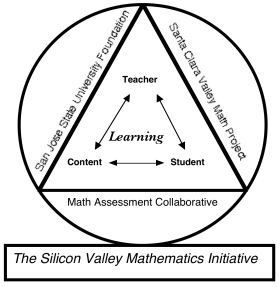
# Common Core Standards: Math

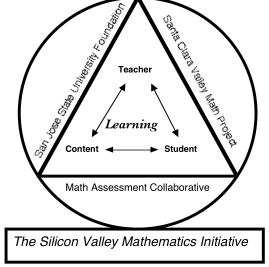
## How The CCSS Will Impact Teaching, Learning And Assessment In Mathematics

**David Foster**
*Silicon Valley Mathematics Initiative*
**www.svmimac.org**



# *Silicon Valley Mathematics Initiative*

## *83 Members - School Districts, Charter School Networks*, and Schools




Albany USD
Alvord SD (Riverside County)
Antioch Unified SD
Aspire Charter School Network
Assumption School (San Leandro)
Bayshore SD
Belmont-Redwood Shores SD
Berryessa SD
Bolinas – Lagunitas SD
Brisbane SD
Buckeye SD
Cambrian SD
Castro Valley USD
Charter School of Morgan Hill
Chicago Public School
Creative Arts Charter (SF)
CSU San Bernardino
Cotati – Rohnert Park
Cupertino SD
Dade County Schools (GA)
Discovery Charter School
Dioceses of Santa Clara
Dublin USD
East Side UHSD
Edmonds Community College
Emery SD
Etiwanda SD (San Bernardino Co)
Gilroy (Brownell MS)
Fairfield-Suisun USD
Fremont USD
Forsyth County School (GA)
Hamilton County (Tn)
Hayward USD
Jefferson ESD
Jefferson HSD
Las Lomitas SD
La Honda-Pescadero Sd
Livermore USD
Los Altos SD
Los Gatos SD
Menlo Park SD
Monterey Peninsula USD
Moreland SD
Mountain SD
National Council of La Raza
New York City PS
New Visions for Public Schools
Oakland Unified SD
Pacifica SD
Pajaro Valley USD
Palo Alto USD
Pittsburgh USD
Portola Valley SD
Ravenswood City SD
Riverside COE
Redwood City Schools
Sacramento City USD
Salinas City Schools
San Carlos Charter Learning Center
San Francisco USD
SMFC (Park School)
San Jose Unified SD
San Leandro USD
San Ramon Valley USD
Santa Clara USD
Santa Cruz City Schools
Saint Michael's School (Poway)
Saint Patrick's School (San Jose)
Saratoga
Scotts Valley USD
SCCOE County Court Schools
Sequoia HSD
SMCOE County Court Schools
South Cook Service District
South San Francisco USD
Sumter County (GA)
The Nueva School
Union SD
University of Illinois, Chicago
Valley Christen (Dublin)
Valdosta City (GA)
Walnut Creek SD
Woodside SD

***Supporting Teaching and Learning of Mathematics Since 1996***

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

# Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

# Problems of the Month


A program to foster school-wide participation in math and problem solving.

George Polya, (1887 - 1985)
Father of Problem Solving;
*"How to Solve It"*, 1945

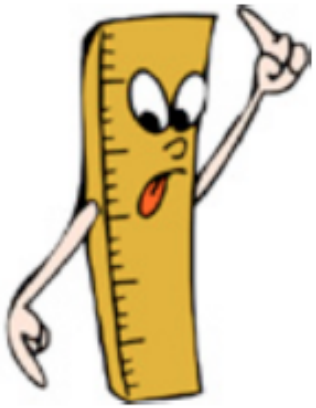
*Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.*

# Why a Problem of the Month?

- George Polya, said, “A problem is not a problem if you can solve it in 24 hours.”
- Doing math is solving non-routine problems.
- Perseverance and learning from mistakes are important attributes of good mathematicians.

# How are the POM be used?

- The POM are used school wide to promote problem solving.
- Each problem is divided into five levels, A-E, to meet the learning development needs of all students.
- A great tool for *Differentiated Instruction*.
- Students, teachers and parents learn to ask questions and persevere in solving non-routine problems.
- The whole school celebrates doing mathematics at school.



# Problem of the Month
# Measuring Up

- Individually read through all the levels (A-E)
- Consider what big idea ties the mathematical ideas together.
- Once you have read all levels, begin working on Level A.
- Work through a level completely to find problems that challenge your thinking.
- The goal is not to necessarily finish all levels.

# Create a Poster on Your Findings on One of the Levels of the POM

- Select a Level of the POM to share your findings.
- Feel free to choose any level.
- The focus of your poster should be more on how your findings can be justified mathematically and your findings make sense.

# Create a Status Check Poster of your Findings on a Level you are still exploring

- Select a Level of the POM you are still exploring.
- The focus of your poster should be more on your processes so far and where you think you want to go next and/or questions/wonderings you have about this level.
- Remember to justify or explain your process you have so far and why they make mathematical sense.

# NORMS FOR A GALLERY WALK

All discussion and conversation in a gallery walk is:

- About what each of us can learn from each other
- Respectful of ALL work

- The FOCUS of a gallery walk is on the MATHEMATICS of the problem:
  - What is the mathematics of the poster
  - Was the thought process the same as yours? If no, what is different?
  - Is the representation the same as yours? If no, what is different?
  - What questions might you pose to the "author[s]" to clarify your own understanding of the mathematics presented?
  - What mathematics contributes to your own understanding?
  - What did you find mathematically interesting?
  - What did you find mathematically challenging?
  - What mathematics presented would you like to engage in?

# Gallery Walk

- Each group will display their poster.
- Pick a docent to stay with your poster(s) and answer clarifying questions about your group's process
- Other group members examine, explore, review the other groups' posters. Please take notes for further discussion.
- There will be time for your group to re-assemble and discuss the information shared in the groups' posters.
- Please mind gallery walk norms and be respectful of the work and information shared.

# Celebrating Problem Solving

School Wide Use of POM's

Teachers facilitate problem solving -- asking good questions, encouraging perseverance, and probing for understanding.

# The entire school does math together

Findings are shared through Group Collaboration, Individual Write-Up, Gallery Walks, and/or Presentations


# Common Core Standards:

## *A New Direction linking Instruction and Assessment*

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

# Mathematical Practice

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2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
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6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

# *States have joined Assessment Consortia funded by RttT*


Partnership for Assessment of Readiness of College and Careers (PARCC) is being managed by Achieve, Inc., a Washington-based non-profit. There are 23 states and DC in PARCC.


The SMARTER-Balanced Assessment Consortium is being managed by San Francisco-based WestEd and its senior program director, Stanley Rabinowitz. SMARTER-Balanced enlisted 31 states.

At this point, both consortia are targeting the first test administration by 2014-15. Both say they will integrate summative or end-of-the-year tests with interim and formative assessments that can guide instruction during the year. Both are promising to include performance-based tasks, such as conducting a science experiment and writing short answers to questions, that are intended to show deeper levels of learning and thinking than multiple choice questions supposedly can measure. Both indicate that technology will play a major role.

# Goals of Assessment

"We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics."

Everybody Counts

SMARTER
Balanced Assessment Consortium

SMARTER

State of Washington OSPI
Office of Superintendent of Public Instruction

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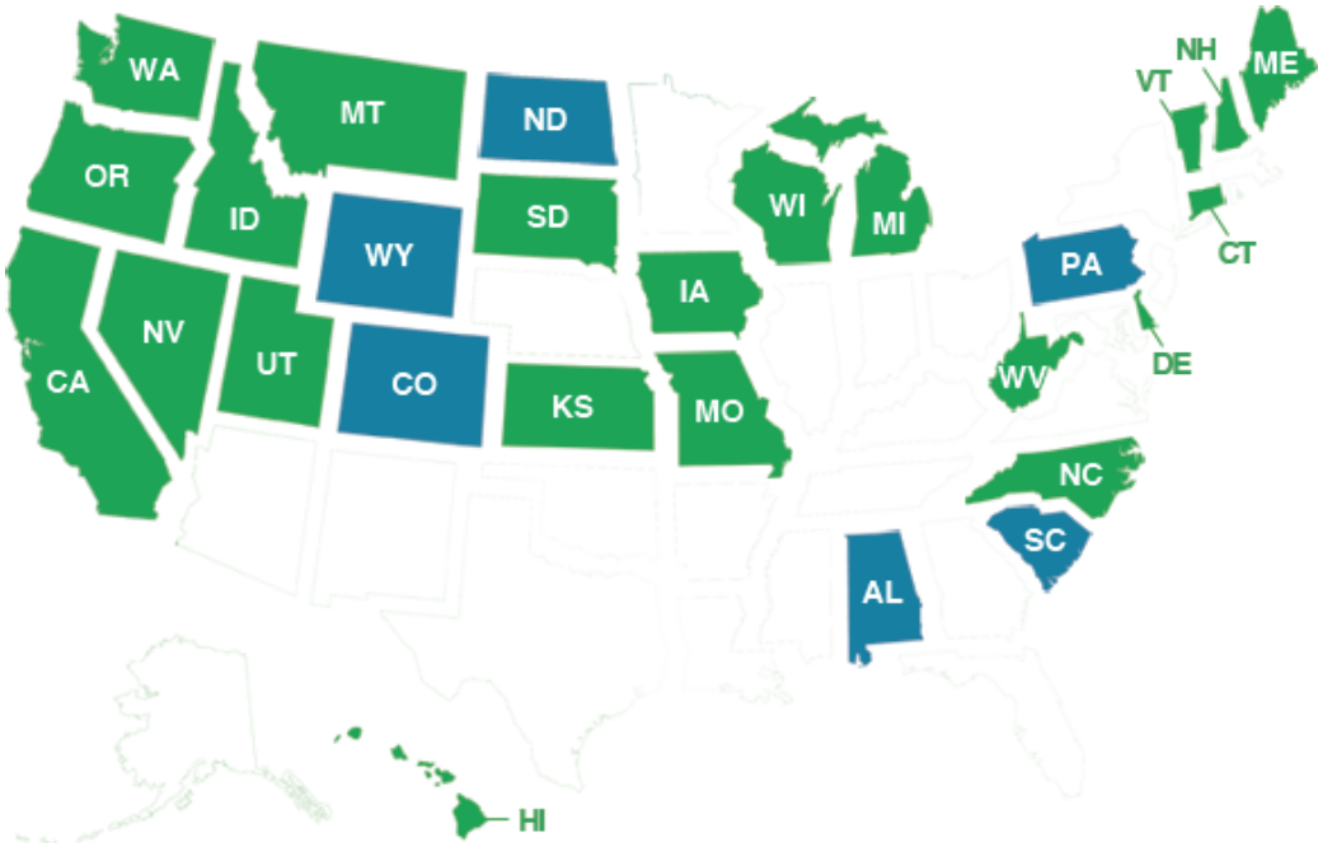
SMARTER Balanced Assessment Consortium SMARTER

## SMARTER Balanced States

States in the SMARTER Balanced Assessment Consortium (as of November 17, 2010):




WestEd

# Current vs. CCSS

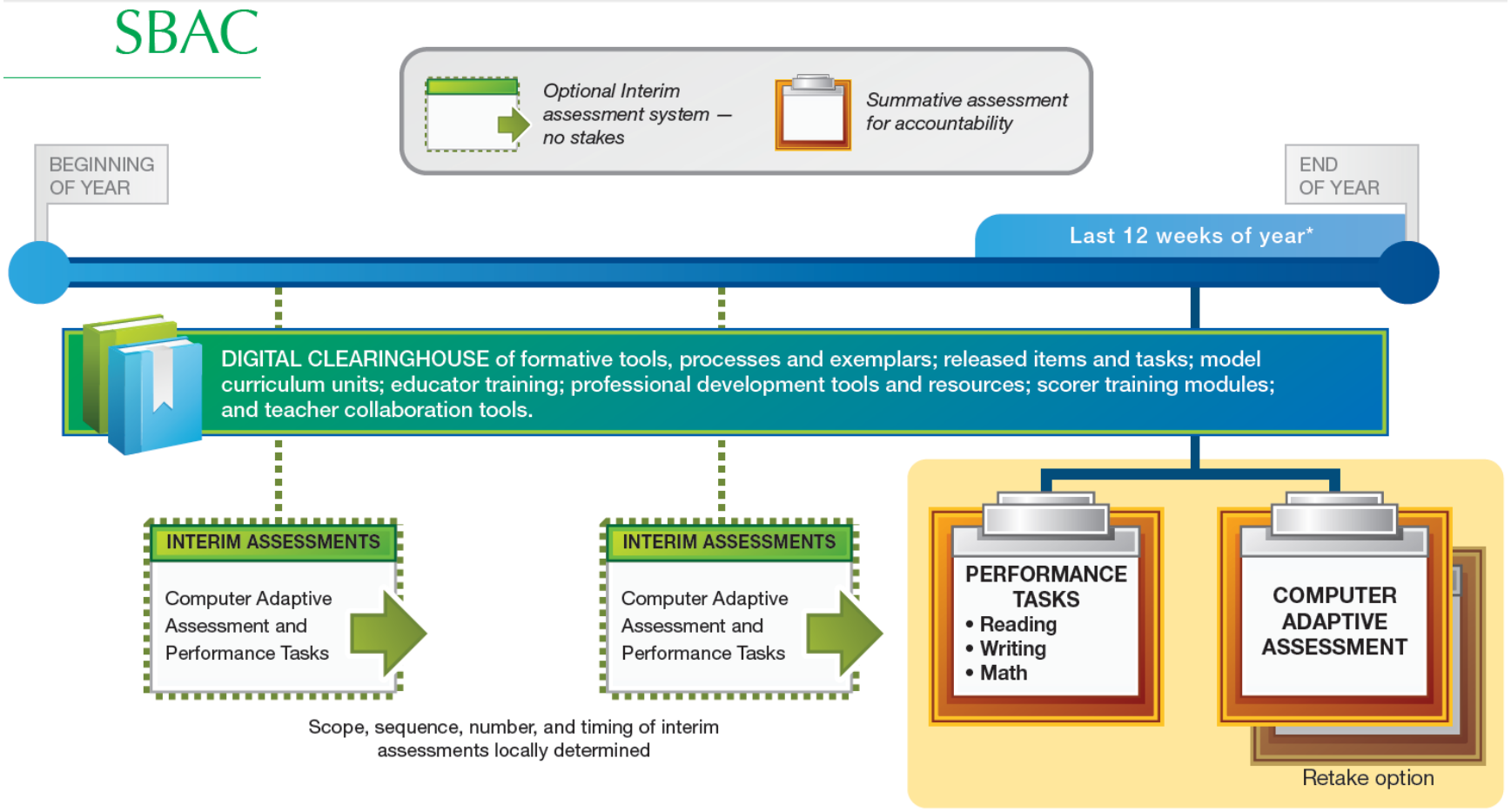
**Current STAR Assessments**

- Grades 2-11, writing at 4th and 7th
- Only paper & pencil option
- Taken around 85% of the instructional days
- Only multiple choice
- Part of the state and federal accountability system

**Proposed CCSS Assessments**

- Grades 3-8 and 11, Grades 9 and 10 available for states that choose to use them
- Delivered via computer (Paper and pencil option available for 3 years) and are computer adaptive
- Taken during the final 12 weeks of school
- Performance tasks and comprehensive end-of -year computer adaptive assessment which will some selected response items
- Accountability system has not been established yet

# The System
# (Possible Scenario)



# Claim-Evidence-Warrant

*A Model for Analyzing Arguments*

(adapted from the work of Stephen Toulmin)

SMARTER
Balanced Assessment Consortium

PARCC States

# Content Specifications
# for the Summative assessment of the
# *Common Core State Standards for Mathematics*

## DRAFT TO ACCOMPANY GOVERNING STATE VOTE ON ASSESSMENT CLAIMS

**March 20, 2012**

Developed with input from content experts and Smarter Balanced Assessment Consortium Staff, Work Group Members, and Technical Advisory Committee

# Acknowledgements

**Alan Schoenfeld**, University of California at Berkeley and **Hugh Burkhardt**, Shell Centre, University of Nottingham served as principal authors of this paper. Sections of the document were also authored by **Jamal Abedi**, University of California at Davis; **Karin Hess**, National Center for the Improvement of Educational Assessment; **Martha Thurlow**, National Center on Educational Outcomes, University of Minnesota

Significant contributions and organization of this second draft were provided by **Shelbi Cole**, Connecticut State Department of Education, and **Jason Zimba**, Student Achievement Partners. The project was facilitated by **Linda Darling-Hammond** at Stanford University.

Others who offered advice and feedback on the document include:
**Rita Crust,** Lead Designer, Mathematics Assessment Resource Service
Past President, Association of State Supervisors of Mathematics
**Brad Findell**, Former Mathematics Initiatives Administrator, Ohio Department of Education
**David Foster,** Director, Silicon Valley Mathematics Initiative
**Henry Pollak,** Adjunct Professor, Columbia University, Teachers College,
Former Head of Mathematics and Statistics, Bell Laboratories
**W. James Popham,** Emeritus Professor, University of California, Los Angeles
**Cathy Seeley,** Senior Fellow, Charles A. Dana Center, The University of Texas at Austin
**Malcolm Swan,** Professor of Mathematics Education, Centre for Research in Mathematic Education,
University of Nottingham

# SMARTER – Balance

# Content Specifications - CCSSM

**Four Major Claims for the SMARTER Balanced Assessment Consortium's assessments of the *Common Core State Standards for Mathematics***

***Claim #1*** **- Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.**

***Claim #2*** **- Students can frame and solve a range of complex problems in pure and applied mathematics.**

***Claim #3*** **- Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.**

***Claim #4*** **- Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

# Types of Tasks in Mathematics

Novice – short items focused on skills and routines

Apprentice – medium performance tasks with scaffolding

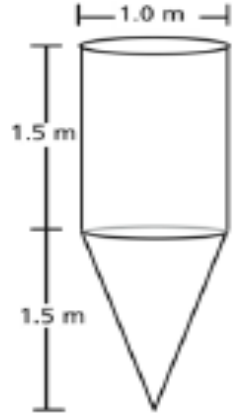
Expert – long tasks with high cognitive load and/or complexity.

# Novice Task Example

5. **Water Tank**

A water tank has shape and dimensions as shown in the diagram.

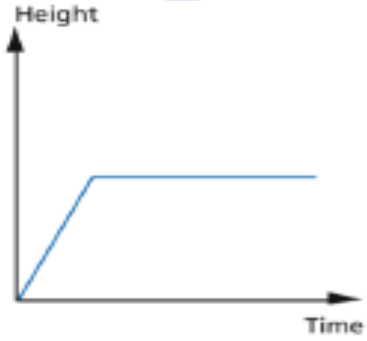
At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.



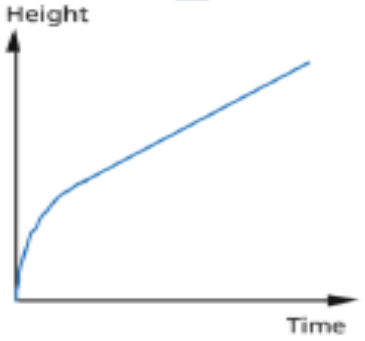

Water tank

**Click on the graph that shows how the height of the water surface changes over time.**

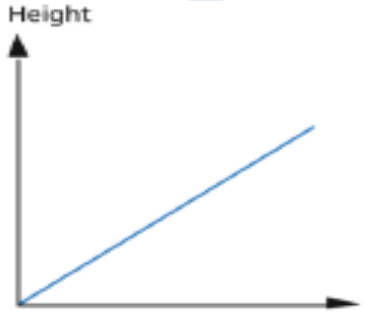
A



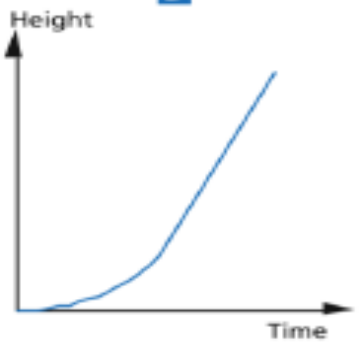

B



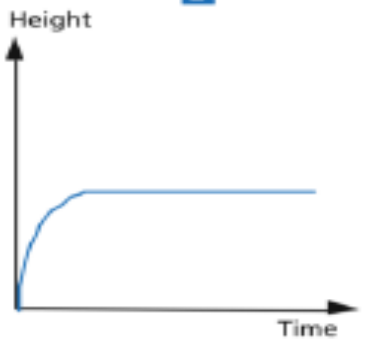

C




D




E




**Click on the graph that shows how the height of the water surface changes over**

# CST – Released Items Algebra 1

**The total cost ($c$) in dollars of renting a sailboat for $n$ days is given by the equation**

$$c = 120 + 60n.$$

**If the total cost was $360, for how many days was the sailboat rented?**

**A** 2

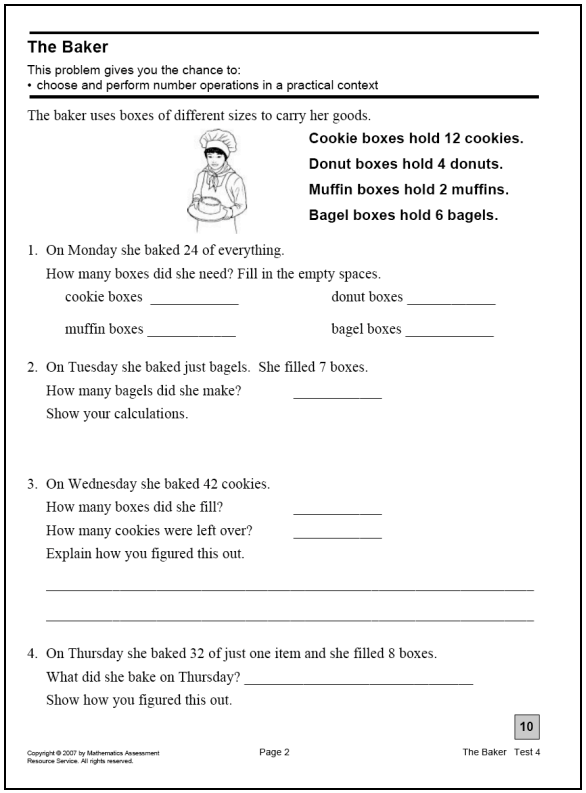
**B** 4

**C** 6

**D** 8

# Performance Assessments

*To Inform Instruction And Measure Higher Level Thinking*



**The Baker**

This problem gives you the chance to:
• choose and perform number operations in a practical context

The baker uses boxes of different sizes to carry her goods.

**Cookie boxes hold 12 cookies.**
**Donut boxes hold 4 donuts.**
**Muffin boxes hold 2 muffins.**
**Bagel boxes hold 6 bagels.**

1. On Monday she baked 24 of everything.
   How many boxes did she need? Fill in the empty spaces.
   cookie boxes _____ donut boxes _____
   muffin boxes _____ bagel boxes _____

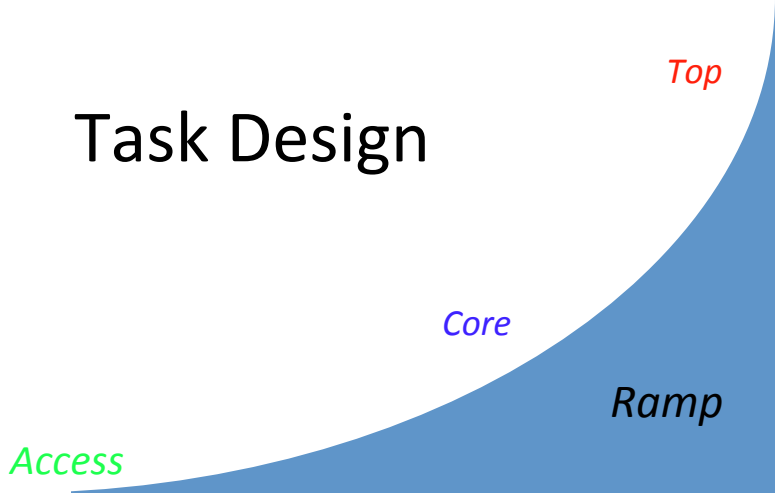
2. On Tuesday she baked just bagels. She filled 7 boxes.
   How many bagels did she make? _____
   Show your calculations.

3. On Wednesday she baked 42 cookies.
   How many boxes did she fill? _____
   How many cookies were left over? _____
   Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.
   What did she bake on Thursday? _____
   Show how you figured this out.

10




**Entry level (access into task)**

**Core Mathematics - (meeting standards)**

**Top of Ramp (conceptually deeper, beyond)**

- The Mathematics Assessment Resource Service (MARS) is an NSF funded collaboration between U.C. Berkeley and the Shell Centre in Nottingham England.
- The Assessments target grades 2- Geometry and are aligned with the State and NCTM National Math Standards.

SHELL CENTRE

BALANCED ASSESSMENT

MARS

# Apprentice Task

## CR 4: Baseball Jerseys

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks 2 local companies to give him a price.

1. 'Print It' will charge $21.50 each for the jerseys.

   Using *n* for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   _____

2. 'Top Print' has a Set-Up cost of $70 and then charges $18 for each jersey.

   Using *n* to stand for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   _____

3. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to order for the price from 'Top Print' to be less than from 'Print It'.
   Explain how you figured it out.

   _____

   _____

   _____

   _____

   _____

4. Bill decides to order 30 jerseys from 'Top Print'.
   How much more would the jerseys have cost if he had bought them from 'Print It'?
   Show all your calculations.

   _____

# Baseball Jerseys

This problem gives you the chance to:

- work with equations that represent real life situations

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks two local companies to give him a price.

1. 'Print It' will charge \$21.50 each for the jerseys.

   Using $n$ for the number of jerseys ordered, and $c$ for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   _____

2. 'Top Print' has a one-time setting up cost of \$70 and then charges \$18 for each jersey.

   Using $n$ to stand for the number of jerseys ordered, and $c$ for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   _____

3. Bill decides to order 30 jerseys from 'Top Print'.
How much more would the jerseys cost if he buys them from 'Print It'?
Show all your calculations. _____

4. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to buy for the price from 'Top Print' to be less than from 'Print It'.
Explain how you figured it out. _____

_____

_____

_____

_____

## Performance Exams

40,000 – 70,000 students per year since 1999

Students in grades 2 through 10th/11th grade are administered performance exams (5 apprentice tasks per exam).

| Task 1: Candies | | Rubric |
|---|---|---|
| The core elements of performance required by this task are:<br>• work with fractions and ratios<br>Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: 2/3 or 6/9 | 1 | 1 |
| 2. Gives correct answer: 3<br>Shows work such as: 1 + 3 = 4  12 ÷ 4 =<br>Accept diagrams. | 1<br>1 | 2 |
| 3 Gives correct answer: 18<br>Shows work such as: 2 + 3 = 5  30 ÷ 5 = 6  6 x 3 =<br>Accept diagrams. | 2<br>1 | 3 |
| 4. Gives correct answer: 6<br>Gives a correct explanation such as: Anthony mixes a ratio of one cup of cream to two cups of chocolate. The ratio stays the same for different amounts. So I wrote the numbers in a chart like this<br>1 to 2 = a total of 3<br>2 to 4 = a total of 6<br>3 to 6 = a total of 9<br>Accept diagrams. | 1<br>1 | 2 |
| | Total Points | 8 |

District scoring leaders are trained in using task specific rubrics

Student tests are hand scored by classroom teachers trained and calibrated using standard protocols.

Random sample of student papers are audited and rescored by SJSU math & CS students. (Two reader correlation >0.95)

EDUCATIONAL DATA SYSTEMS

Student results are collected, analyzed, and reported by an independent data contractor.

# Spring 2011 Trends Grade to Grade

| Grade 2 | MARS 1 | MARS 2 | MARS 3 | MARS 4 | Total |
|---|---|---|---|---|---|
| Far Below | 1.0% | 0.6% | 0.1% | 0.0% | 1.7% |
| Below Basic | 1.9% | 4.1% | 1.1% | 0.1% | 7.2% |
| Basic | 0.8% | 5.3% | 4.6% | 0.6% | 11.3% |
| Proficient | 0.4% | 5.1% | 16.2% | 6.5% | 28.2% |
| Advanced | 0.2% | 1.6% | 15.2% | 34.6% | 51.6% |
| Total | 4.3% | 16.7% | 37.2% | 41.8% | 100.0% |

| Grade 2 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 13.7% | 6.5% | 20.2% |
| CST AT or ^ | 7.3% | 72.5% | 79.8% |
| Totals | 21.0% | 79.0% | 100.0% |

# Spring 2011 Trends Grade to Grade

| Grade 3 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 16.4% | 4.5% | 20.9% |
| CST AT or ^ | 12.7% | 66.3% | 79.0% |
| Totals | 29.1% | 70.8% | 99.9% |

| Grade 4 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 15.6% | 5.8% | 21.4% |
| CST AT or ^ | 12.9% | 65.8% | 78.7% |
| Totals | 28.5% | 71.6% | 100.1% |

| Grade 5 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 17.3% | 6.0% | 29.7% |
| CST AT or ^ | 12.4% | 64.4% | 70.4% |
| Totals | 23.3% | 76.8% | 100.1% |

| Grade 6 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 34.7% | 3.8% | 38.5% |
| CST AT or ^ | 21.7% | 39.6% | 61.3% |
| Totals | 56.4% | 43.4% | 99.8% |

# Spring 2011 Trends Grade to Grade

| Grade 7 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 38.1% | 0.4% | 38.5% |
| CST AT or ^ | 38.1% | 23.5% | 61.6% |
| Totals | 76.2% | 23.9% | 100.1% |

| Grade 8 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 55.1% | 2.8% | 57.9% |
| CST AT or ^ | 25.0% | 17.0% | 42.0% |
| Totals | 80.1% | 19.8% | 99.9% |

| Course 1 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 31.9% | 4.1% | 36.0% |
| CST AT or ^ | 21.5% | 42.0% | 63.5% |
| Totals | 53.4% | 46.1% | 99.5% |

| Course 2 | MARS Below | MARS At or ^ | Total |
|---|---|---|---|
| CST Below | 15.4% | 0.0% | 15.4% |
| CST AT or ^ | 36.0% | 48.7% | 84.7% |
| Totals | 51.4% | 48.7% | 100.1% |

# 8th Grade Geometry
# California's Highest Achieving Students

| Geometry | MARS Below | MARS At or Above | Total |
|---|---|---|---|
| CST Below | 15.3% | 0.0% | 15.3% |
| CST AT or Above | 36.0% | 48.7% | 84.7% |
| Totals | 51.3% | 48.7% | 100% |

# Expert Tasks

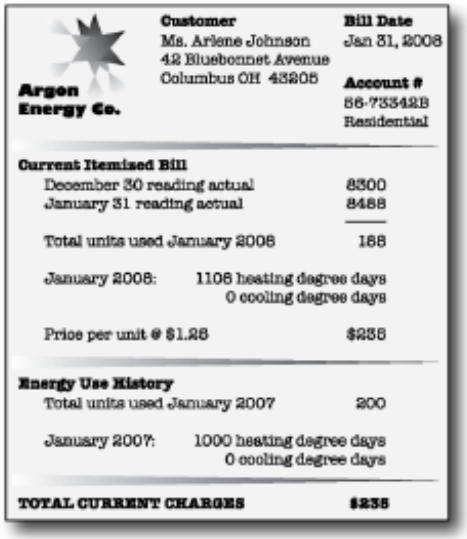
*The main point in mathematics teaching is to develop the tactics of problem solving.*

George Polya

# Gas Bills, Heating Degree Days, and Energy Efficiency

Here is a typical story about an Ohio family concerned with saving money and energy by better insulating their house.

Kevin and Shana Johnson's mother was surprised by some very high gas heating bills during the winter months of 2007. To improve the energy efficiency of her house, Ms. Johnson found a contractor who installed new insulation and sealed some of her windows. He charged her $600 for this work and told her he was pretty sure that her gas bills would go down by "at least 10 percent each year." Since she had spent nearly $1,500 to keep her house warm the previous winter, she expected her investment would conserve enough energy to save at least $150 each winter (10% of $1,500) on her gas bills.



**Argon Energy Co.**

**Customer**
Ms. Arlene Johnson
42 Bluebonnet Avenue
Columbus OH 43205

**Bill Date**
Jan 31, 2008

**Account #**
86-73342B
Residential

**Current Itemized Bill**

| | |
|---|---|
| December 30 reading actual | 8300 |
| January 31 reading actual | 8488 |
| Total units used January 2008 | 188 |
| January 2008: | 1106 heating degree days<br>0 cooling degree days |
| Price per unit @ $1.28 | $235 |

**Energy Use History**

| | |
|---|---|
| Total units used January 2007 | 200 |
| January 2007: | 1000 heating degree days<br>0 cooling degree days |

**TOTAL CURRENT CHARGES** **$235**

Ms. Johnson's gas bill in January 2007 was $240. When she got the bill for January 2008, she was stunned that the new bill was $235. If the new insulation was going to save only $5 each month, it was going to take a very long time to earn back the $600 she had spent. So she called the insulation contractor to see if he had an explanation for what might have gone wrong. The contractor pointed out that the month of January had been very cold this year *and* that the rates had gone up from last year. He said her bill was probably at least 10% less than it would have been without the new insulation and window sealing.

Ms. Johnson compared her January bill from 2008 to her January bill from 2007. She found out that she had used 200 units of heat in January of 2007 and was charged $1.20 per unit (total = $240). In 2008, she had used 188 units of heat but was charged $1.25 per unit (total = $235) because gas prices were higher in 2008. She found out the average temperature in Ohio in January 2007 had been 32.9 degrees, and in January of 2008, the average temperature was more than 4 degrees colder, 28.7 degrees. Ms. Johnson realized she was doing well to have used less energy (188 units versus 200 units), especially in a month when it had been colder than the previous year.

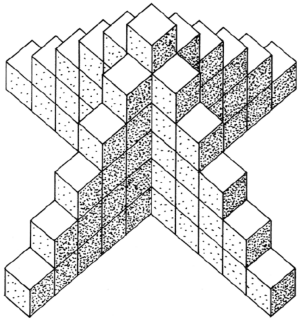
Since she used gas for heating only, Ms. Johnson wanted a better estimate of the savings due to the additional insulation and window sealing. She asked Kevin and Shana to look into whether the "heating degree days" listed on the bill might provide some insight.

Ohio Performance Assessment

Ohio Department of Education • Stanford University • ESC of Central Ohio

Pilot Project

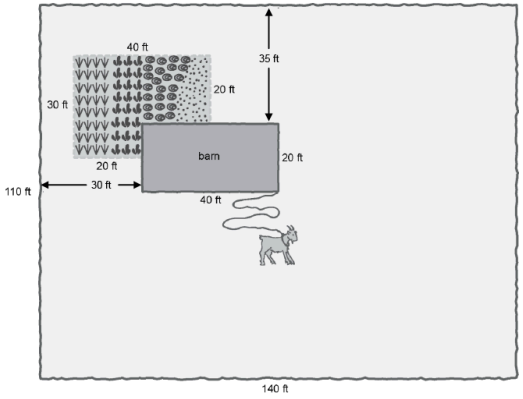
# SALLY'S SUGAR STACK



## Grazing Area

A farmer tethers her goat to the corner of a 40-by-20-foot barn in a fenced lot that is 140-by-110 feet. She also has an herb garden next to part of the barn. The goat is tethered on a 50-foot rope to the corner of the barn farthest from the herb garden.

After leaving the goat out on the rope for one day, the farmer discovers that a large area of her herb garden has been nibbled to the ground! Where can she tether the goat so that her herb garden is not within reach of the goat, but without decreasing the grazing area of the goat? The original grazing area includes all the grass area the goat could reach including that section of the herb garden that the goat ate.




## Open for Business

Malena is a student who wants to raise $5,000 to tour South America next summer. To raise the money, she decides to open her own business on eBay.


The owner of an electronics shop offers to sell Malena some of his products at the wholesale price. She needs to decide which items to sell and how to price those items in order to maximize her profit.

She does some market research and finds the information provided in the table below about some of the items she is considering selling. Her research results include the cost to buy these items from the wholesale supplier, the retail price at which different items were sold at different times, and the number of items sold at these different prices during the month.

SCALE

Stanford Center for Assessment, Learning, & Equity

SRN LEADS STANFORD UNIVERSITY

| *Mathematics Claim #1* |
| --- |
| **Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.** |

The measurement instrument will be novice [short] tasks. The items assessing this claim will account for 40% of the total score administered by computer.

| *Mathematics Claim #2* |
| --- |
| **Students can frame and solve a range of complex problems in pure and applied mathematics.** |

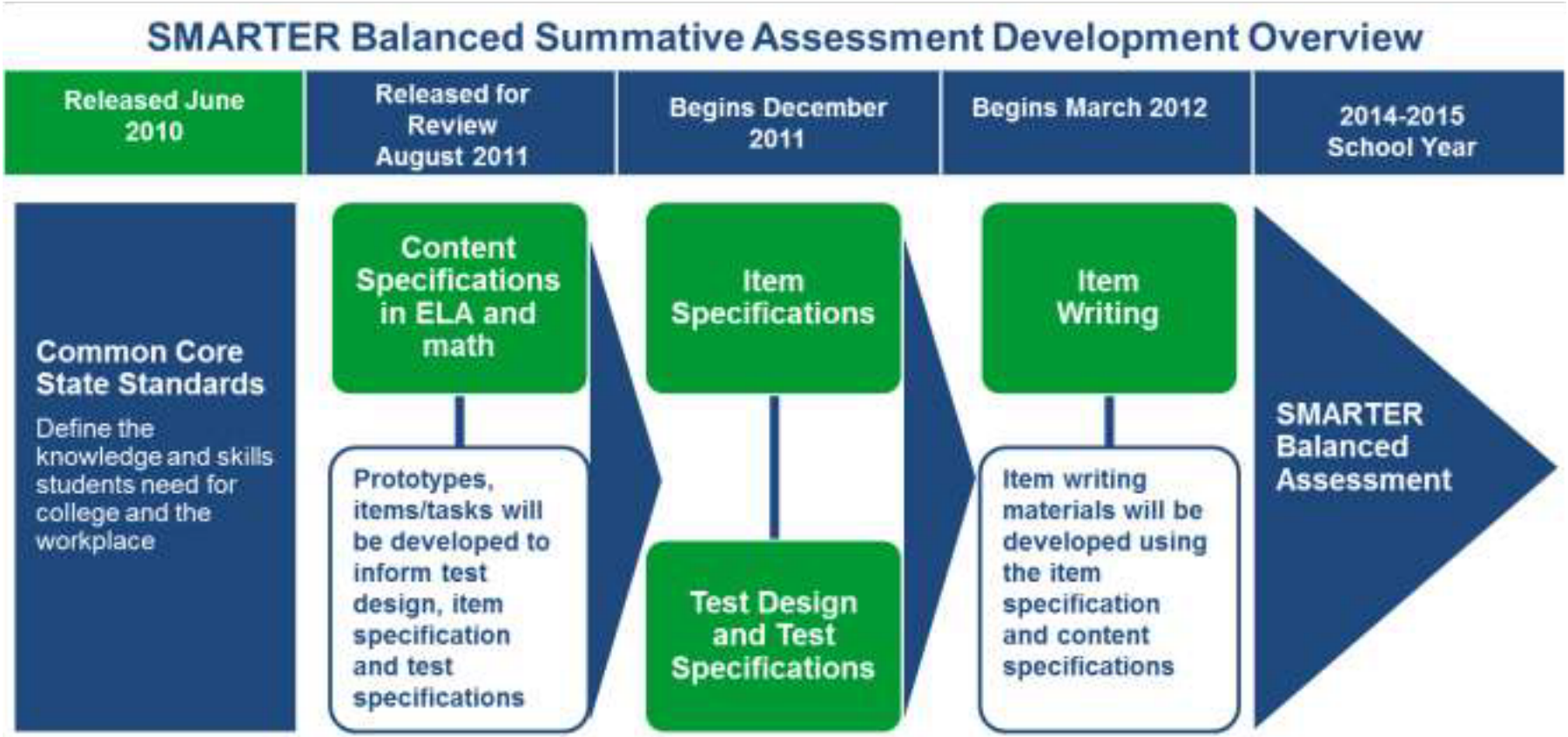
The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.

| *Mathematics Claim #3* |
| --- |
| **Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.** |

The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.

| *Mathematics Claim #4* |
| --- |
| **Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.** |

The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.


SMARTER Balanced Summative Assessment Development Overview
Released June 2010
Released for Review August 2011
Begins December 2011
Begins March 2012
2014-2015 School Year
Common Core State Standards
Define the knowledge and skills students need for college and the workplace
Content Specifications in ELA and math
Prototypes, items/tasks will be developed to inform test design, item specification and test specifications
Item Specifications
Test Design and Test Specifications
Item Writing
Item writing materials will be developed using the item specification and content specifications
SMARTER Balanced Assessment

## *Problem Sources*

**Part I: Short items**

1: MARS
2: MARS
3: SBAC
4: MARS
5: PISA
6: MARS
7: PISA
8: MARS
9: MARS
10: MARS
11: SBAC
12: SBAC
13: SBAC

**Part II: Selected Response Tasks**

CR 1: SBAC
CR 2: MARS
CR 3: MARS
CR 4: MARS
CR 5: MARS
CR 6: MARS
CR 7: MARS
CR 8: MARS
CR 9: MARS
CR 10: MARS

**Part III: Extended Performance Task**

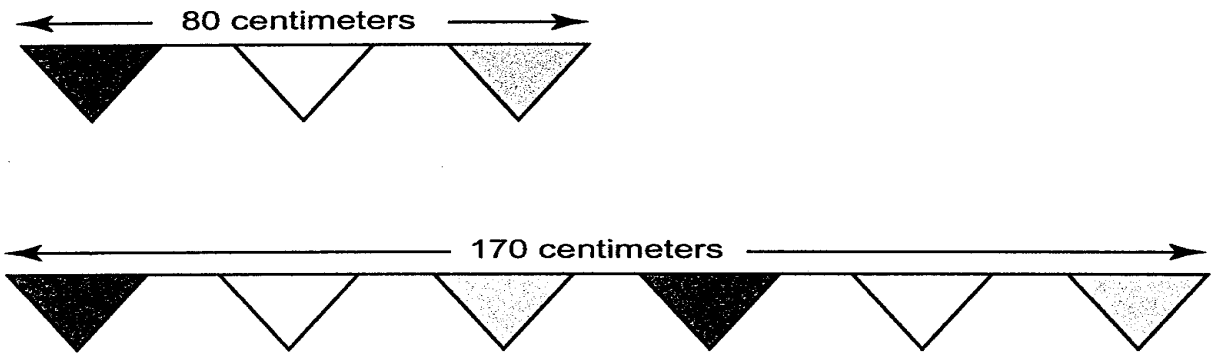
Ohio Department of Education and the Stanford University School Redesign Network

# Teaching for Meaning

## Party Flags

This problem gives you the chance to:

- find sizes by interpreting a diagram
- express a function by a formula




Erica is putting up lines of colored flags for a party.

The flags are all the same size and are spaced equally along the line.

1. Calculate the length of the sides of each flag, and the space between flags.

Show all your work clearly.

2. How long will a line of $n$ flags be?

Write down a formula to show how long a line of $n$ flags would be.

Algebra students had been working on system of linear equations for weeks.

$$6x + 5y = 170$$

$$3x + 2y = 80$$

$$6x + 5y = 170$$

$$-6x + -4y = -160$$

$$y = 10$$

# The Findings from Party Flags

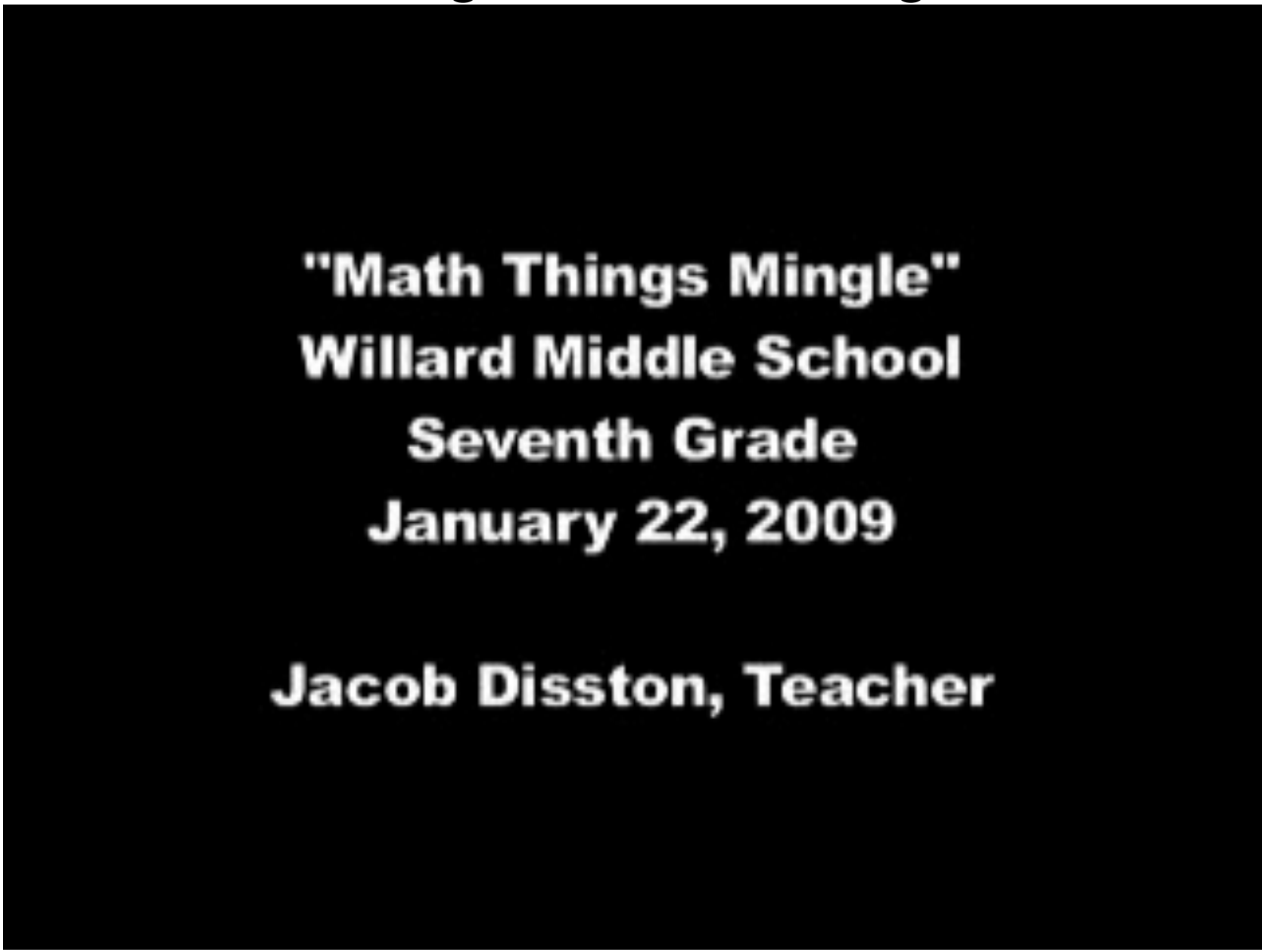
- The task may be approached as a system of simultaneous equations, almost no algebra students used such an approach.
- 49%  of algebra students had no success.
- 44% accurately found the two lengths (most commonly by an estimation strategy only using one constraint).
- 21% correctly used both constraints (the length of three flags is 80 cm. and the length of 6 flags is 170 cm.).
- 7% of the students were able to develop a valid generalization for n flags.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

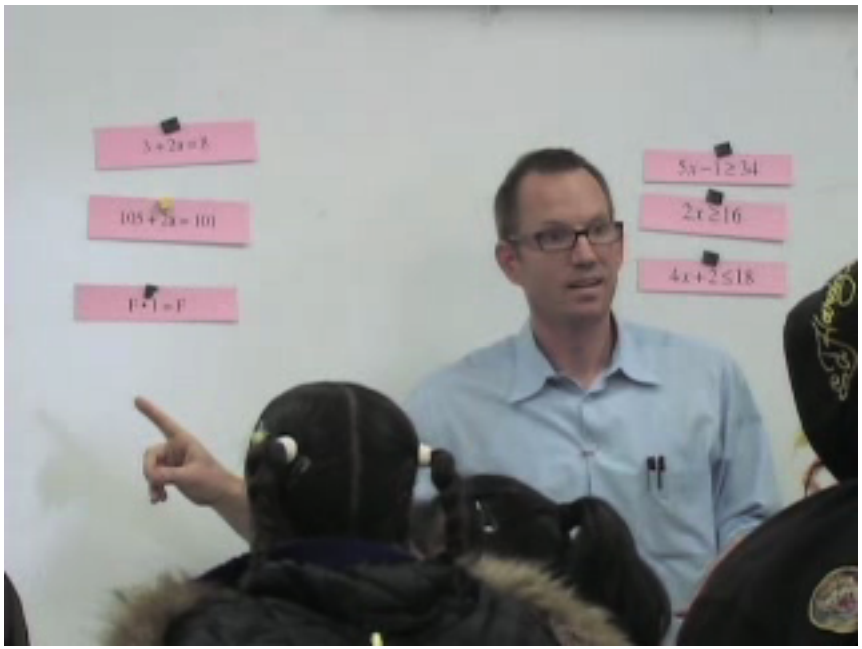
# Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

# Teaching for Understanding?



# Discuss the Lesson




In small groups discuss:

- How did the students make sense of the mathematics?
- What mathematical ideas did student struggle to understand?
- What did student seems to learn?
- What role did the teacher play in the lesson?
- Which mathematical practices were the students engaged in during the lesson?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Press Esc to exit full screen mode.
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05:29

# Examining Student Thinking and Informing Instruction

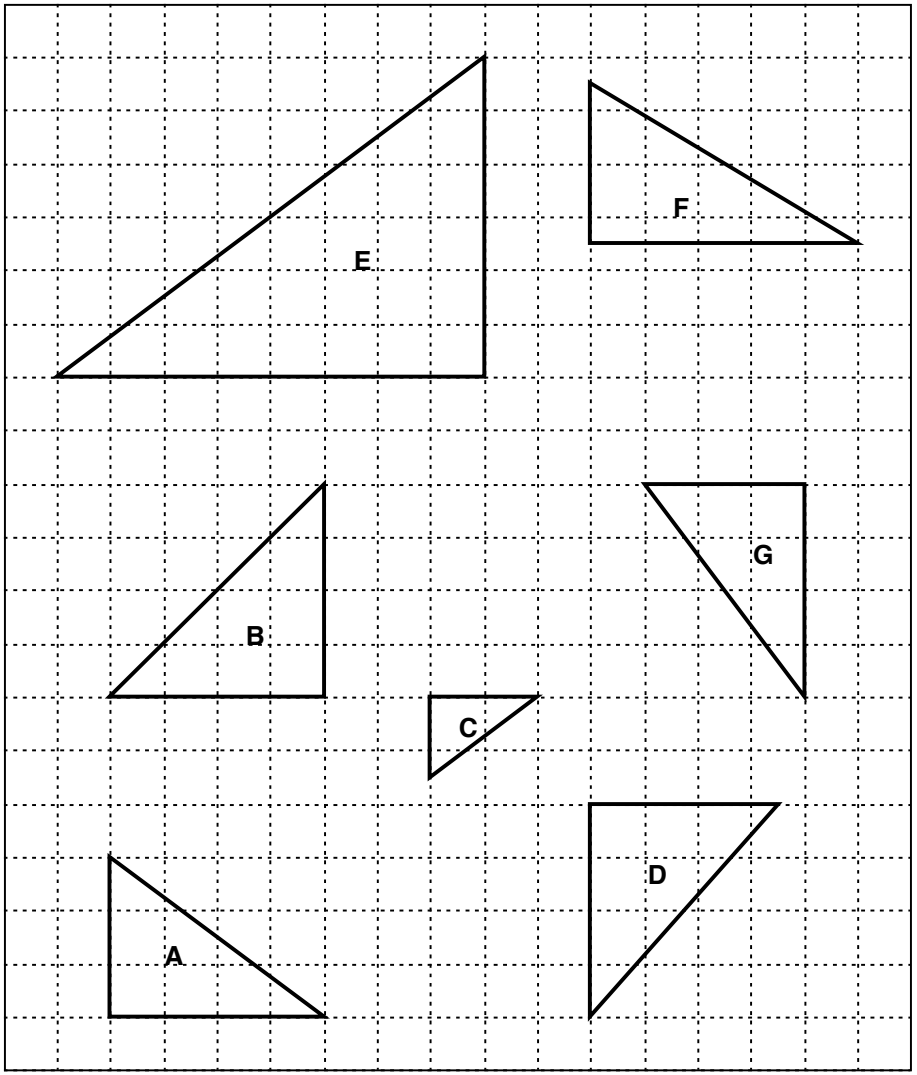
# 7th Grade Geometry Task

**Triangles**

This problem gives you the chance to:

- reason about similar figures and scale factor

Here are some right triangles.




1. Which of the triangles on the opposite page are congruent to triangle A?

Explain your reasons.

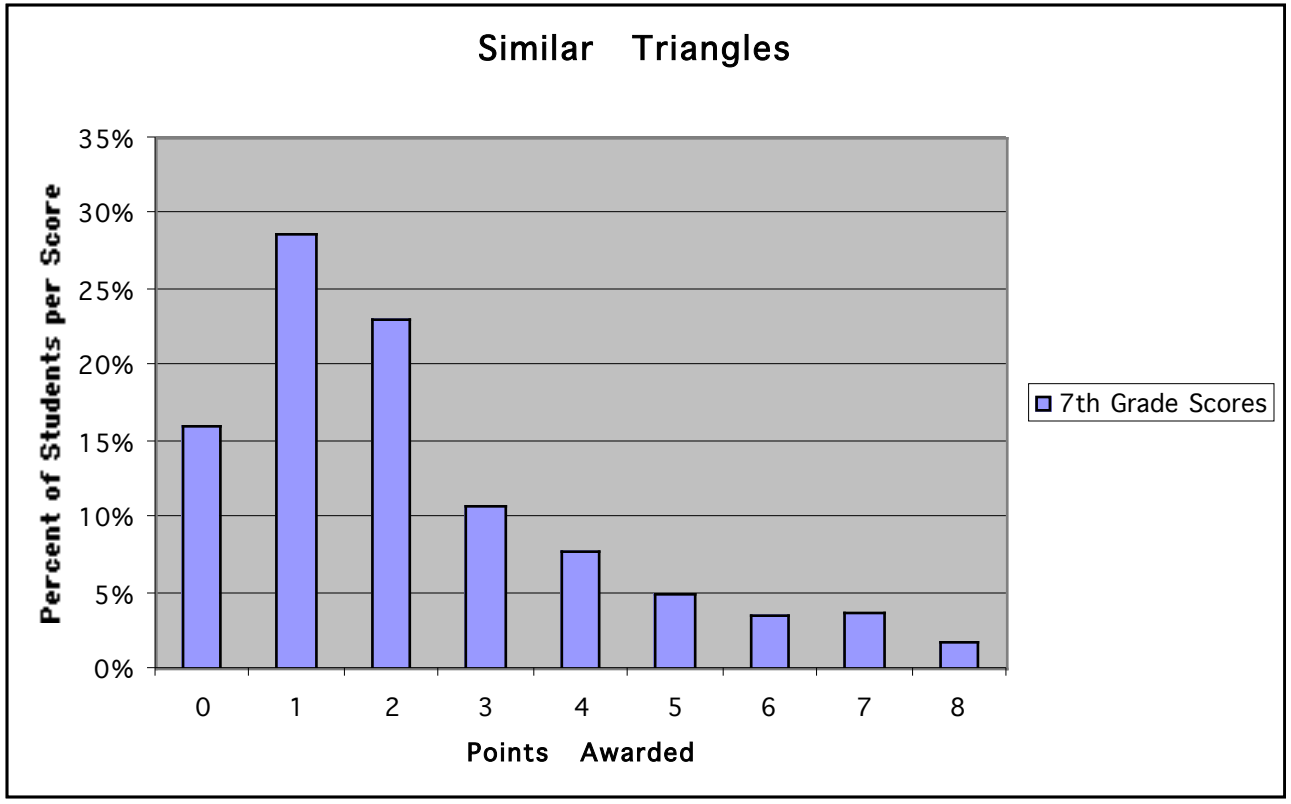
2. Which of the triangles on the opposite page are similar to triangle A?

Explain how you decided.

3. If triangle A is enlarged by a scale factor of 3, what will the area of the new triangle be?

Show your work.

# Score Distribution for 7th Grade Similar Triangles



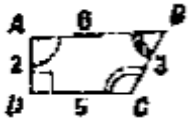
# Misconception students illustrated in their work on Similar Triangles

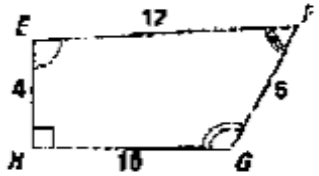
- Students thought the orientation of the figured mattered in whether figures were similar (they both face the same way).
- Students believe that all triangles are similar or all rectangles are similar.
- Students misinterpreted how to measure length of figures on graph paper.
- Students added the scale factor, instead of multiplying to find proportional enlargements of the lengths.
- Students seldom identified that a similar figure could be smaller in size. (go from large triangle to small triangle)

# Similarity

## Goal: Using Properties of Similar Figures

If the corresponding angles of two figures are congruent and the ratios of the lengths of their corresponding sides are equal, the figures are similar. Similar figures are the same shape, but are not necessarily the same size. So, two congruent figures are always similar, but two similar figures are not necessarily congruent. In the diagram below, quadrilateral $ABCD$ is similar to quadrilateral $EFGH$. You can write this statement as quadrilateral $ABCD \sim$ quadrilateral $EFGH$.




**CORRESPONDING ANGLES**

$\angle A \cong \angle E \quad \angle C \cong \angle G$

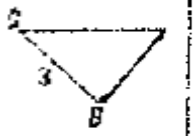
$\angle B \cong \angle F \quad \angle D \cong \angle H$

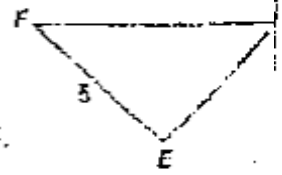
**CORRESPONDING SIDES**

$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH} =$

### EXAMPLE: Properties of Similarity

$\triangle ABC \sim \triangle DEF$. Describe the relationships among the angles and sides of the triangle.




**Solution**

Corresponding angles are congruent. That is, $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$.

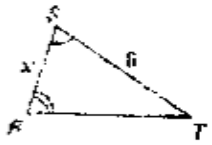
The ratios of the lengths of corresponding sides are equal. The lengths of corresponding sides $\overline{BC}$ and $\overline{EF}$ are given, so this ratio is 3 : 5.

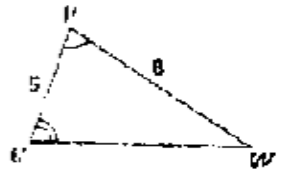
$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} = \frac{3}{5}$$

Find the length of $\overline{RS}$.

**Solution**

Because two angles of $\triangle RST$ are congruent to two angles of $\triangle UVW$, $\triangle RST \sim \triangle UVW$ by the AA similarity postulate. Write and solve a proportion to find the length of $\overline{RS}$.



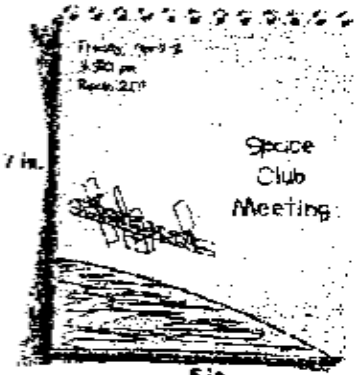

| | |
|---|---|
| $\frac{RS}{UV} = \frac{ST}{VW}$ | Write proportion. |
| $\frac{x}{5} = \frac{6}{8}$ | Substitute. |
| $8x = 30$ | Cross product property |
| $x = 3.75$ | Divide each side by 3. |

**ANSWER** ▶ The length of $\overline{RS}$ is 3.75 units.

If two polygons are similar, the ratio of the lengths of two corresponding side lengths is called the **scale factor**. In the triangles above, the scale factor of $\triangle RST$ to $\triangle UVW$ is $\frac{6}{8}$, or $\frac{3}{4}$.

### EXAMPLE: Using a Scale Factor

You are designing a poster to advertise the next meeting of the Space Club. You begin by sketching the design shown at the right. The scale factor of the actual poster to your sketch is 4 : 1. Find the height and the width of the actual poster.




**Solution**

Use the scale factor to find the height $h$ and the width $w$ of the poster.

$$\frac{\text{Poster height}}{\text{Sketch height}} = \text{Scale factor} \qquad \frac{\text{Poster width}}{\text{Sketch width}} = \text{Scale factor}$$

$$\frac{h}{7 \text{ inches}} = \frac{4}{1} \qquad \frac{w}{5 \text{ inches}} = \frac{4}{1}$$

$$h = 4(7) \qquad w = 4(5)$$

$$h = 28 \qquad w = 20$$

**ANSWER** ▶ The poster has a height of 28 inches and a width of 20 inches.

# MATH TALKS

# Math Talks

- A daily ritual with the entire class for the purpose of developing conceptual understanding of and efficiency with numbers, operations and other mathematics such as geometry and algebra. (no more than 10 minutes per day)

Math Talks are used to:

- Support active student discourse and discussions
- Review and practice procedures and concepts
- Introduce a concept before diving into the lesson of the day
- Support students in deepening their understanding of the number, operations and algebraic thinking.
- Explore mathematical connections and relationships
- Encourage students to construct viable arguments and critique the reasoning of others
- Support students in using precise mathematical language in sharing their different strategies and approaches

29 x 31

# Today's Number

36

**Possible Solutions**

$18 + 18$

$3^2 \cdot 2^2$

$9+9+9+9$

$25.65+10.35$

$9 \div 1/4$

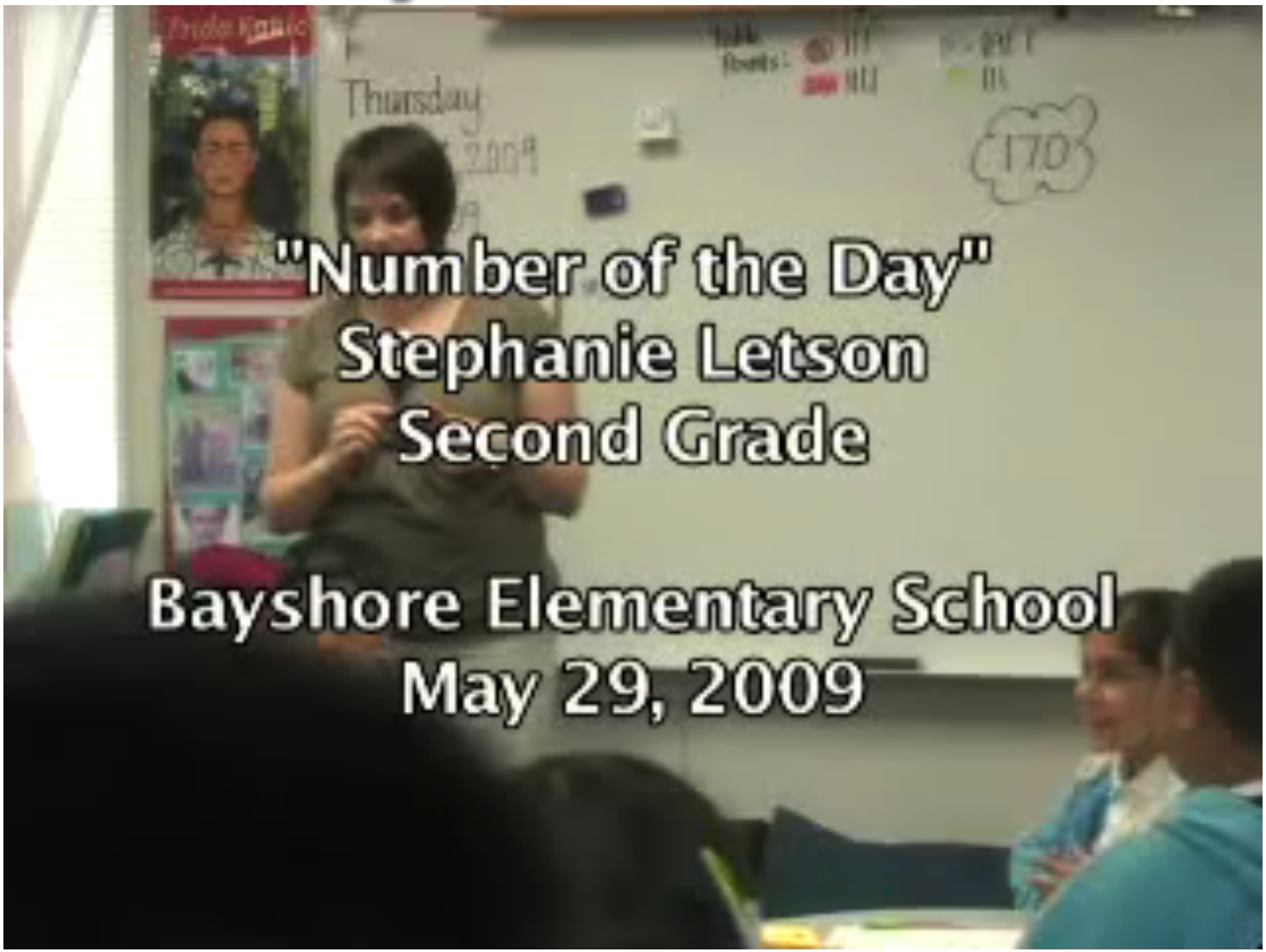
$-15 + 51$

$3\sqrt{144}$

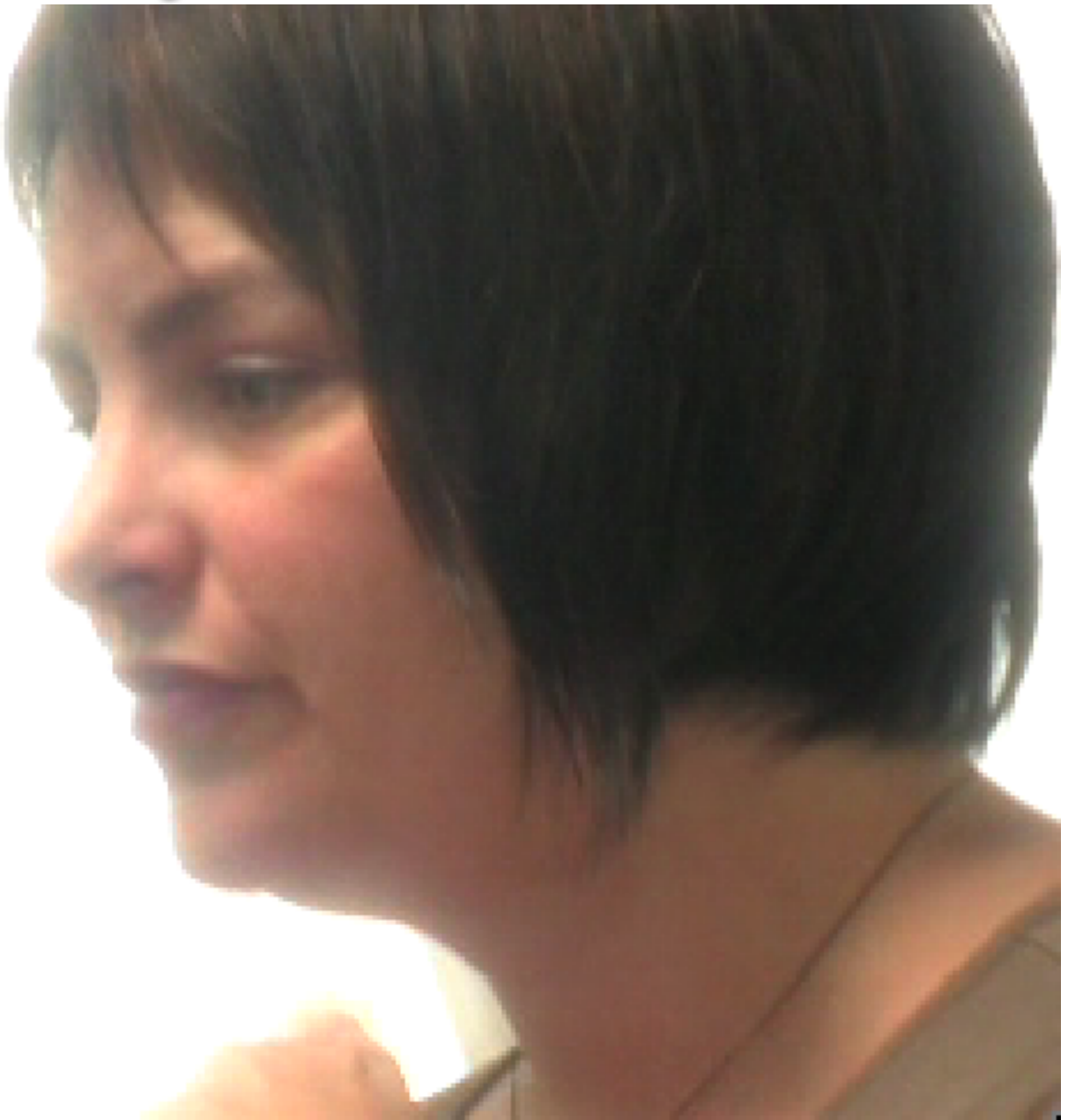
**Today's Number with Constraints**

- More than one operation
- Using Two digit numbers (3 digits etc.)
- Using Fractions, Decimals, Percents
- Using sets of numbers and operations
- Using exponents, square roots
- Using integers (sign numbers)
- Using a set of numbers and different operations.

# Today's Number



# Today's Number



# Discuss the Number Talk Video

- Who did the math thinking during the number talk?
- What specific mathematics did the students demonstrate they understood?
- What did the teacher do to support the student discourse?
- What recording techniques did the teacher employ that supported learning in the class?

# Curriculum inspired by the CCSS

MAP's Formative Assessment Lessons and Professional Development Modules

BILL & MELINDA GATES *foundation*

MARS

MARS Team

Mathematics Assessment Resource Service

**Assessment For Learning**

Formative Assessment Lessons (2 days) for High School and Middle School

CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

# Proportion and Non-Proportion Situations

# Getting Things in Proportion

**Q1. Leon**

Leon has $40.

**How many Mexican Pesos can Leon buy with his dollars?**

Explain how you figure this out.

| Exchange Rate |
| --- |
| $1 US = 12 Mexican Pesos |

**Q2. Minna**

This is the call plan for Minna's cell phone:

*$15 a month plus free texts plus $0.20 per minute of call time.*

Minna made 30 minutes of calls this month, and 110 texts.

**How much does she have to pay the phone company?**

Explain how you figure this out.

**Q3. Nuala**

Nuala drives to her grandma's.

She drives at 20 miles per hour.

The journey takes 50 minutes.

**How long would the journey take if Nuala drove at 40 miles per hour?**

Explain how you figure this out.

**Q4. Orhan**

Orhan mixes some purple paint.

He uses three pints of blue paint for every five pints of red paint.

Orhan wants to mix more paint exactly the same color.

He has 17 ½ pints of red paint.

**How much blue paint does he need?**

Explain how you figure this out.


Q5. Here are two statements about the math in Q1 to Q4 above.

For each question, decide which statements are **always true**. Explain your answers.

| **Statement** | **Q1 Leon** | **Q2 Minna** | **Q3 Nuala** | **Q4 Orhan** |
|---|---|---|---|---|
| If you double one quantity, you double the other. | $ : Mexican Pesos | Minutes : $ | Speed : Time | Blue paint : Red paint |
| The ratio:<br>*first quantity: second quantity*<br>is always the same. | $ : Mexican Pesos | Minutes : $ | Speed : Time | Blue paint : Red paint |

| Common issues | Suggested questions and prompts |
|---|---|
| **Student uses mental or jotted strategies**<br>For example: The student has (correctly or incorrectly) calculated solutions, but written very little. | • Explain in more detail how you figured out your solution.<br>• Help your reader to understand your solution. |
| **Student uses informal strategies**<br>For example: The student has used doubling and halving with addition. | • Can you think of a method that could be used for any quantity, e.g. 23 cans of paint?<br>• You had to do a lot of work to figure out that answer. Can you think of a really efficient way of solving this kind of problem?<br>• |
| **When answering proportional questions, student identifies the problem structure as additive rather than multiplicative**<br>For example: The student calculates 3 + 12½ (Q4). | • How many cans of blue paint would you use for one single red can? How can you use that in your solution? |
| **When answering proportional questions, student uses method of cross multiplying proportions (Q4)**<br>For example: The student writes $\frac{17\frac{1}{2}}{x}=\frac{5}{3}$, which is correct, but manipulates the equation incorrectly.<br>Or the student writes $\frac{x}{5}=\frac{17\frac{1}{2}}{3}$, which is incorrect. | • Which of these numbers relate to red paint? Blue paint? Explain how the method works. |
| **Student does not recognize when quantities vary in direct proportion**<br>For example: The student does not answer Q5.<br>Or: The student says that the paint question (Q4) is not a proportional relationship.<br>Or: The student claims that the cell phone question (Q2) is a proportional relationship. | • Which of these properties does the function have? It is a linear function; if one quantity is zero so is the other; if one quantity doubles, so does the other.<br>• What properties must proportional relationships have? Does this relationship have all those properties? |
| **Student does not justify her claims**<br>For example: The student distinguishes correctly between proportional and other functions, but does not explain how she made the distinctions. | • What properties do proportional relationships have?<br>• How do you know this relationship scales? |
| **Student completes the task** | • Write a different question in which the quantities vary in direct proportion. Now answer your own question. |

# Proportion and Non-Proportion Situations

Projector Resources

# Buying Cola

20 ounces of soda costs $1.20

Ross wants to buy …… ounces of soda.

Ross will have to pay $ .....

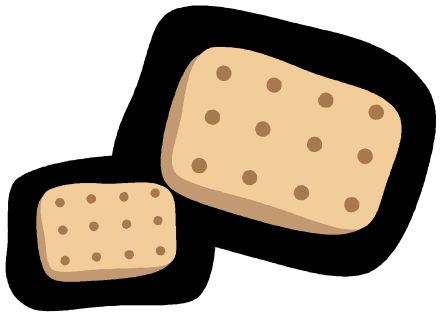
# Properties of Proportion Situations

- One quantity is a multiple of the other.
- If the first quantity is zero,
  the second quantity is zero.
- If you double one quantity,
  the other also doubles.
- The graph of the relationship
  is a straight line through the origin.

# Recipe

Shortbread Cookies (makes ……. Cookies)

- 2 cups of flour
- 1 stick butter
- ½ cup sugar



In order to make ……. cookies, I will need

……. cups of flour

……. sticks of butter

……. cups of sugar.

# Working Together

Choose one of the cards to work on together.

1. Pick easy numbers to fill in the blanks
   Answer the question you have written.
   Write all your reasoning on the card.

2. Using the same card, pick harder numbers to fill in the blanks.
   Answer your new question together.
   Write all your reasoning on the card.

3. Decide whether the quantities vary in direct proportion.
   Write your answer and your reasoning on the card.

When you have finished one card, choose another.

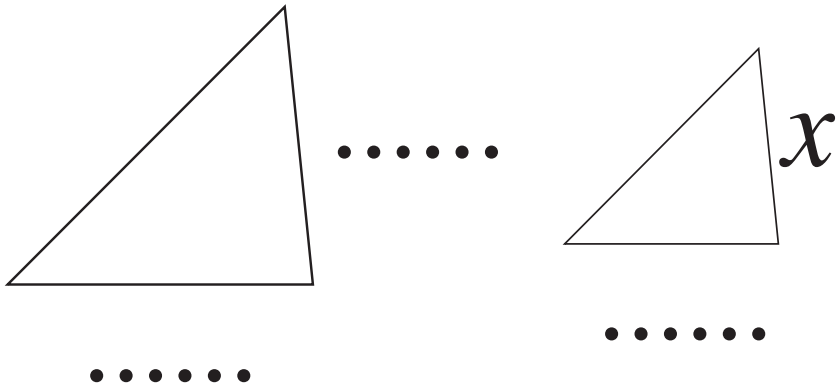
# Analyzing Each Other's Work

Read one of the solutions carefully.

- Is there anything you don't understand?
- Compare the work to your own solution to the question.
- Have you used the same methods?
- Do you have the same numerical answers?
- Do you see notice any errors?
- Do you agree about which question involves direct proportion?
- Now read and analyze the other solution.

# Triangles

These triangles are similar.




Calculate the length marked *x*.