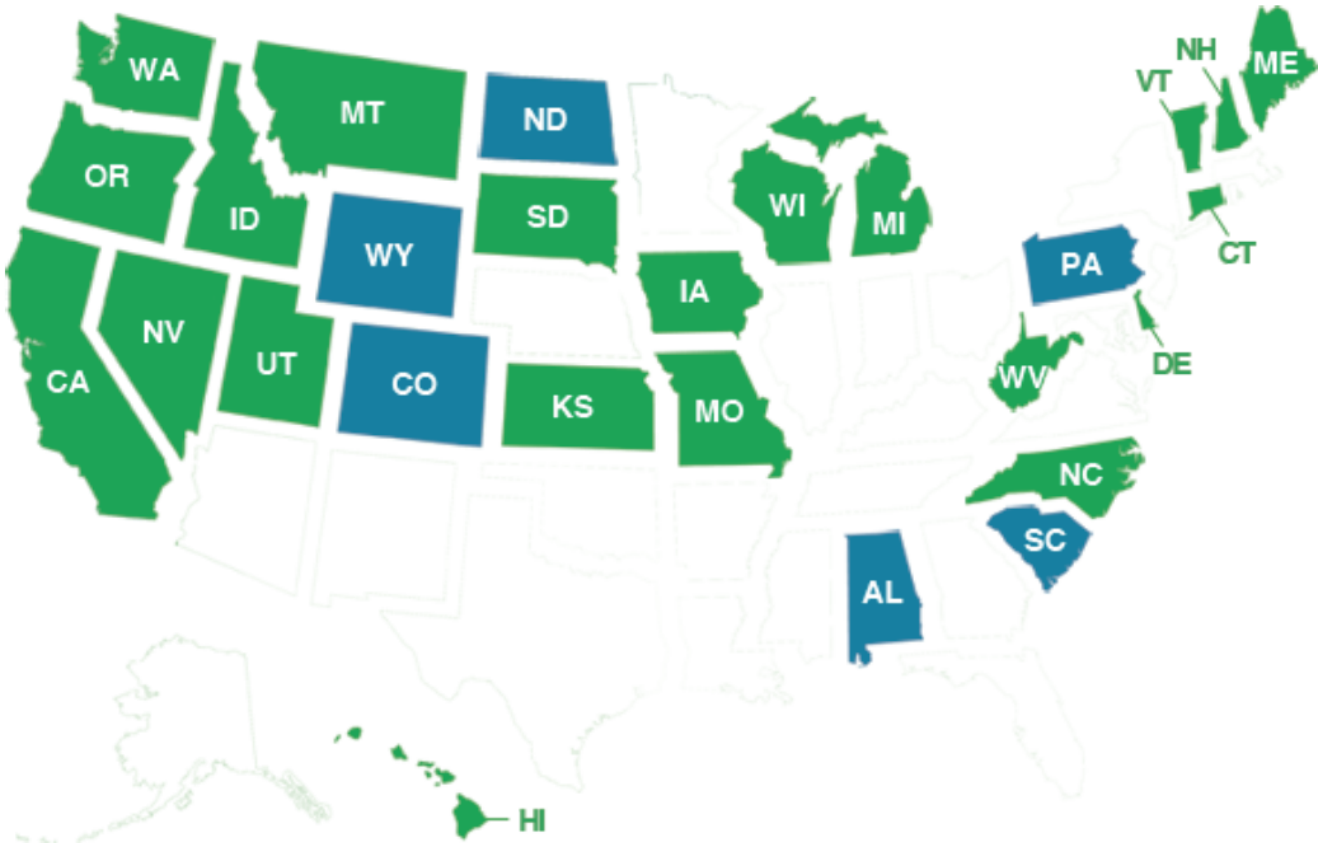
State of Washington OSPI
Office of Superintendent of Public Instruction

Home | Certification | Offices & Programs | Teaching & Lea

SMARTER Balanced Assessment Consortium SMARTER

SMARTER Balanced States

States in the SMARTER Balanced Assessment Consortium (as of November 17, 2010):



WestEd

Current vs. CCSS

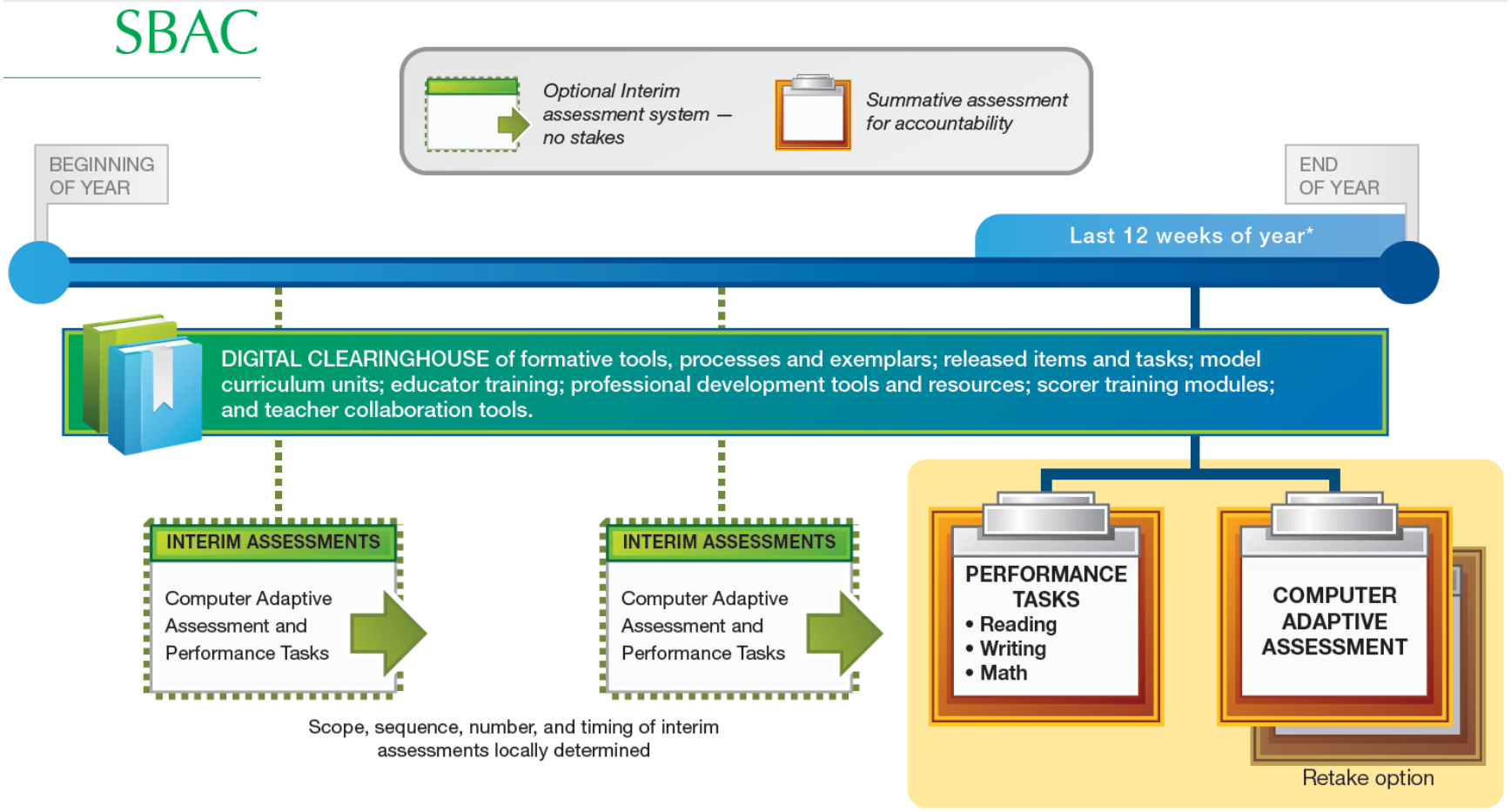
Current STAR Assessments

- Grades 2-11, writing at 4th and 7th
- Only paper & pencil option
- Taken around 85% of the instructional days
- Only multiple choice
- Part of the state and federal accountability system

Proposed CCSS Assessments

- Grades 3-8 and 11, Grades 9 and 10 available for states that choose to use them
- Delivered via computer (Paper and pencil option available for 3 years) and are computer adaptive
- Taken during the final 12 weeks of school
- Performance tasks and comprehensive end-of -year computer adaptive assessment which will some selected response items
- Accountability system has not been established yet

The System
(Possible Scenario)



Claim-Evidence-Warrant

A Model for Analyzing Arguments

(adapted from the work of Stephen Toulmin)

SMARTER
Balanced Assessment Consortium

PARCC States

Content Specifications
for the Summative assessment of the
Common Core State Standards for Mathematics

DRAFT TO ACCOMPANY GOVERNING STATE VOTE ON ASSESSMENT CLAIMS

March 20, 2012

Developed with input from content experts and Smarter Balanced Assessment Consortium Staff, Work Group Members, and Technical Advisory Committee

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Past President, Association of State Supervisors of Mathematics
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David Foster, Director, Silicon Valley Mathematics Initiative
Henry Pollak, Adjunct Professor, Columbia University, Teachers College,
Former Head of Mathematics and Statistics, Bell Laboratories
W. James Popham, Emeritus Professor, University of California, Los Angeles
Cathy Seeley, Senior Fellow, Charles A. Dana Center, The University of Texas at Austin
Malcolm Swan, Professor of Mathematics Education, Centre for Research in Mathematic Education,
University of Nottingham

SMARTER – Balance

Content Specifications - CCSSM

Four Major Claims for the SMARTER Balanced Assessment Consortium's assessments of the *Common Core State Standards for Mathematics*

Claim #1 **- Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.**

Claim #2 **- Students can frame and solve a range of complex problems in pure and applied mathematics.**

Claim #3 **- Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.**

Claim #4 **- Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

Types of Tasks in Mathematics

Novice – short items focused on skills and routines

Apprentice – medium performance tasks with scaffolding

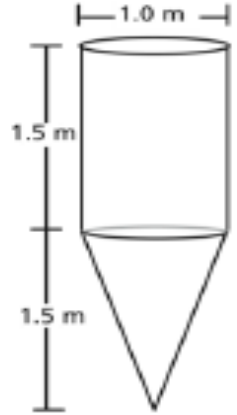
Expert – long tasks with high cognitive load and/or complexity.

Novice Task Example

5. **Water Tank**

A water tank has shape and dimensions as shown in the diagram.

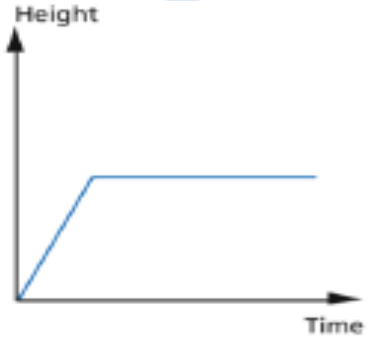
At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.



Water tank

Click on the graph that shows how the height of the water surface changes over time.

A



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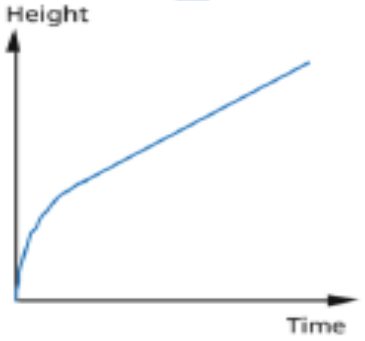

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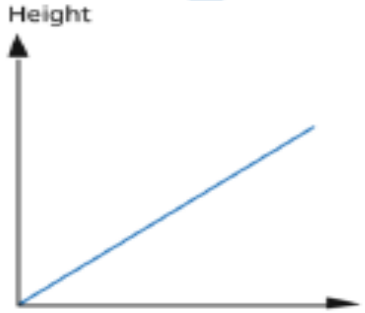

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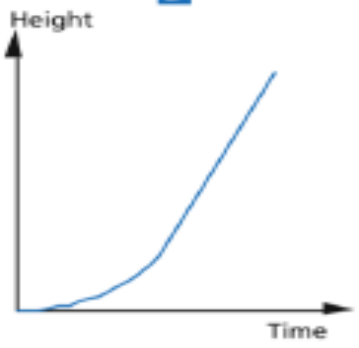

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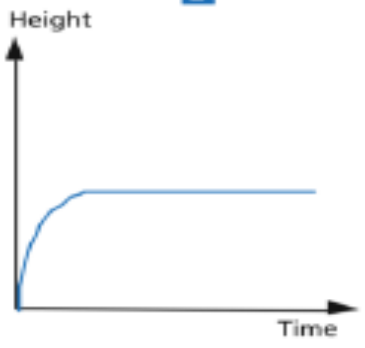


Click on the graph that shows how the height of the water surface changes over

CST – Released Items Algebra 1

The total cost (c) in dollars of renting a sailboat for n days is given by the equation

$$c = 120 + 60n.$$

If the total cost was $360, for how many days was the sailboat rented?

A 2

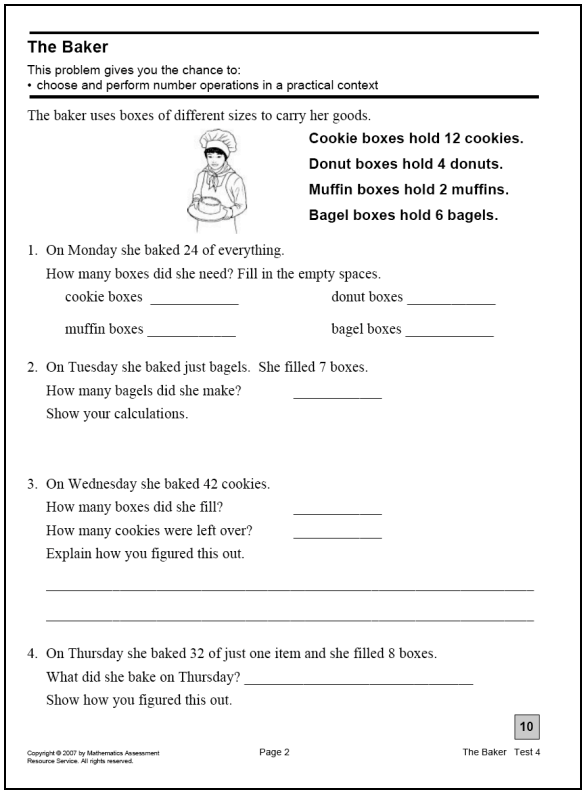
B 4

C 6

D 8

Performance Assessments

To Inform Instruction And Measure Higher Level Thinking



The Baker

This problem gives you the chance to:
• choose and perform number operations in a practical context

The baker uses boxes of different sizes to carry her goods.

Cookie boxes hold 12 cookies.
Donut boxes hold 4 donuts.
Muffin boxes hold 2 muffins.
Bagel boxes hold 6 bagels.

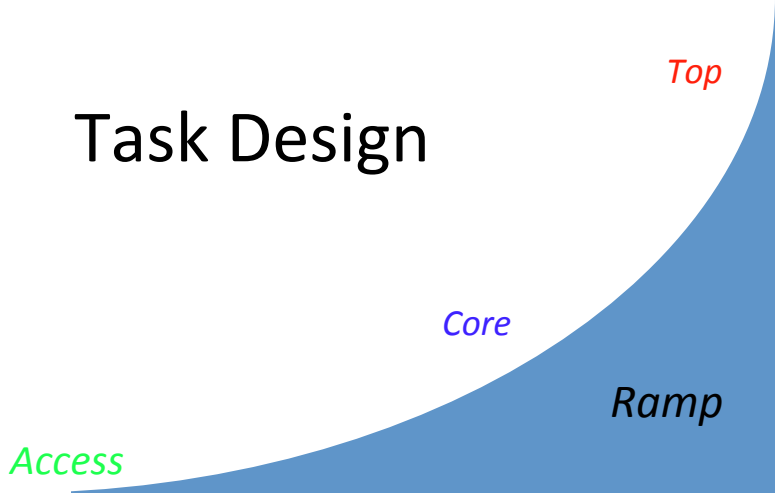
1. On Monday she baked 24 of everything.
   How many boxes did she need? Fill in the empty spaces.
   cookie boxes _____ donut boxes _____
   muffin boxes _____ bagel boxes _____

2. On Tuesday she baked just bagels. She filled 7 boxes.
   How many bagels did she make? _____
   Show your calculations.

3. On Wednesday she baked 42 cookies.
   How many boxes did she fill? _____
   How many cookies were left over? _____
   Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.
   What did she bake on Thursday? _____
   Show how you figured this out.

10



Entry level (access into task)

Core Mathematics - (meeting standards)

Top of Ramp (conceptually deeper, beyond)

- The Mathematics Assessment Resource Service (MARS) is an NSF funded collaboration between U.C. Berkeley and the Shell Centre in Nottingham England.
- The Assessments target grades 2- Geometry and are aligned with the State and NCTM National Math Standards.

SHELL CENTRE

BALANCED ASSESSMENT

MARS

Apprentice Task

CR 4: Baseball Jerseys

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks 2 local companies to give him a price.

1. 'Print It' will charge $21.50 each for the jerseys.

   Using *n* for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   _____

2. 'Top Print' has a Set-Up cost of $70 and then charges $18 for each jersey.

   Using *n* to stand for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   _____

3. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to order for the price from 'Top Print' to be less than from 'Print It'.
   Explain how you figured it out.

   _____

   _____

   _____

   _____

   _____

4. Bill decides to order 30 jerseys from 'Top Print'.
   How much more would the jerseys have cost if he had bought them from 'Print It'?
   Show all your calculations.

   _____

Baseball Jerseys

This problem gives you the chance to:

- work with equations that represent real life situations

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks two local companies to give him a price.

1. 'Print It' will charge \$21.50 each for the jerseys.

   Using n for the number of jerseys ordered, and c for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   _____

2. 'Top Print' has a one-time setting up cost of \$70 and then charges \$18 for each jersey.

   Using n to stand for the number of jerseys ordered, and c for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   _____

3. Bill decides to order 30 jerseys from 'Top Print'.
How much more would the jerseys cost if he buys them from 'Print It'?
Show all your calculations. _____

4. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to buy for the price from 'Top Print' to be less than from 'Print It'.
Explain how you figured it out. _____

_____

_____

_____

_____

Performance Exams

40,000 – 70,000 students per year since 1999

Students in grades 2 through 10th/11th grade are administered performance exams (5 apprentice tasks per exam).

Task 1: Candies		Rubric
The core elements of performance required by this task are: • work with fractions and ratios Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: 2/3 or 6/9	1	1
2. Gives correct answer: 3 Shows work such as: 1 + 3 = 4 12 ÷ 4 = Accept diagrams.	1 1	2
3 Gives correct answer: 18 Shows work such as: 2 + 3 = 5 30 ÷ 5 = 6 6 x 3 = Accept diagrams.	2 1	3
4. Gives correct answer: 6 Gives a correct explanation such as: Anthony mixes a ratio of one cup of cream to two cups of chocolate. The ratio stays the same for different amounts. So I wrote the numbers in a chart like this 1 to 2 = a total of 3 2 to 4 = a total of 6 3 to 6 = a total of 9 Accept diagrams.	1 1	2
	Total Points	8

District scoring leaders are trained in using task specific rubrics

Student tests are hand scored by classroom teachers trained and calibrated using standard protocols.

Random sample of student papers are audited and rescored by SJSU math & CS students. (Two reader correlation >0.95)

EDUCATIONAL DATA SYSTEMS

Student results are collected, analyzed, and reported by an independent data contractor.

Spring 2011 Trends Grade to Grade

Grade 2	MARS 1	MARS 2	MARS 3	MARS 4	Total
Far Below	1.0%	0.6%	0.1%	0.0%	1.7%
Below Basic	1.9%	4.1%	1.1%	0.1%	7.2%
Basic	0.8%	5.3%	4.6%	0.6%	11.3%
Proficient	0.4%	5.1%	16.2%	6.5%	28.2%
Advanced	0.2%	1.6%	15.2%	34.6%	51.6%
Total	4.3%	16.7%	37.2%	41.8%	100.0%

Grade 2	MARS Below	MARS At or ^	Total
CST Below	13.7%	6.5%	20.2%
CST AT or ^	7.3%	72.5%	79.8%
Totals	21.0%	79.0%	100.0%

Spring 2011 Trends Grade to Grade

Grade 3	MARS Below	MARS At or ^	Total
CST Below	16.4%	4.5%	20.9%
CST AT or ^	12.7%	66.3%	79.0%
Totals	29.1%	70.8%	99.9%

Grade 4	MARS Below	MARS At or ^	Total
CST Below	15.6%	5.8%	21.4%
CST AT or ^	12.9%	65.8%	78.7%
Totals	28.5%	71.6%	100.1%

Grade 5	MARS Below	MARS At or ^	Total
CST Below	17.3%	6.0%	29.7%
CST AT or ^	12.4%	64.4%	70.4%
Totals	23.3%	76.8%	100.1%

Grade 6	MARS Below	MARS At or ^	Total
CST Below	34.7%	3.8%	38.5%
CST AT or ^	21.7%	39.6%	61.3%
Totals	56.4%	43.4%	99.8%

Spring 2011 Trends Grade to Grade

Grade 7	MARS Below	MARS At or ^	Total
CST Below	38.1%	0.4%	38.5%
CST AT or ^	38.1%	23.5%	61.6%
Totals	76.2%	23.9%	100.1%

Grade 8	MARS Below	MARS At or ^	Total
CST Below	55.1%	2.8%	57.9%
CST AT or ^	25.0%	17.0%	42.0%
Totals	80.1%	19.8%	99.9%

Course 1	MARS Below	MARS At or ^	Total
CST Below	31.9%	4.1%	36.0%
CST AT or ^	21.5%	42.0%	63.5%
Totals	53.4%	46.1%	99.5%

Course 2	MARS Below	MARS At or ^	Total
CST Below	15.4%	0.0%	15.4%
CST AT or ^	36.0%	48.7%	84.7%
Totals	51.4%	48.7%	100.1%

8th Grade Geometry
California's Highest Achieving Students

Geometry	MARS Below	MARS At or Above	Total
CST Below	15.3%	0.0%	15.3%
CST AT or Above	36.0%	48.7%	84.7%
Totals	51.3%	48.7%	100%

Expert Tasks

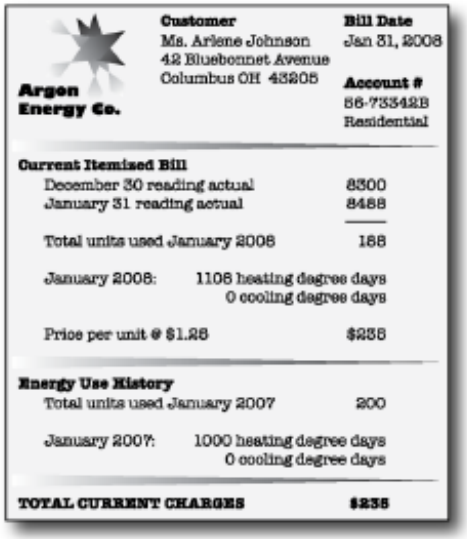
The main point in mathematics teaching is to develop the tactics of problem solving.

George Polya

Gas Bills, Heating Degree Days, and Energy Efficiency

Here is a typical story about an Ohio family concerned with saving money and energy by better insulating their house.

Kevin and Shana Johnson's mother was surprised by some very high gas heating bills during the winter months of 2007. To improve the energy efficiency of her house, Ms. Johnson found a contractor who installed new insulation and sealed some of her windows. He charged her $600 for this work and told her he was pretty sure that her gas bills would go down by "at least 10 percent each year." Since she had spent nearly $1,500 to keep her house warm the previous winter, she expected her investment would conserve enough energy to save at least $150 each winter (10% of $1,500) on her gas bills.



Argon Energy Co.

Customer
Ms. Arlene Johnson
42 Bluebonnet Avenue
Columbus OH 43205

Bill Date
Jan 31, 2008

Account #
86-73342B
Residential

Current Itemized Bill

December 30 reading actual	8300
January 31 reading actual	8488
Total units used January 2008	188
January 2008:	1106 heating degree days 0 cooling degree days
Price per unit @ $1.28	$235

Energy Use History

Total units used January 2007	200
January 2007:	1000 heating degree days 0 cooling degree days

TOTAL CURRENT CHARGES **$235**

Ms. Johnson's gas bill in January 2007 was $240. When she got the bill for January 2008, she was stunned that the new bill was $235. If the new insulation was going to save only $5 each month, it was going to take a very long time to earn back the $600 she had spent. So she called the insulation contractor to see if he had an explanation for what might have gone wrong. The contractor pointed out that the month of January had been very cold this year *and* that the rates had gone up from last year. He said her bill was probably at least 10% less than it would have been without the new insulation and window sealing.

Ms. Johnson compared her January bill from 2008 to her January bill from 2007. She found out that she had used 200 units of heat in January of 2007 and was charged $1.20 per unit (total = $240). In 2008, she had used 188 units of heat but was charged $1.25 per unit (total = $235) because gas prices were higher in 2008. She found out the average temperature in Ohio in January 2007 had been 32.9 degrees, and in January of 2008, the average temperature was more than 4 degrees colder, 28.7 degrees. Ms. Johnson realized she was doing well to have used less energy (188 units versus 200 units), especially in a month when it had been colder than the previous year.

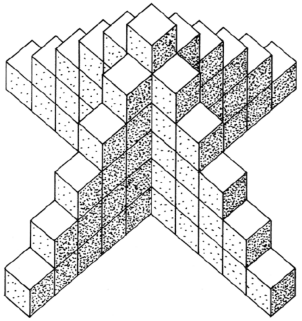
Since she used gas for heating only, Ms. Johnson wanted a better estimate of the savings due to the additional insulation and window sealing. She asked Kevin and Shana to look into whether the "heating degree days" listed on the bill might provide some insight.

Ohio Performance Assessment

Ohio Department of Education • Stanford University • ESC of Central Ohio

Pilot Project

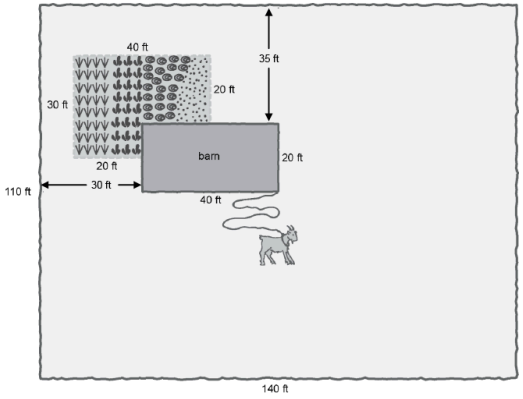
SALLY'S SUGAR STACK



Grazing Area

A farmer tethers her goat to the corner of a 40-by-20-foot barn in a fenced lot that is 140-by-110 feet. She also has an herb garden next to part of the barn. The goat is tethered on a 50-foot rope to the corner of the barn farthest from the herb garden.

After leaving the goat out on the rope for one day, the farmer discovers that a large area of her herb garden has been nibbled to the ground! Where can she tether the goat so that her herb garden is not within reach of the goat, but without decreasing the grazing area of the goat? The original grazing area includes all the grass area the goat could reach including that section of the herb garden that the goat ate.



Open for Business

Malena is a student who wants to raise $5,000 to tour South America next summer. To raise the money, she decides to open her own business on eBay.

The owner of an electronics shop offers to sell Malena some of his products at the wholesale price. She needs to decide which items to sell and how to price those items in order to maximize her profit.

She does some market research and finds the information provided in the table below about some of the items she is considering selling. Her research results include the cost to buy these items from the wholesale supplier, the retail price at which different items were sold at different times, and the number of items sold at these different prices during the month.

SCALE

Stanford Center for Assessment, Learning, & Equity

SRN LEADS STANFORD UNIVERSITY

Mathematics Claim #1
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

The measurement instrument will be novice [short] tasks. The items assessing this claim will account for 40% of the total score administered by computer.

Mathematics Claim #2
Students can frame and solve a range of complex problems in pure and applied mathematics.

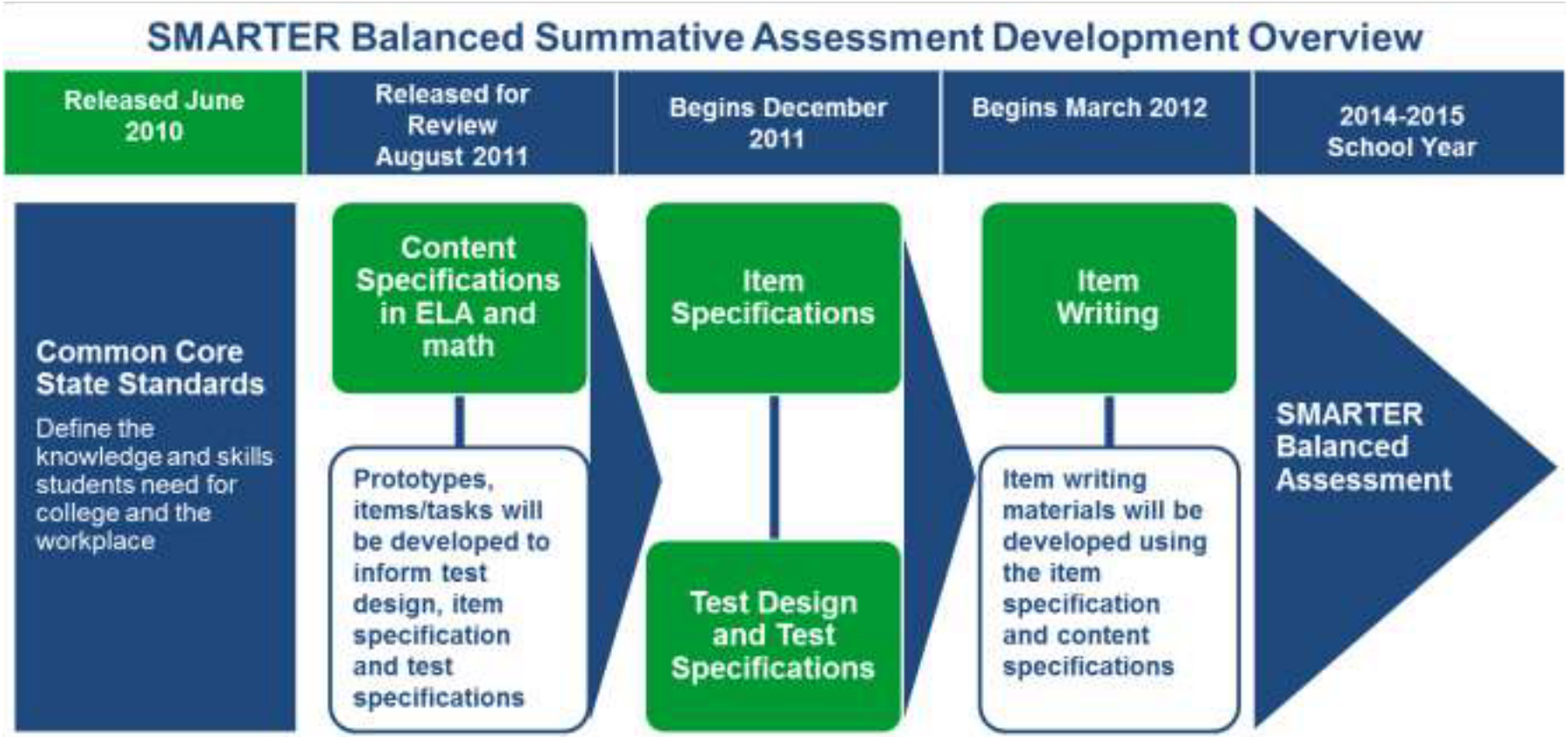
The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.

Mathematics Claim #3
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.

Mathematics Claim #4
Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.

The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.


SMARTER Balanced Summative Assessment Development Overview
Released June 2010
Released for Review August 2011
Begins December 2011
Begins March 2012
2014-2015 School Year
Common Core State Standards
Define the knowledge and skills students need for college and the workplace
Content Specifications in ELA and math
Prototypes, items/tasks will be developed to inform test design, item specification and test specifications
Item Specifications
Test Design and Test Specifications
Item Writing
Item writing materials will be developed using the item specification and content specifications
SMARTER Balanced Assessment

Problem Sources

Part I: Short items

1: MARS
2: MARS
3: SBAC
4: MARS
5: PISA
6: MARS
7: PISA
8: MARS
9: MARS
10: MARS
11: SBAC
12: SBAC
13: SBAC

Part II: Selected Response Tasks

CR 1: SBAC
CR 2: MARS
CR 3: MARS
CR 4: MARS
CR 5: MARS
CR 6: MARS
CR 7: MARS
CR 8: MARS
CR 9: MARS
CR 10: MARS

Part III: Extended Performance Task

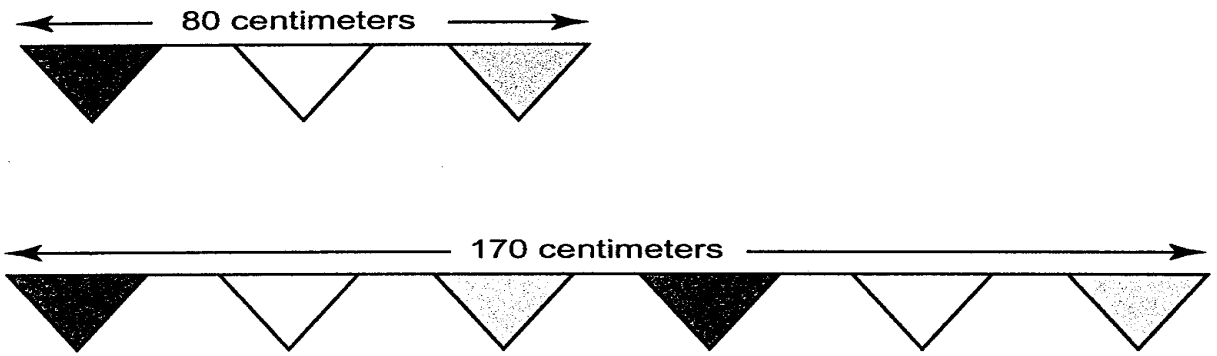
Ohio Department of Education and the Stanford University School Redesign Network

Teaching for Meaning

Party Flags

This problem gives you the chance to:

- find sizes by interpreting a diagram
- express a function by a formula



Erica is putting up lines of colored flags for a party.

The flags are all the same size and are spaced equally along the line.

1. Calculate the length of the sides of each flag, and the space between flags.

Show all your work clearly.

2. How long will a line of n flags be?

Write down a formula to show how long a line of n flags would be.

Algebra students had been working on system of linear equations for weeks.

$$6x + 5y = 170$$

$$3x + 2y = 80$$

$$6x + 5y = 170$$

$$-6x + -4y = -160$$

$$y = 10$$

The Findings from Party Flags

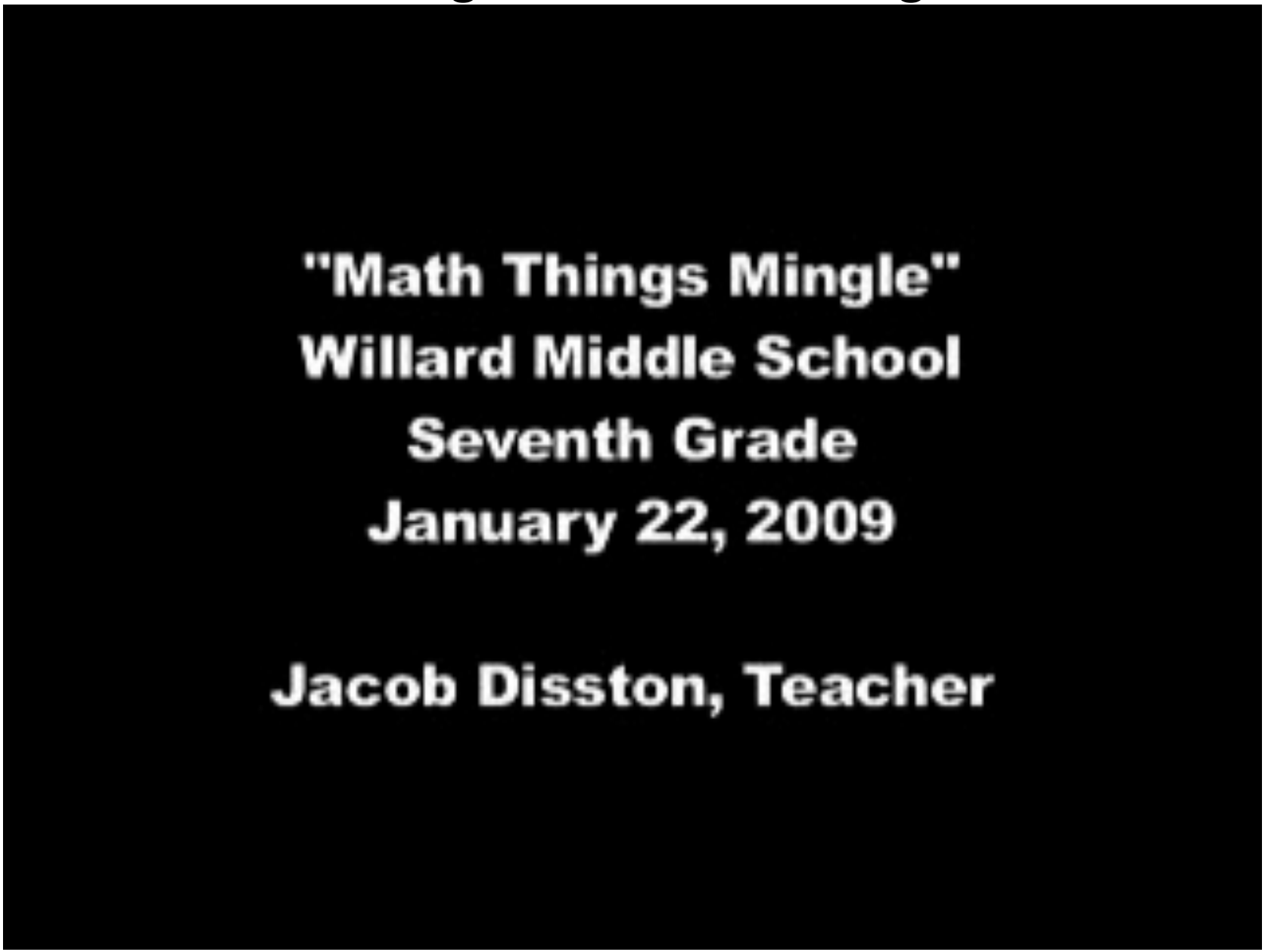
- The task may be approached as a system of simultaneous equations, almost no algebra students used such an approach.
- 49% of algebra students had no success.
- 44% accurately found the two lengths (most commonly by an estimation strategy only using one constraint).
- 21% correctly used both constraints (the length of three flags is 80 cm. and the length of 6 flags is 170 cm.).
- 7% of the students were able to develop a valid generalization for n flags.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

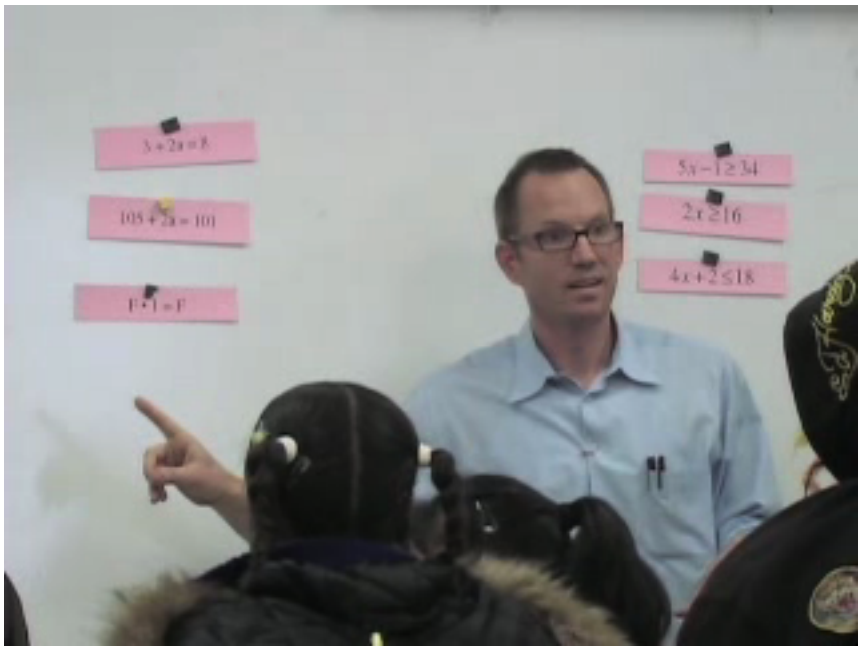
Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Teaching for Understanding?



Discuss the Lesson



In small groups discuss:

- How did the students make sense of the mathematics?
- What mathematical ideas did student struggle to understand?
- What did student seems to learn?
- What role did the teacher play in the lesson?
- Which mathematical practices were the students engaged in during the lesson?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Problems of the Month

A program to foster school-wide participation in math and problem solving.

George Polya, (1887 - 1985)
Father of Problem Solving;
"How to Solve It", 1945

Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.

Why a Problem of the Month?

- George Polya, said, “A problem is not a problem if you can solve it in 24 hours.”
- Doing math is solving non-routine problems.
- Perseverance and learning from mistakes are important attributes of good mathematicians.

How are the POM be used?

- The POM are used school wide to promote problem solving.
- Each problem is divided into five levels, A-E, to meet the learning development needs of all students.
- A great tool for *Differentiated Instruction*.
- Students, teachers and parents learn to ask questions and persevere in solving non-routine problems.
- The whole school celebrates doing mathematics at school.

Celebrating Problem Solving

School Wide Use of POM's

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Mathematical Practice

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8. **Look for and express regularity in repeated reasoning.**

Teachers facilitate problem solving -- asking good questions, encouraging perseverance, and probing for understanding.

The entire school does math together

Findings are shared through Group Collaboration, Individual Write-Up, Gallery Walks, and/or Presentations

Problem of the Month

Party Time

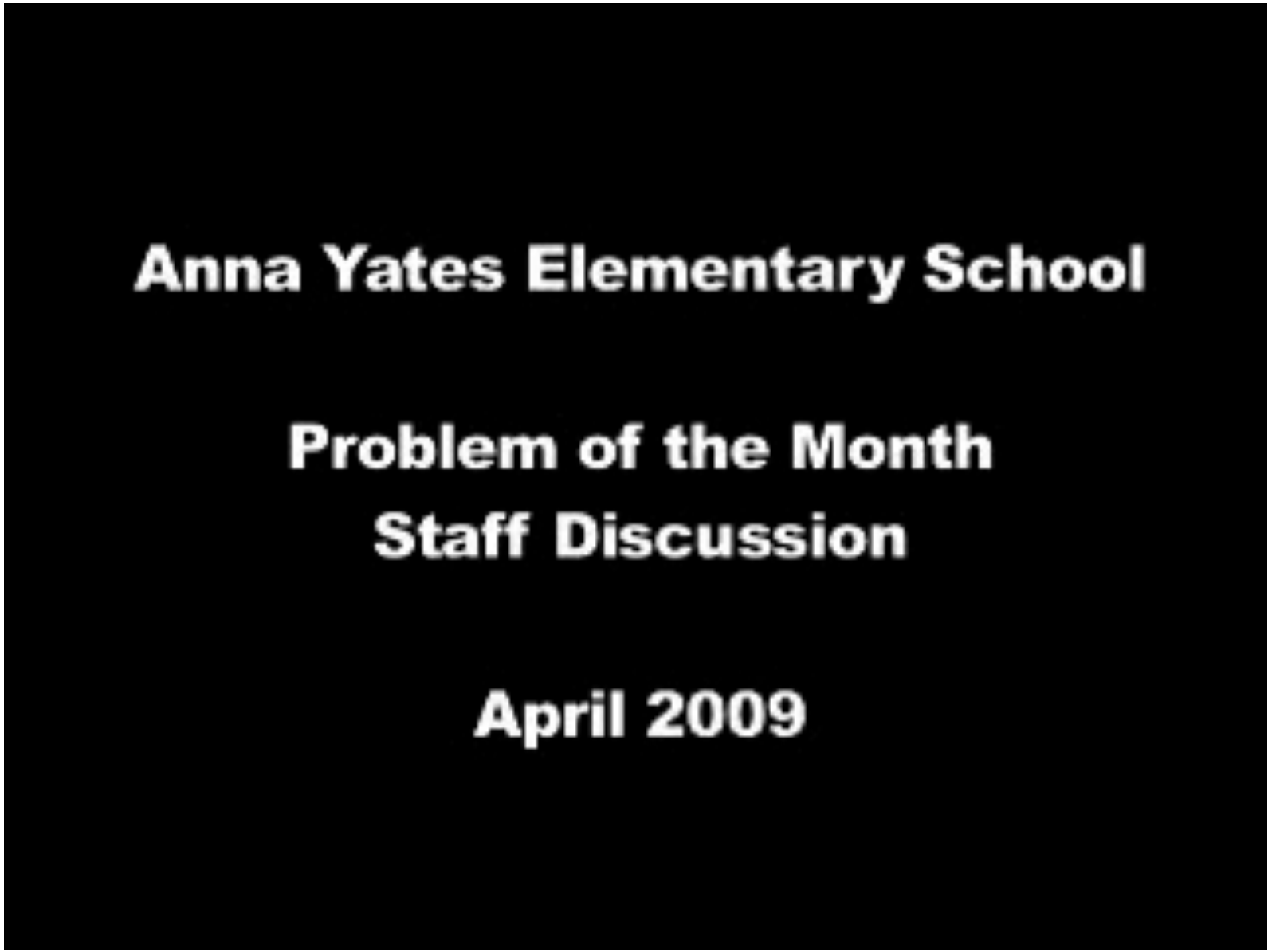
Level A

Cindy had a party. She invited two guests. Her guests each invited four guests, and then those guests each invited three guests.

How many people were at Cindy's party?

Explain how you determined your solution.

Teacher Discussion about using the Party Time POM



Problem of the Month

Party Time

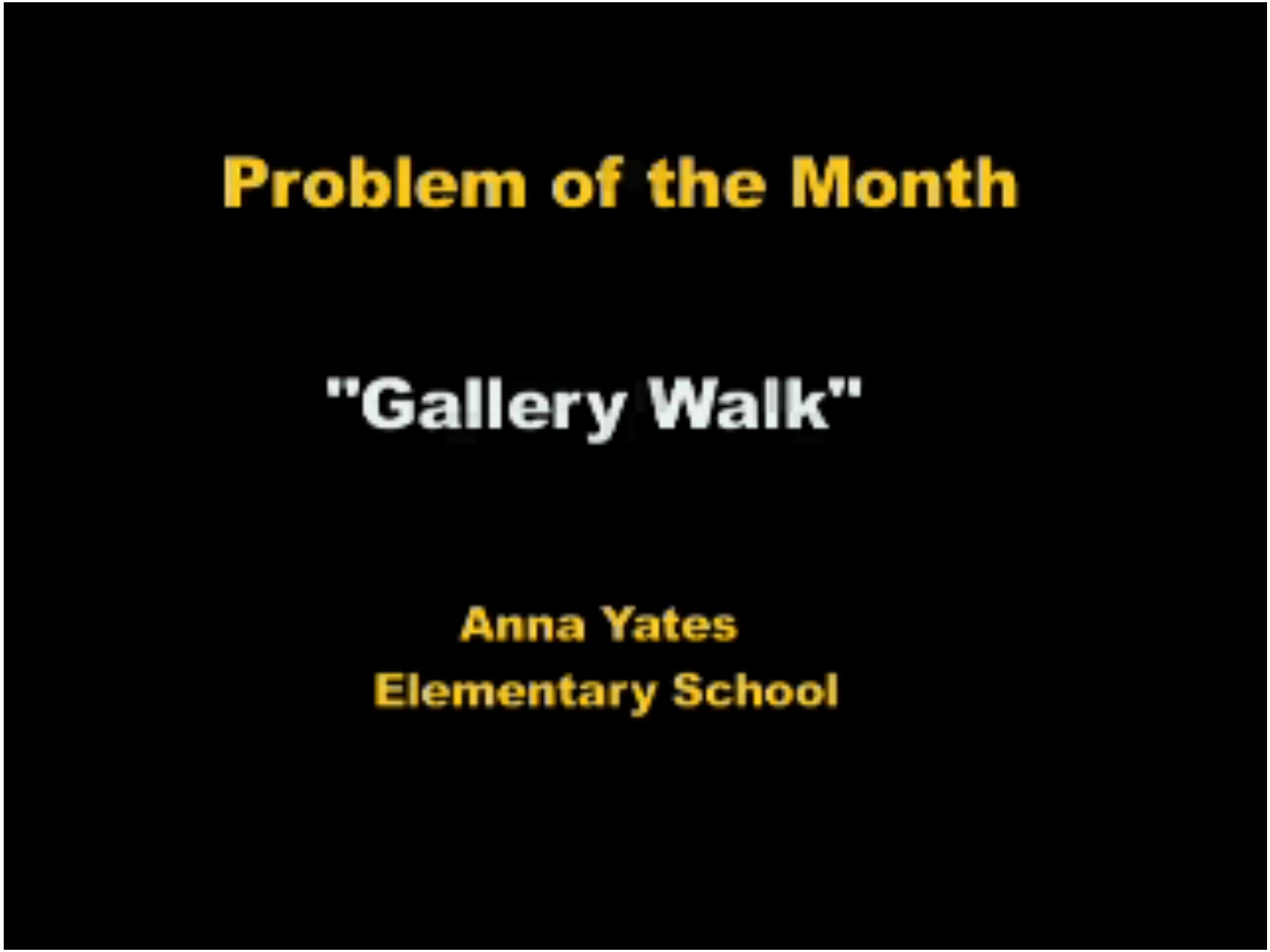
Level C

Mia, Jake, Carol, Barbara, Ford and Jeff are all going to a costume party. Figure out which person is wearing what costume and when they arrived at the party.

- The person that arrived fourth was wearing bathing suit.
- Barbara was the last to arrive.
- Jake and Mia arrived and stayed together.
- The first person was dressed as a French Maid.
- Superman arrived right before Barbara.
- The Potato Heads were always together at the party.
- Ford was a Surfer Dude.
- The French Maid was not Carol.
- The Vampire arrived after Superman.

Second Grade - Working on Party Time Level C

Gallery Walk for Party Time



"Mathematics is not a careful march down a well-cleared highway, but a journey into a strange wilderness, where the explorers often get lost."

Fermat's Enigma, p. 71

MATH TALKS

Math Talks

- A daily ritual with the entire class for the purpose of developing conceptual understanding of and efficiency with numbers, operations and other mathematics such as geometry and algebra. (no more than 10 minutes per day)

Math Talks are used to:

- Support active student discourse and discussions
- Review and practice procedures and concepts
- Introduce a concept before diving into the lesson of the day
- Support students in deepening their understanding of the number, operations and algebraic thinking.
- Explore mathematical connections and relationships
- Encourage students to construct viable arguments and critique the reasoning of others
- Support students in using precise mathematical language in sharing their different strategies and approaches

Today's Number

36

Possible Solutions

$18 + 18$

$3^2 \cdot 2^2$

$9+9+9+9$

$25.65+10.35$

$9 \div 1/4$

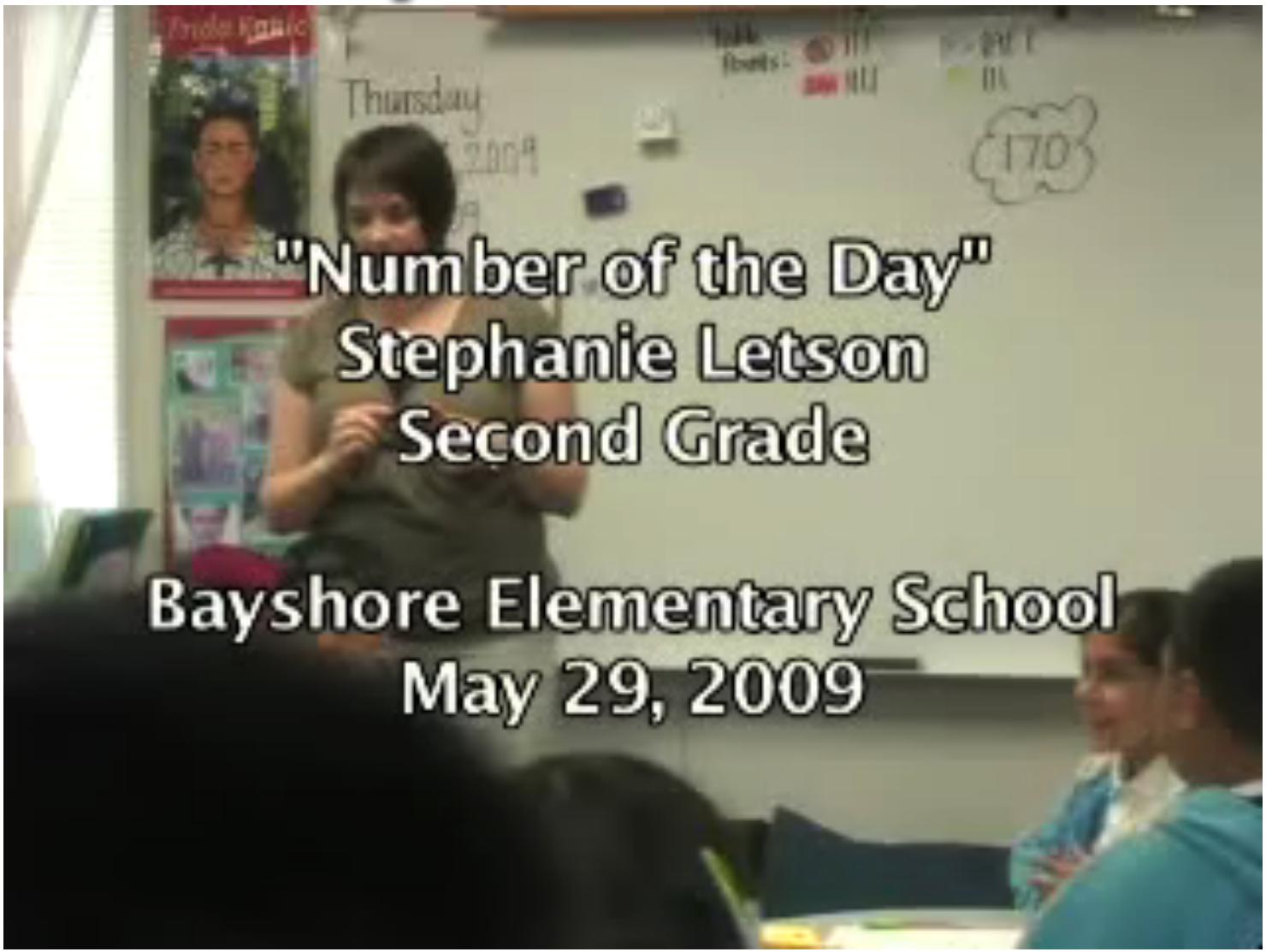
$-15 + 51$

$3\sqrt{144}$

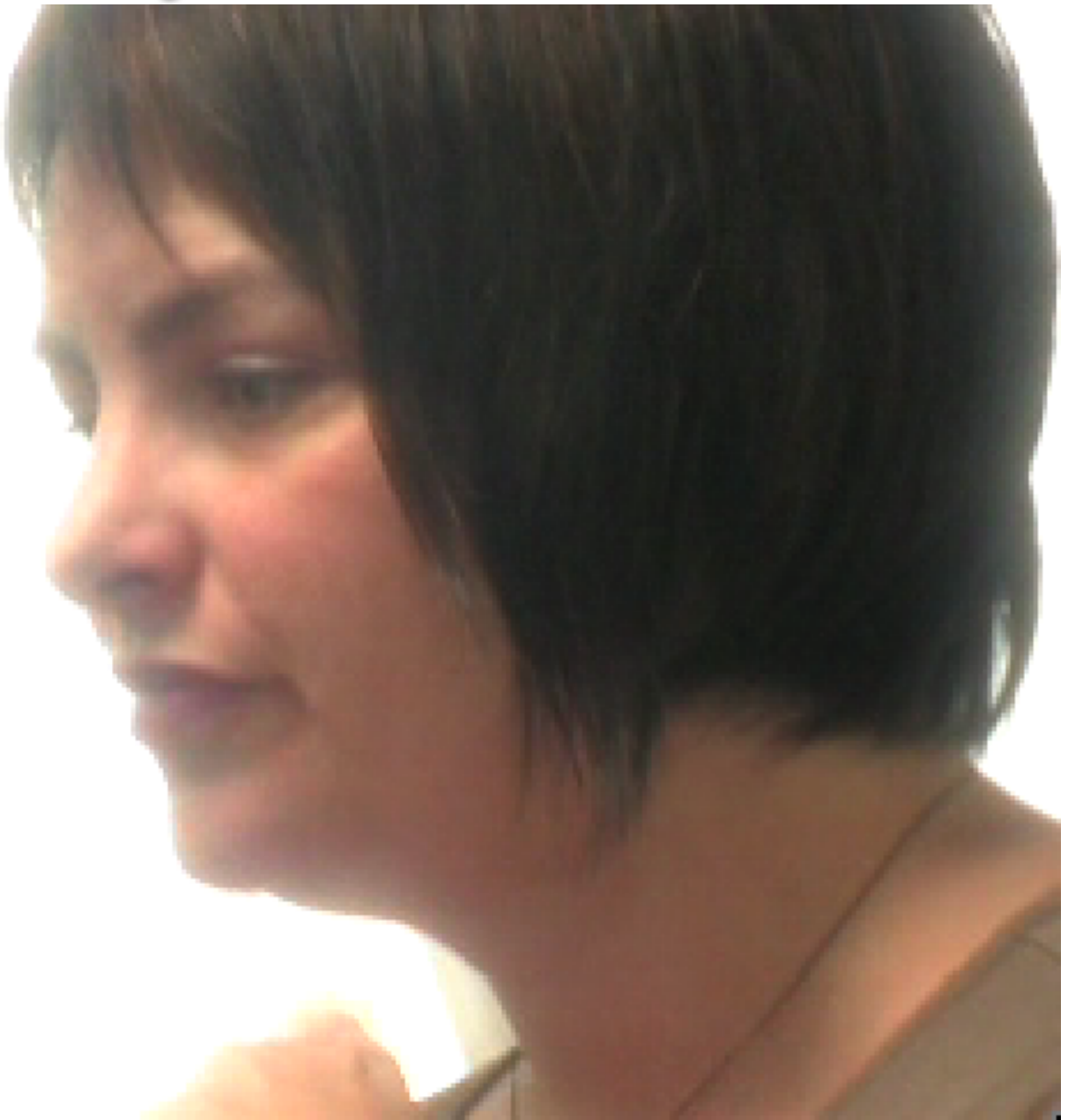
Today's Number with Constraints

- More than one operation
- Using Two digit numbers (3 digits etc.)
- Using Fractions, Decimals, Percents
- Using sets of numbers and operations
- Using exponents, square roots
- Using integers (sign numbers)
- Using a set of numbers and different operations.

Today's Number



Today's Number



Discuss the Number Talk Video

- Who did the math thinking during the number talk?
- What specific mathematics did the students demonstrate they understood?
- What did the teacher do to support the student discourse?
- What recording techniques did the teacher employ that supported learning in the class?

Curriculum inspired by the CCSS

MAP's Formative Assessment Lessons and Professional Development Modules

BILL & MELINDA GATES *foundation*

MARS

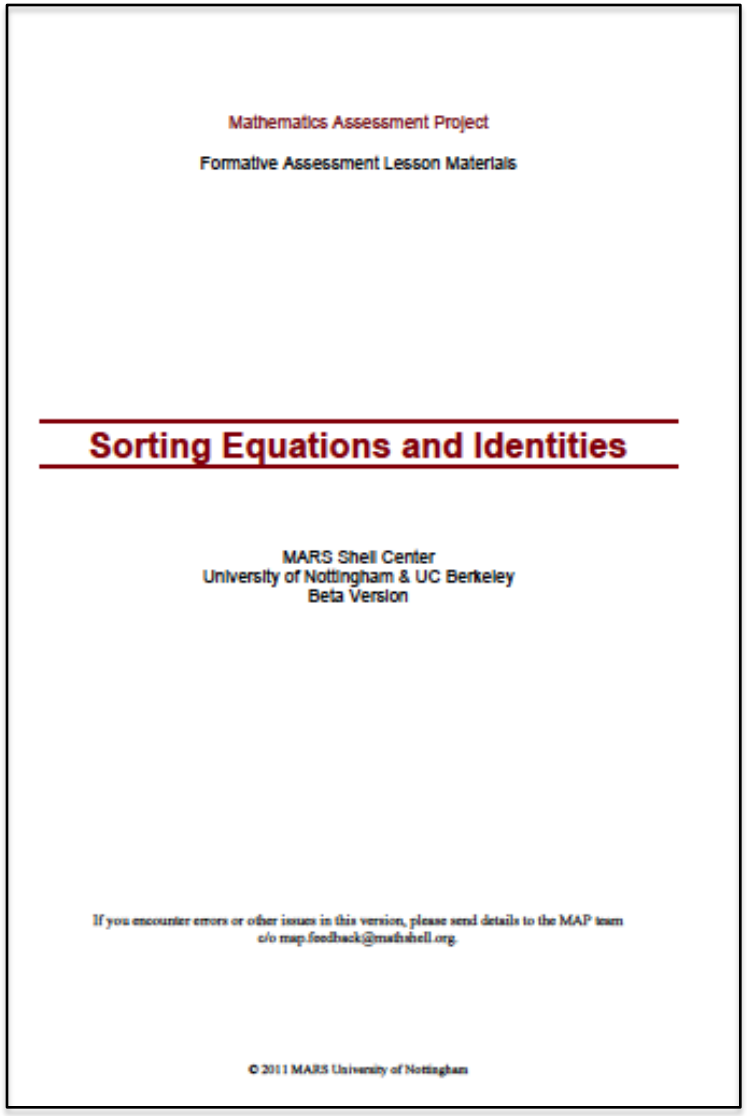
MARS Team

Mathematics Assessment Resource Service

Assessment For Learning

Formative Assessment Lessons (2 days) for High School and Middle School

Sorting Equations and Identities



Mathematics Assessment Project

Formative Assessment Lesson Materials

Sorting Equations and Identities

MARS Shell Center
University of Nottingham & UC Berkeley
Beta Version

If you encounter errors or other issues in this version, please send details to the MAP team
c/o map.feedback@mathshell.org.

Sorting Equations and Identities

Mathematical goals

This lesson unit is intended to help you assess how well students are able to:

- Recognize the differences between equations and identities.
- Substitute numbers into algebraic statements in order to test their validity in special cases.
- Resist common errors when manipulating expressions such as $2(x-3) = 2x-3$; $(x+3)^2 = x^2+3^2$.
- Carry out correct algebraic manipulations.

It also aims to encourage discussion on some common misconceptions about algebra.

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

A-SSE: Interpret the structure of expressions.
Write expressions in equivalent forms to solve problems.
A-REI: Solve equations and inequalities in one variable.

This lesson involves a range of *mathematical practices*, with emphasis on:

3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

Pre-Assessment Task

Equations and Identities

1. Write down an example of an equation that has:

   (a) One solution. .....

   (b) Two solutions. .....

   (c) An infinite number of solutions. .....

   (d) No solutions. .....

2. For each of the following statements, indicate whether it is "Always true," "Never true," or "Sometimes true." Circle the correct answer. If you choose "Sometimes true" then state on the line below when it is true. The first one is done for you as an example.

$x+2=3$	Always true	Never true	Sometimes true
	It is true when $x=1$.		
$x-12=x+30$	Always true	Never true	Sometimes true
	It is true when _____.		
$2(x+6)=2x+12$	Always true	Never true	Sometimes true
	It is true when _____.		
$3(x-2)=3x-2$	Always true	Never true	Sometimes true
	It is true when _____		
$(x+4)^2=x^2+4^2$	Always true	Never true	Sometimes true
	It is true when _____.		
$x^2+4=0$	Always true	Never true	Sometimes true
	It is true when _____.		

3. Which of the equations in question 2 are also identities?

   .....

   .....

   In your own words, explain what is meant by an identity.

   .....

   .....

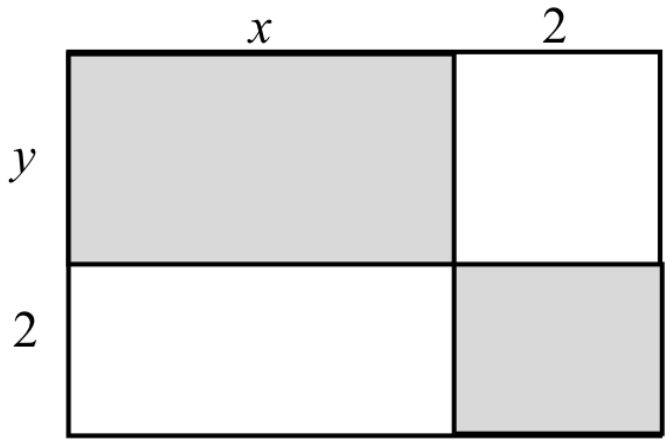
Common issues:	Suggested questions and prompts:
Student writes expressions rather than equations For example: The student writes $y + 3$ for an equation with an infinite number of solutions.	• *What is the difference between an equation and an expression?* • *How can you change your expression to an equation?*
Student fails to include a variable in their equation For example: The student has written $5 + 5 = 10$ as an example of an equation with one solution.	• *Can you include an unknown number or a variable in the equation so that we can look at all possible values of that unknown?*
Student fails to provide an example of an equation with an infinite number of solutions	• *What would an equation with an infinite number of solutions look like?*
Student provides a quadratic with non-integer solutions as an example of an equation with no solutions For example: The student gives $x^2 + 8x + 13 = 0$ as an answer to Q1d. The student has assumed that, because it won't factorize there are no solutions.	• *Can a quadratic equation that will not factorize still have solutions/cross the x-axis? How can you check whether or not a quadratic equation has solutions?*
Student assumes that $-(x^2)$ is the same as $(-x)^2$ For example: The student classifies $x^2 + 4 = 0$ as true when $x = -2$.	• *What does $(-x)^2$ mean? What kind of number do we get when we multiply two negative numbers together?* • *Is x^2 positive or negative?*
Student correctly answers all the questions The student needs an extension task.	• *Use algebra to justify one of your answers to Question 2.* • *Draw a diagram to justify one of your answers to Question 2.*

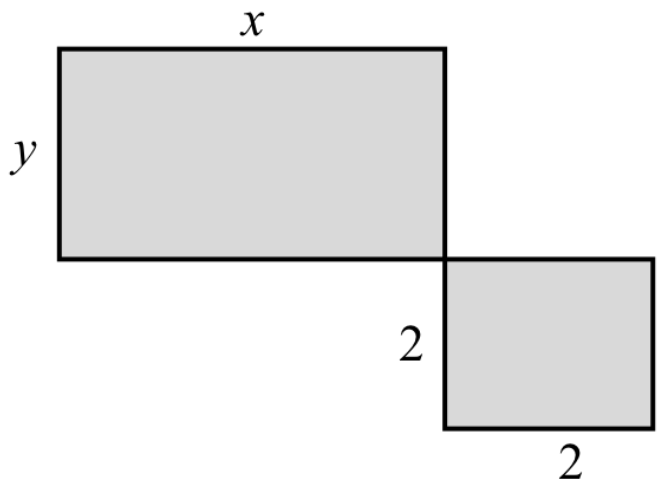
Is this Always, Sometimes or Never True?

$$(X + 2)(Y + 2) = XY + 4$$

Always, Sometimes, or Never True?

$$(x + 2)(y + 2) = xy + 4$$





Card Set: Always, Sometimes, or Never True?

1 $x-6=6-x$	2 $x+6=y+6$
3 $\frac{x}{6}=\frac{6}{x}$	4 $6+2x=8x$
5 $2(x-3)=2x-3$	6 $2(x+3)=2x+6$
7 $\frac{x+6}{2}=x+3$	8 $x^2=2x$
9 $(x+3)^2=x^2+3^2$	10 $(x-6)^2=(6-x)^2$
11 $(3x)^2=9x^2$	12 $x^2-1=(x+1)(x-1)$
13 $x^2+6=0$	14 $(x+1)(x+4)=x^2+14$

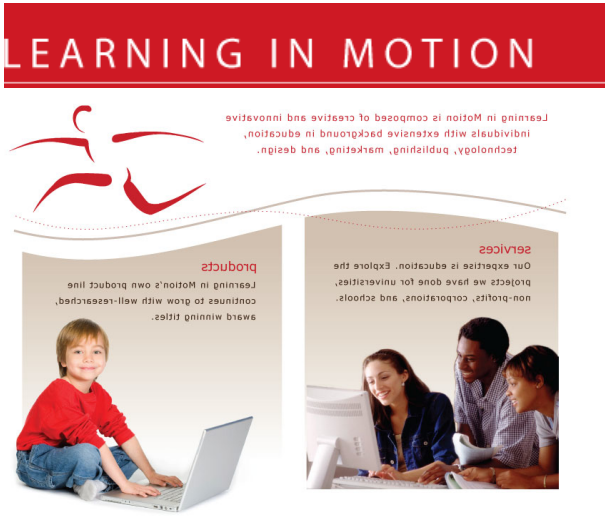
Always, Sometimes, or Never True?

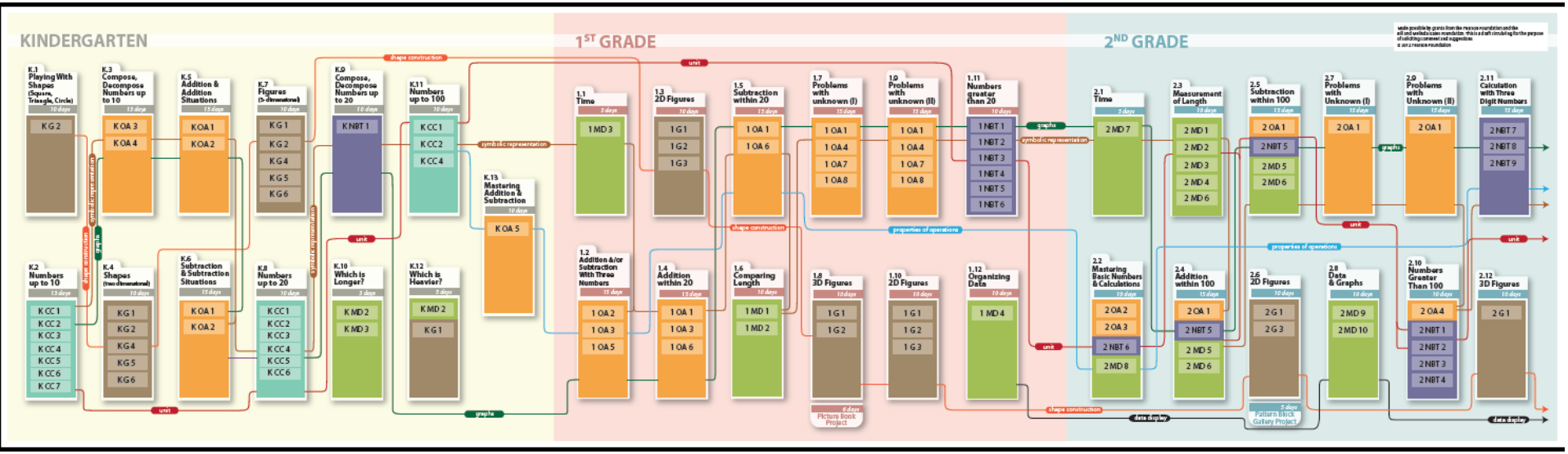
- In your groups, take turns to place a card in a column and justify your answer to your partner.
- If you think the equation is sometimes true, you will need to find values of x for which it is true and values of x for which it is not true.
- If you think the equation is always true or never true, you will need to explain how we can be sure that this is the case.
- Another member of the group should then either explain that reasoning again in his or her own words, or challenge the reasons you gave.
- When the entire group agrees, glue the card onto your poster. Write the reason for your choice of category next to the card.

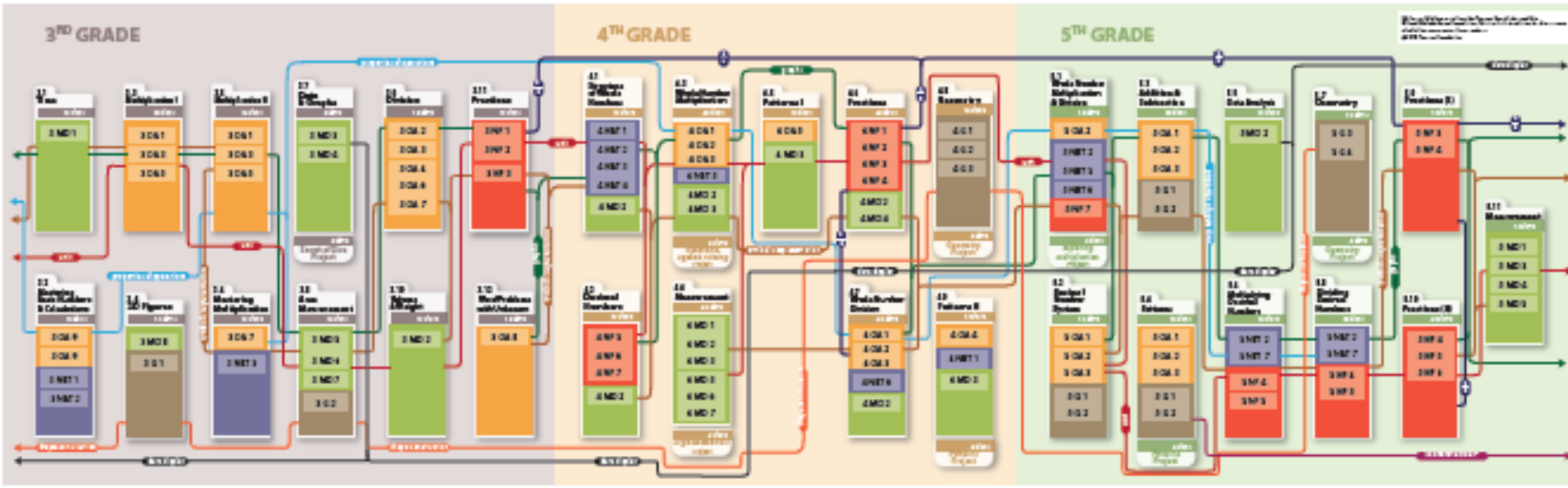
New K-12 Math Curriculum Inspired by The Common Core State Standards

BILL & MELINDA GATES foundation

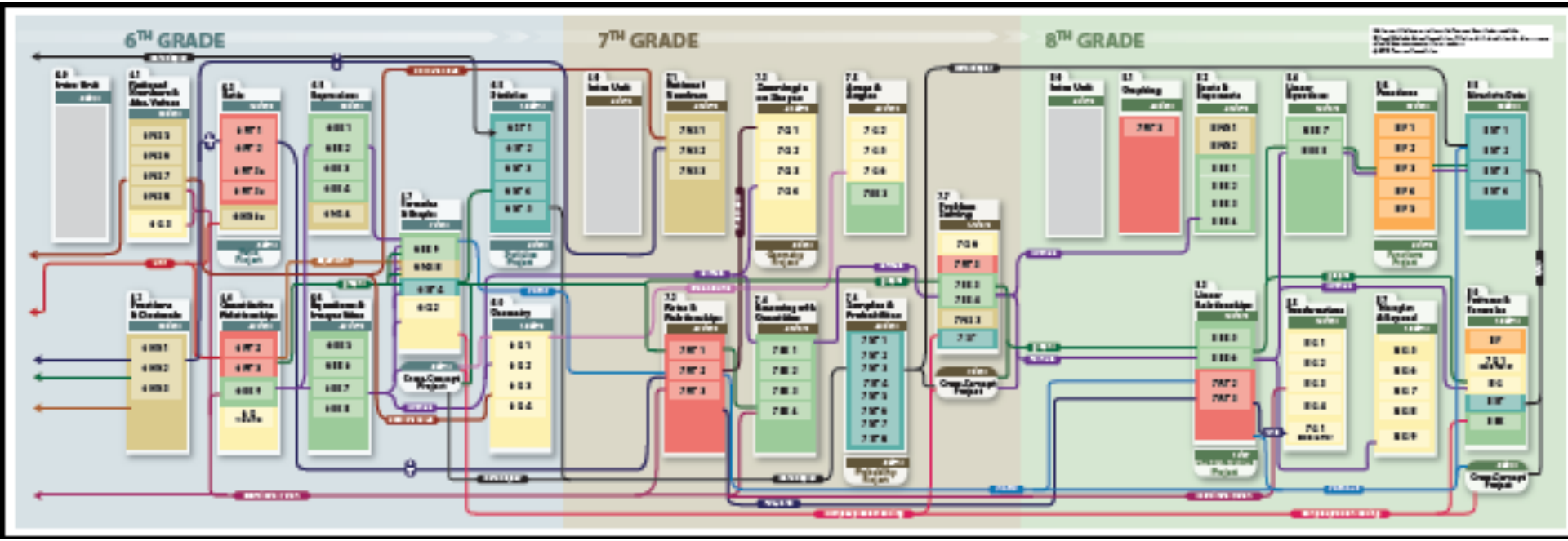
PEARSON FOUNDATION



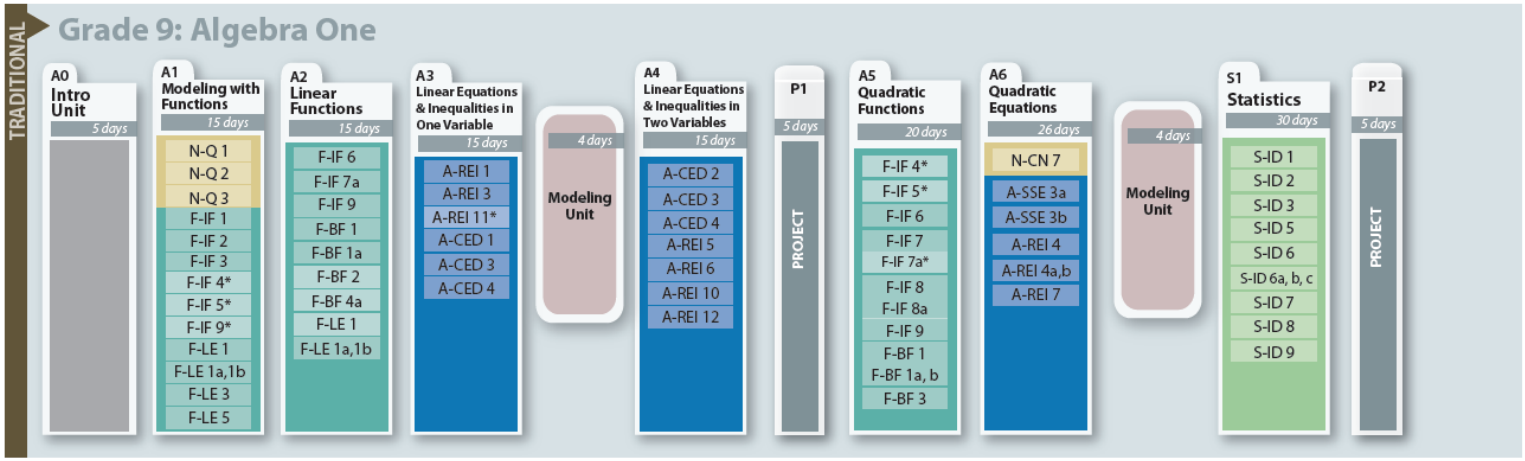

KINDERGARTEN
1ST GRADE
2ND GRADE
K.1 Playing With Shapes (Square, Triangle, Circle)
K.2 Numbers up to 10
K.3 Compose, Decompose Numbers up to 10
K.4 Shapes (two dimensional)
K.5 Addition & Addition Situations
K.6 Subtraction & Subtraction Situations
K.7 Figures (3-dimensional)
K.8 Numbers up to 20
K.9 Compose, Decompose Numbers up to 20
K.10 Which is Longer?
K.11 Numbers up to 100
K.12 Which is Heavier?
K.13 Mastering Addition & Subtraction
1.1 Time
1.2 Addition &/or Subtraction With Three Numbers
1.3 2D Figures
1.4 Addition within 20
1.5 Subtraction within 20
1.6 Comparing Length
1.7 Problems with unknown (I)
1.8 3D Figures
1.9 Problems with unknown (II)
1.10 2D Figures
1.11 Numbers greater than 20
1.12 Organizing Data
Picture Book Project
2.1 Time
2.2 Mastering Basic Numbers & Calculations
2.3 Measurement of Length
2.4 Addition within 100
2.5 Subtraction within 100
2.6 2D Figures
2.7 Problems with Unknown (I)
2.8 Data & Graphs
2.9 Problems with Unknown (II)
2.10 Numbers Greater Than 100
2.11 Calculation with Three Digit Numbers
2.12 3D Figures
Pattern Block Gallery Project
shape construction
symbolic representation
graphs
unit
properties of operations
data display

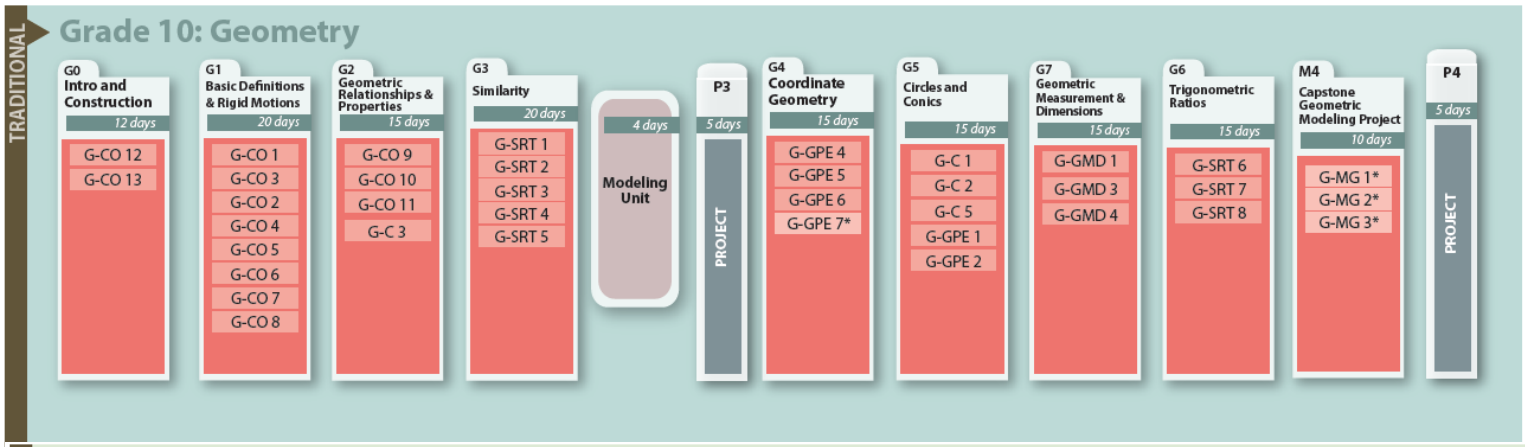

3RD GRADE
4TH GRADE
5TH GRADE

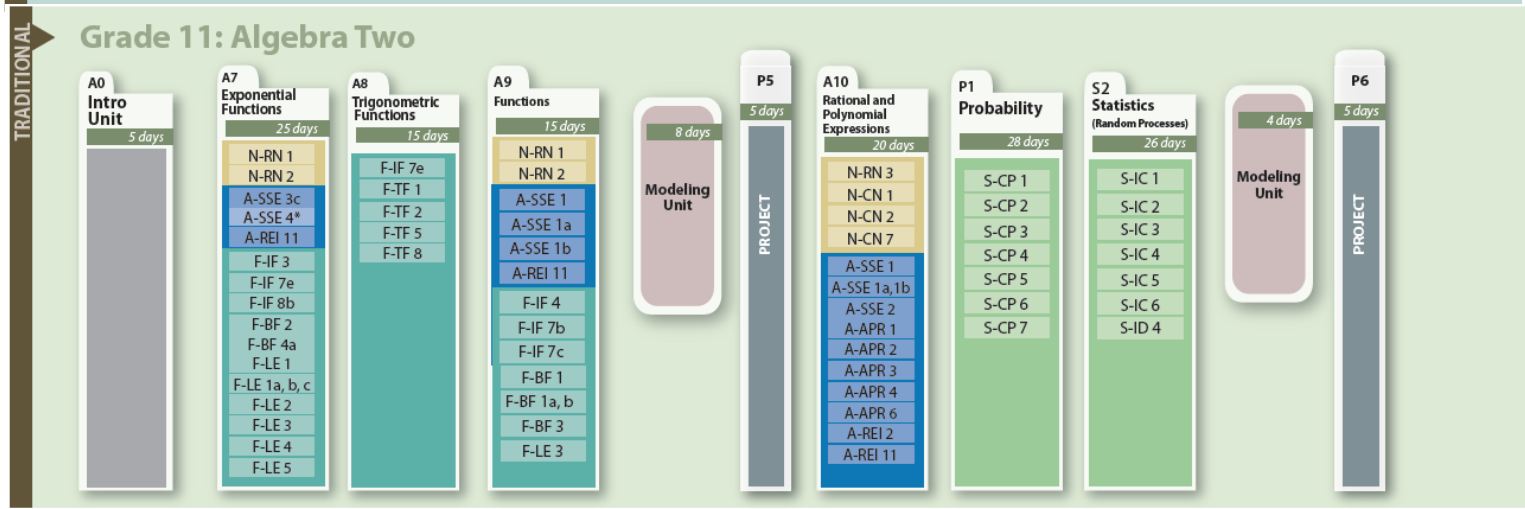
Middle School Curriculum



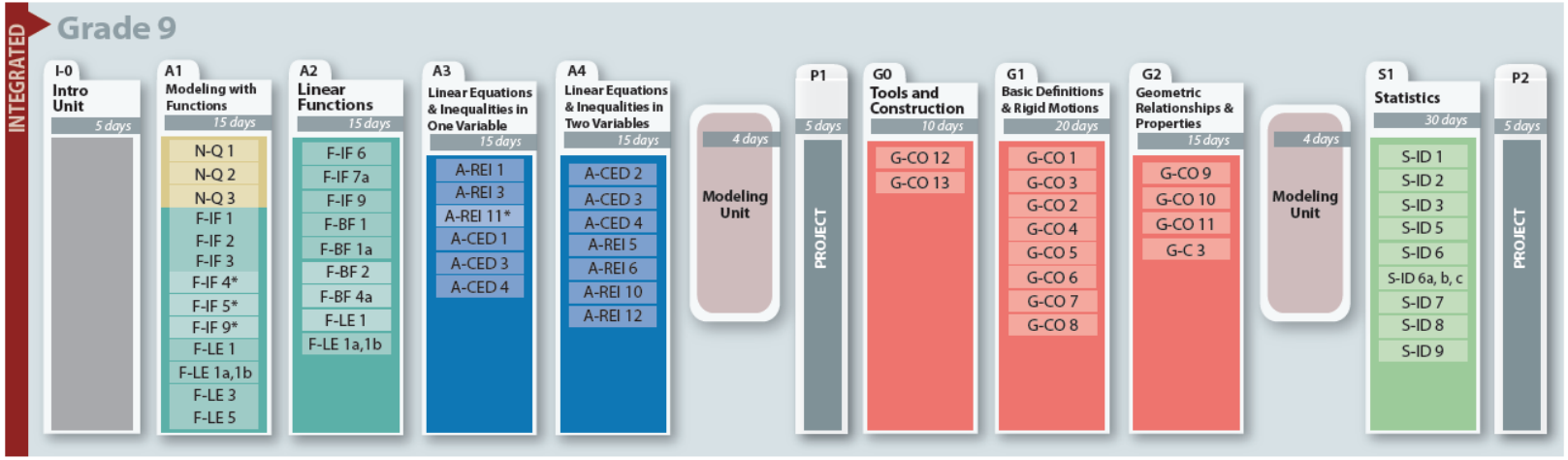
TRADITIONAL

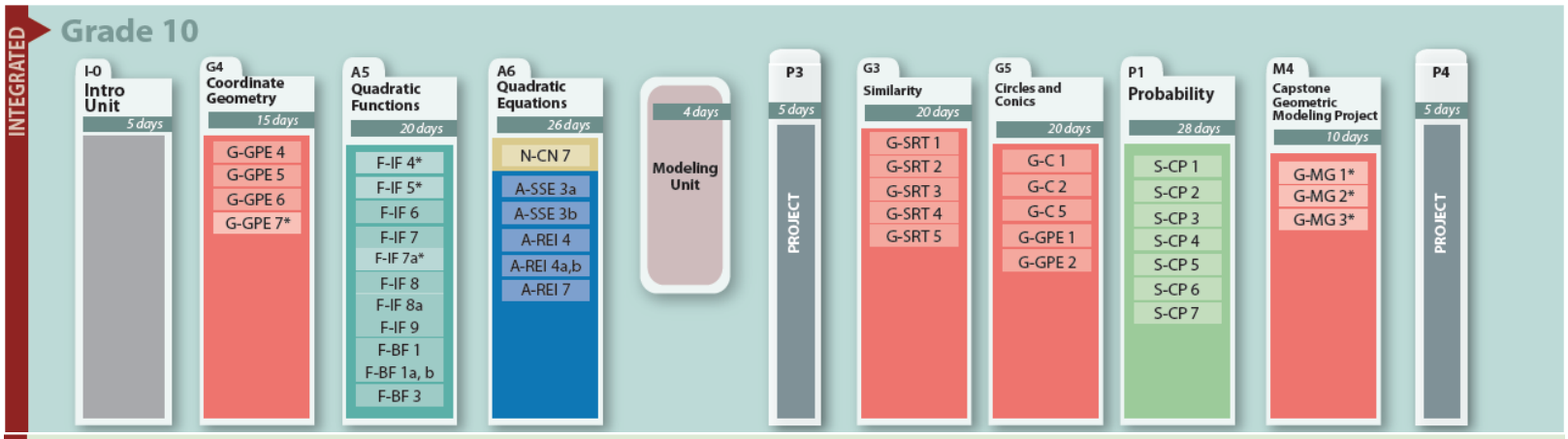


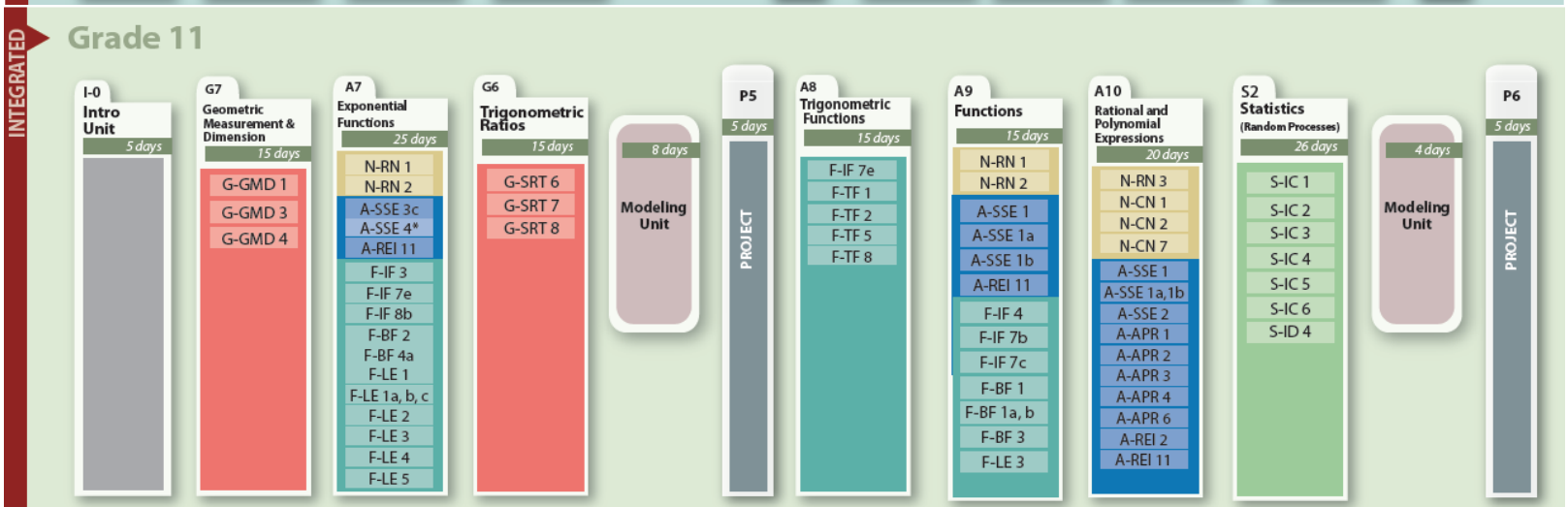




INTEGRATED


INTEGRATED
Grade 9
I-0
Intro Unit
5 days
A1
Modeling with Functions
15 days
N-Q 1
N-Q 2
N-Q 3
F-IF 1
F-IF 2
F-IF 3
F-IF 4*
F-IF 5*
F-IF 9*
F-LE 1
F-LE 1a,1b
F-LE 3
F-LE 5
A2
Linear Functions
15 days
F-IF 6
F-IF 7a
F-IF 9
F-BF 1
F-BF 1a
F-BF 2
F-BF 4a
F-LE 1
F-LE 1a,1b
A3
Linear Equations & Inequalities in One Variable
15 days
A-REI 1
A-REI 3
A-REI 11*
A-CED 1
A-CED 3
A-CED 4
A4
Linear Equations & Inequalities in Two Variables
15 days
A-CED 2
A-CED 3
A-CED 4
A-REI 5
A-REI 6
A-REI 10
A-REI 12
4 days
Modeling Unit
P1
5 days
PROJECT
G0
Tools and Construction
10 days
G-CO 12
G-CO 13
G1
Basic Definitions & Rigid Motions
20 days
G-CO 1
G-CO 3
G-CO 2
G-CO 4
G-CO 5
G-CO 6
G-CO 7
G-CO 8
G2
Geometric Relationships & Properties
15 days
G-CO 9
G-CO 10
G-CO 11
G-C 3
4 days
Modeling Unit
S1
Statistics
30 days
S-ID 1
S-ID 2
S-ID 3
S-ID 5
S-ID 6
S-ID 6a, b, c
S-ID 7
S-ID 8
S-ID 9
P2
5 days
PROJECT


INTEGRATED
Grade 10
I-0
Intro Unit
5 days
G4
Coordinate Geometry
15 days
G-GPE 4
G-GPE 5
G-GPE 6
G-GPE 7*
A5
Quadratic Functions
20 days
F-IF 4*
F-IF 5*
F-IF 6
F-IF 7
F-IF 7a*
F-IF 8
F-IF 8a
F-IF 9
F-BF 1
F-BF 1a, b
F-BF 3
A6
Quadratic Equations
26 days
N-CN 7
A-SSE 3a
A-SSE 3b
A-REI 4
A-REI 4a,b
A-REI 7
4 days
Modeling Unit
P3
5 days
PROJECT
G3
Similarity
20 days
G-SRT 1
G-SRT 2
G-SRT 3
G-SRT 4
G-SRT 5
G5
Circles and Conics
20 days
G-C 1
G-C 2
G-C 5
G-GPE 1
G-GPE 2
P1
Probability
28 days
S-CP 1
S-CP 2
S-CP 3
S-CP 4
S-CP 5
S-CP 6
S-CP 7
M4
Capstone Geometric Modeling Project
10 days
G-MG 1*
G-MG 2*
G-MG 3*
P4
5 days
PROJECT


INTEGRATED
Grade 11
I-0
Intro Unit
5 days
G7
Geometric Measurement & Dimension
15 days
G-GMD 1
G-GMD 3
G-GMD 4
A7
Exponential Functions
25 days
N-RN 1
N-RN 2
A-SSE 3c
A-SSE 4*
A-REI 11
F-IF 3
F-IF 7e
F-IF 8b
F-BF 2
F-BF 4a
F-LE 1
F-LE 1a, b, c
F-LE 2
F-LE 3
F-LE 4
F-LE 5
G6
Trigonometric Ratios
15 days
G-SRT 6
G-SRT 7
G-SRT 8
8 days
Modeling Unit
P5
5 days
PROJECT
A8
Trigonometric Functions
15 days
F-IF 7e
F-TF 1
F-TF 2
F-TF 5
F-TF 8
A9
Functions
15 days
N-RN 1
N-RN 2
A-SSE 1
A-SSE 1a
A-SSE 1b
A-REI 11
F-IF 4
F-IF 7b
F-IF 7c
F-BF 1
F-BF 1a, b
F-BF 3
F-LE 3
A10
Rational and Polynomial Expressions
20 days
N-RN 3
N-CN 1
N-CN 2
N-CN 7
A-SSE 1
A-SSE 1a,1b
A-SSE 2
A-APR 1
A-APR 2
A-APR 3
A-APR 4
A-APR 6
A-REI 2
A-REI 11
S2
Statistics
(Random Processes)
26 days
S-IC 1
S-IC 2
S-IC 3
S-IC 4
S-IC 5
S-IC 6
S-ID 4
4 days
Modeling Unit
P6
5 days
PROJECT

Inside Mathematics Website

http://www.insidemathematics.org

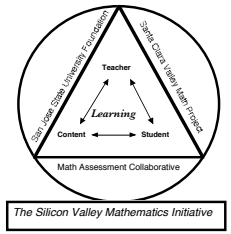
Mathematics Assessment Pro

UC Berkeley & Shell Centre for Mathematical Education

MARS Team
Mathematics Assessment Resource Service

http://map.mathshell.org/materials/lessons.php

Silicon Valley Mathematics Initiative



http://www.svmimac.org

"Don't be encumbered by history-- go off and do something wonderful."

Dr. Robert N. Noyce

Inventor of the Silicon Chip

Co-founder of Intel

Inside Mathematics Website

http://www.insidemathematics.org

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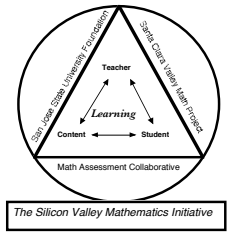


http://www.svmimac.org

1. Make sense of problems and persevere in solving them.

Traffic jam

Last Sunday an accident caused a traffic jam 12 miles long on a freeway

When the accident was cleared, the cars drove away from the front, one car every two seconds. Estimate how long it took before the last car moved.

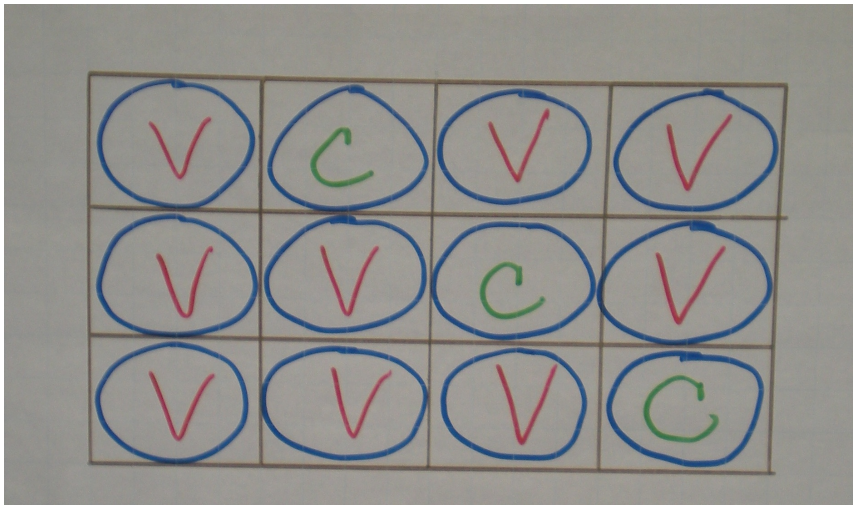
2. Reason Abstractly and Quantitatively

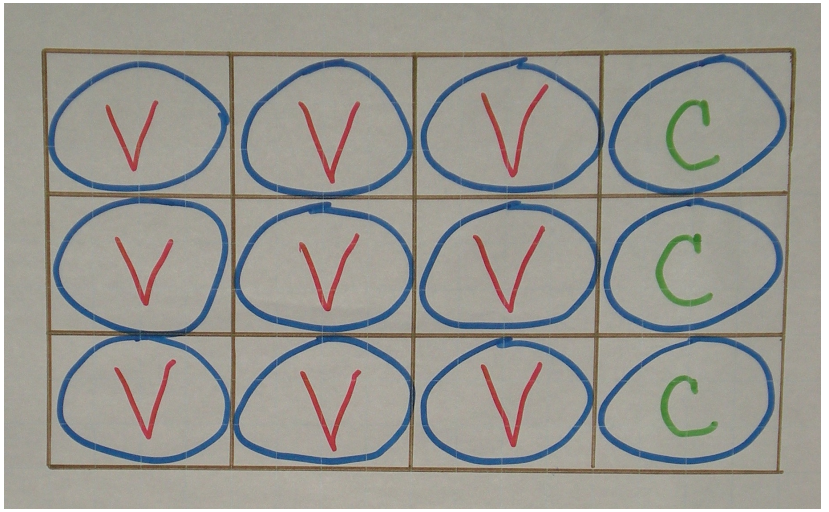
Where are a+b, b-a and a-b?

What can you say about where a/b is?

3. Construct viable arguments and critique the reasoning of others.

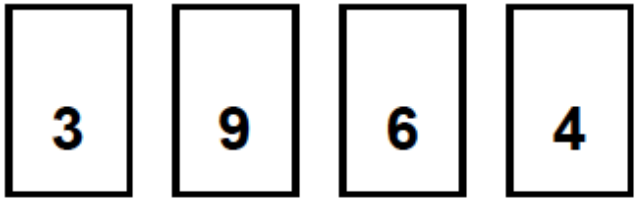
Valerie shares some of the 12 candies. She gives Cindy 1 candy for every 3 candies she eats herself. How many does she give Cindy?



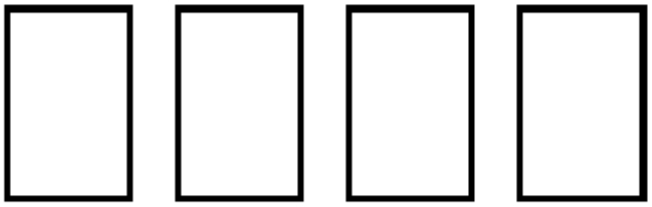


3. Construct viable arguments and critique the reasoning of others.

Jane and Tom are playing number card games. They have the four cards shown below.



5. Show how they arrange the four cards to make the number that is nearest to **5000**.



Explain how you figured it out.

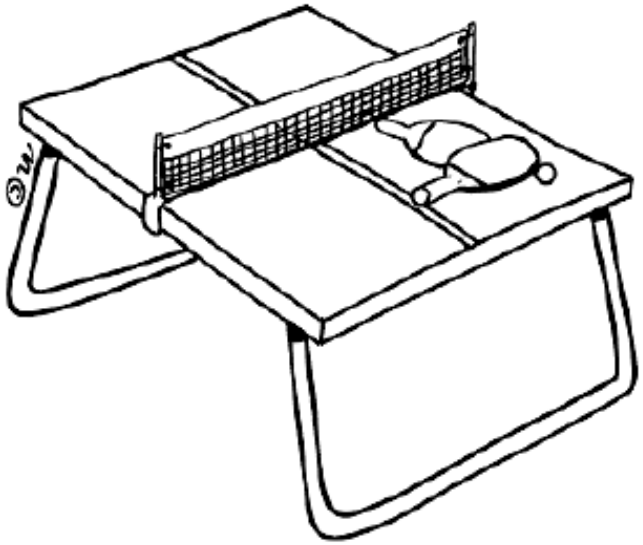
_____

_____

_____

4. Model with mathematics

PLANNING AND ORGANIZING: A TABLE TENNIS TOURNAMENT

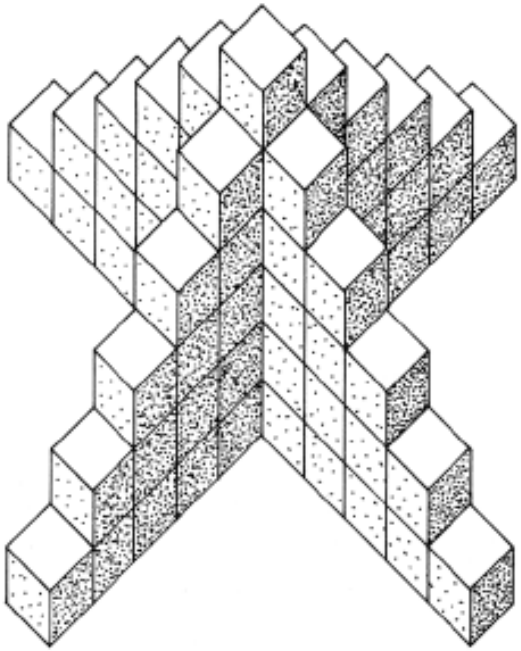


You have the job of organising a table tennis league.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.

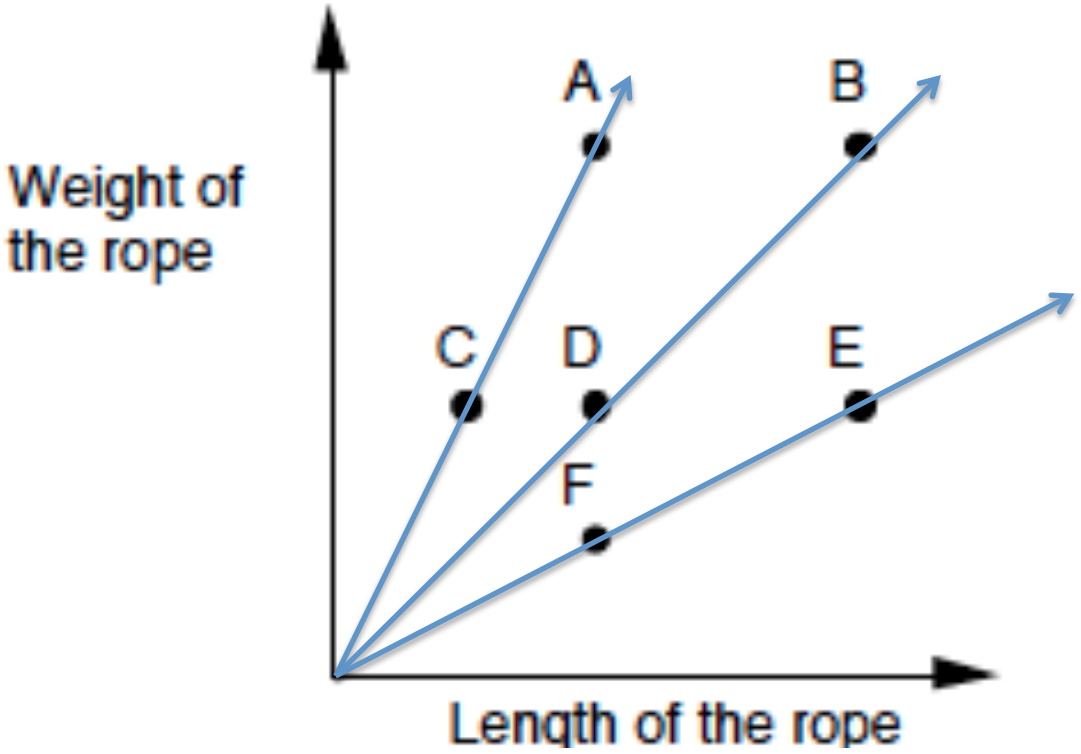
4. Model with mathematics

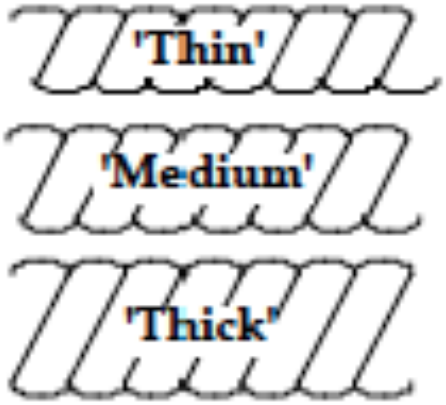
SKELETON TOWER



1. How many cubes are needed to build this tower?
2. How many cubes are needed to build a tower like this, but 12 cubes high? Explain how you work out your answer.
3. How would you calculate the number of cubes needed for a tower n cubes high?

5. Use appropriate tools strategically





Which ropes are ‘Thin’?
Which ropes are ‘Medium’?
Which ropes ‘Thick’?

Explain your reasoning.

6. Attend to Precision

- Precision in Calculations
- Precision in Vocabulary
- Precision in Argumentation

6. Attend to Precision

Imagine that you have just discovered this ancient floor tiling pattern in Syria.

You telephone New York to tell them about this exciting discovery.

Describe the pattern as accurately as you can, so that someone else can draw it without seeing it.

Describe the shapes as completely as you can.

_____

_____

_____

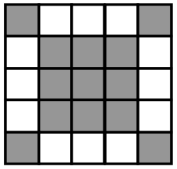
_____

7. Look for and make use of structure

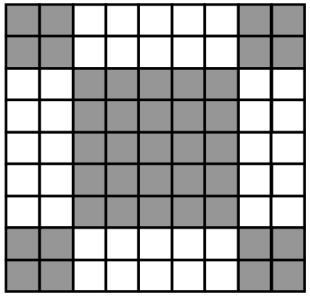
Sidewalk Patterns

In Prague some sidewalks are made of small square blocks of stone.

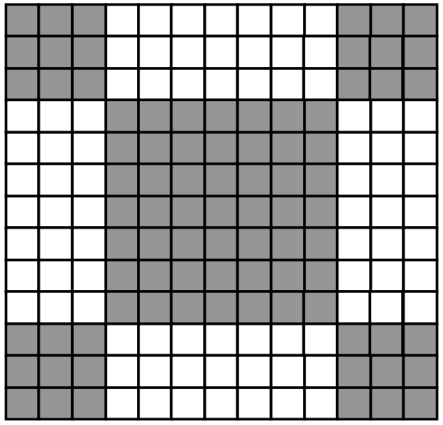
The blocks are in different shades to make patterns that are in various sizes.



Pattern



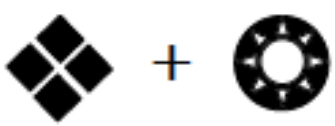
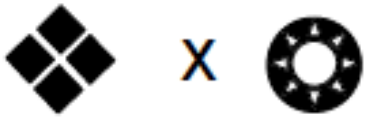
Pattern number



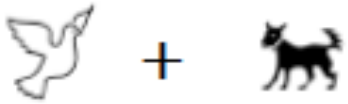
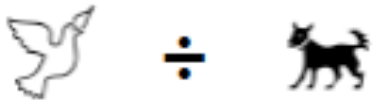
Pattern number 3

Write formulas for the number W of white tiles, and G of grey tiles, in terms of the pattern number n.

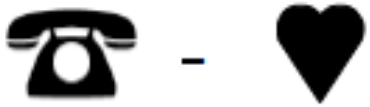
7. Look for and make use of structure

 and 

____ + ____ = 7 and ____ x ____ = 12

 and 

____ + ____ = 10 and ____ ÷ ____ = 4

 and 

____ ÷ ____ = 3 and ____ - ____ = 6

8. Look for and express regularity in repeated reasoning

Explain why the answer to:

$$4/5 \div 2/3$$

Makes sense knowing why 45 ÷ 8 makes sense.

8. Look for and express regularity in repeated reasoning

When Aaron plays the numbers game, he needs to decide which numbers belong in each set. Here is another game for you to play.

2. All of these numbers are Grogs.

123	789	456	345

None of these numbers are Grogs.

121	81	246	5678

Which of the numbers below are Grogs? Draw a circle around each Grog.

234 56 678 989

Explain how you know which numbers are Grogs and which are not.

_____

_____