Driving

DRIVING

If I drive at … miles per hour, my journey will take … hours.

How long will my journey take if I drive at … miles per hour?

Gasoline

If you buy … gallons of gasoline it costs …

How much will … gallons cost?

Cell Phone

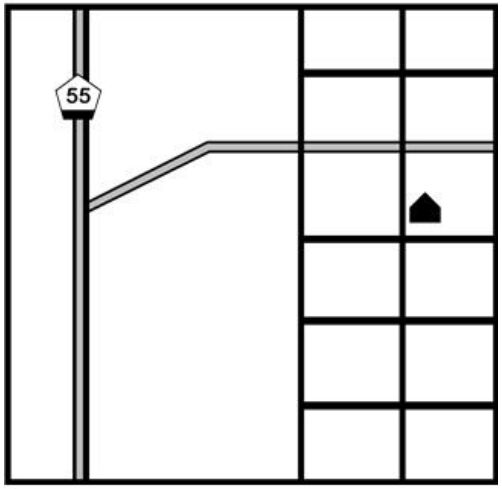
A cell phone company charges $... per month

plus $... per call minute.

I used … call minutes last month.

How much did that cost?

MAP

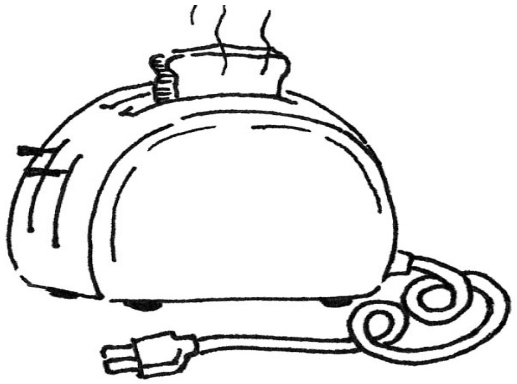


A road … inches long on a map is … miles long in real life.

A river is … inches long on the map.

How long is the river in real life?

Toast



My toaster has two slots for bread.

It takes … minutes to make … slices of toast.

How long does it take to make … slices of toast?

Smoothie

To make three strawberry smoothies, you need:

… cups of apple juice

… bananas

… cups of strawberries

How many bananas are needed for … smoothies?

Line

A straight line passes through the points (0, 0) and (…, …).

It also passes through the point (…, y)

Calculate the value of y.

Math Theme: Proportional Reasoning

Caterpillar Problem

A class has a tank full of caterpillars. It takes five leaves each day to feed two caterpillars. How many leaves does it take to feed 12 caterpillars.

Show how you figured it out.

After Solving the Task, Consider:

- All the ways students may approach the problem
- Other mathematically valid methods to solve the problem

Discuss the different approaches and solution methods within your group

Caterpillars 6th Grade

Video Clip: Group Discourse

- What was the mathematical approach the students were discussing?
- How were they making sense of the problem? Where was their confusion?
- What understandings of proportional reasoning was evident?
- What is the value of listening to student thinking?

Solution Strategies for Caterpillars

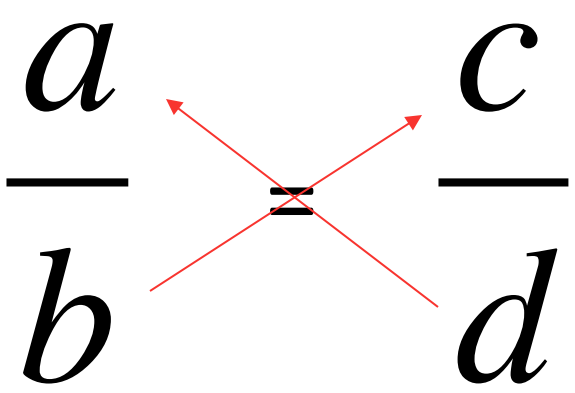
- Drawing/counting by exhaustion (counting by 5 leaves and 2 caterpillars)
- T-table/counting by exhaustion
- Finding the scale factor of 6 (12/2) then multiplying 6 • 5 leaves to find 30 leaves. (Function approach $y = 12/2\ x$)
- Finding the unit rate (1 caterpillar eats 2.5 leaves) then multiply by 12 or counting 12 sets. (Function approach $y = 2.5\ x$)
- Determine that 5 leaves are eaten each day. So 12 caterpillars eat the 5 leaves making 60 -- but those are shared by a pair of caterpillars, so only half of that total are actually eaten. (reasoning about cross multiplication $5/2 = x/12$)

Proportional Reasoning

The concepts of proportionality, of proportional relationships, and of proportional reasoning in general are central to an understanding of middle school and high school mathematics, but most existing courses do not bring out these ideas with sufficient clarity.

Dick Stanley, Susan Hudson Hull, et. al

Cross Multiplication is Cul-De-Sac Reasoning



Proportional Reasoning Should be Taught from a Functional Approach

$$y = k \bullet x$$

$$k = \frac{a}{b}$$

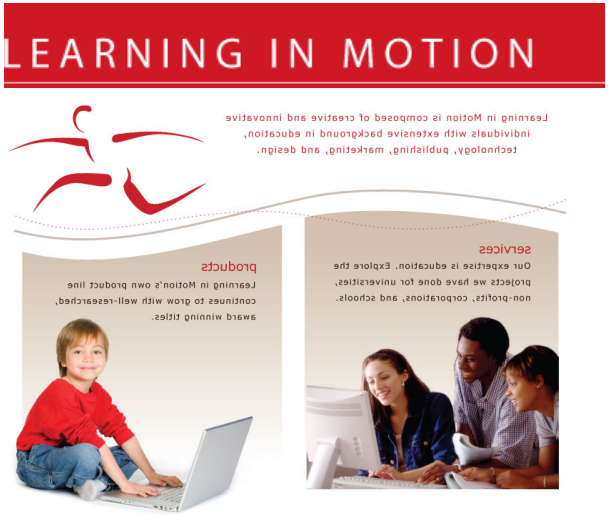
THE CONSTANT OF PROPORTIONALITY

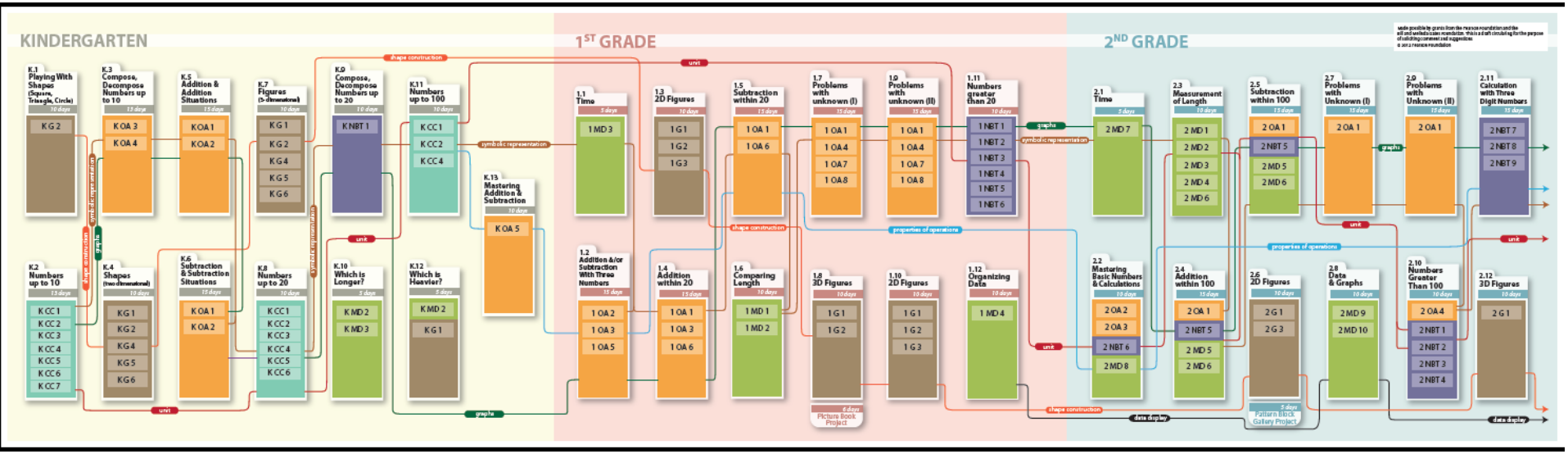
In situations involving proportional relationships, one quantity is a constant multiple of the other; symbolically, $y = k x$. The constant multiplier k is called **the constant of proportionality.** It is the essential quantity connected with any situation involving proportionality. Still, its central character is sometimes missed by students since it shows up in such a variety of different forms depending on the context. Specifically, the constant of proportionality is **the constant ratio** in a proportion, **the constant slope** of a graph, **the constant rate** in rate situations, **the scale** in a map, **the percent** (divided by 100) that designates the part in part-whole situations, **the scale factor** linking two similar figures, **the common trigonometric ratio** of sides in a class of similar right triangles, and so forth.

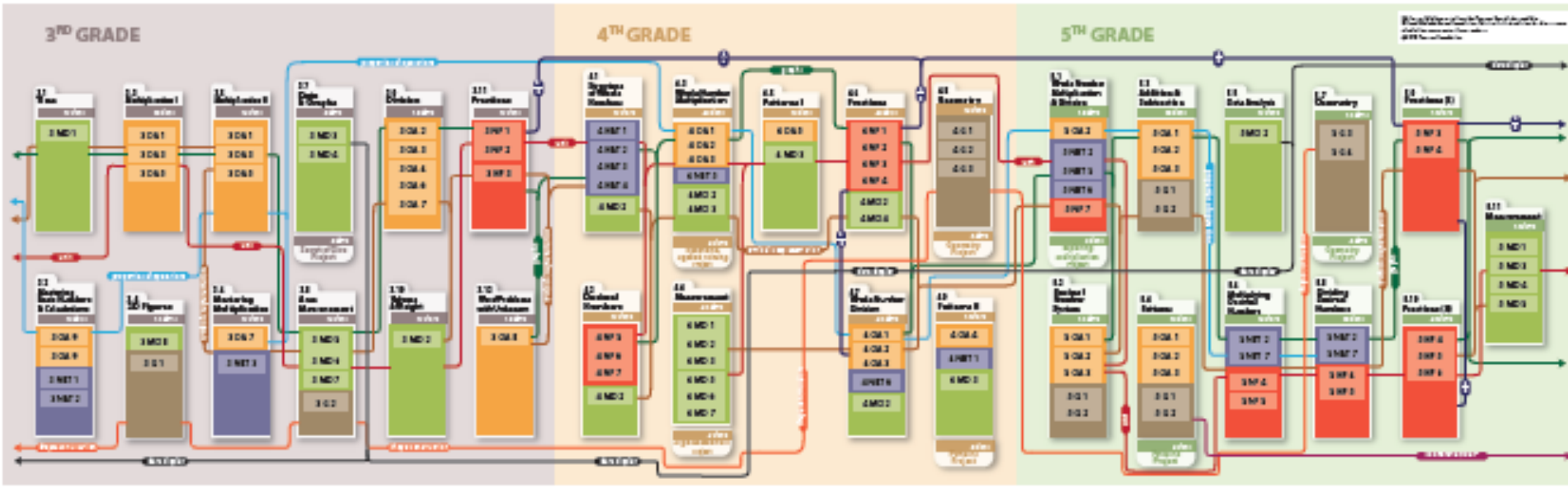
New K-12 Math Curriculum Inspired by The Common Core State Standards

BILL & MELINDA GATES foundation

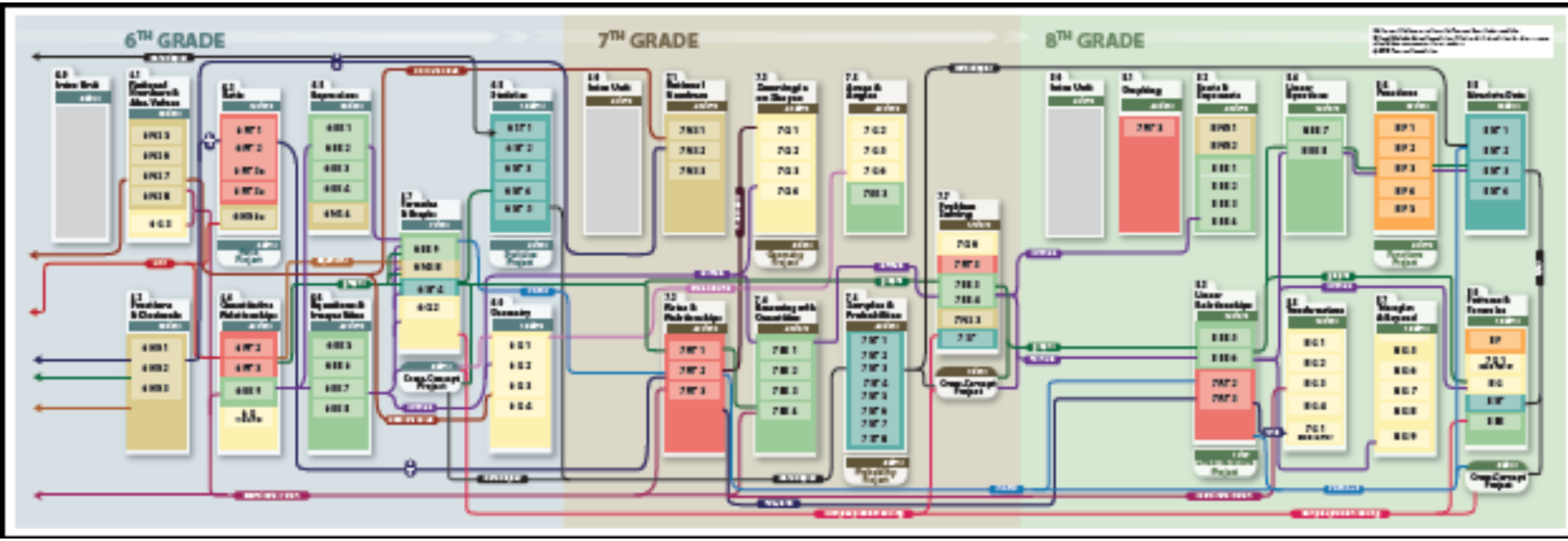
PEARSON FOUNDATION



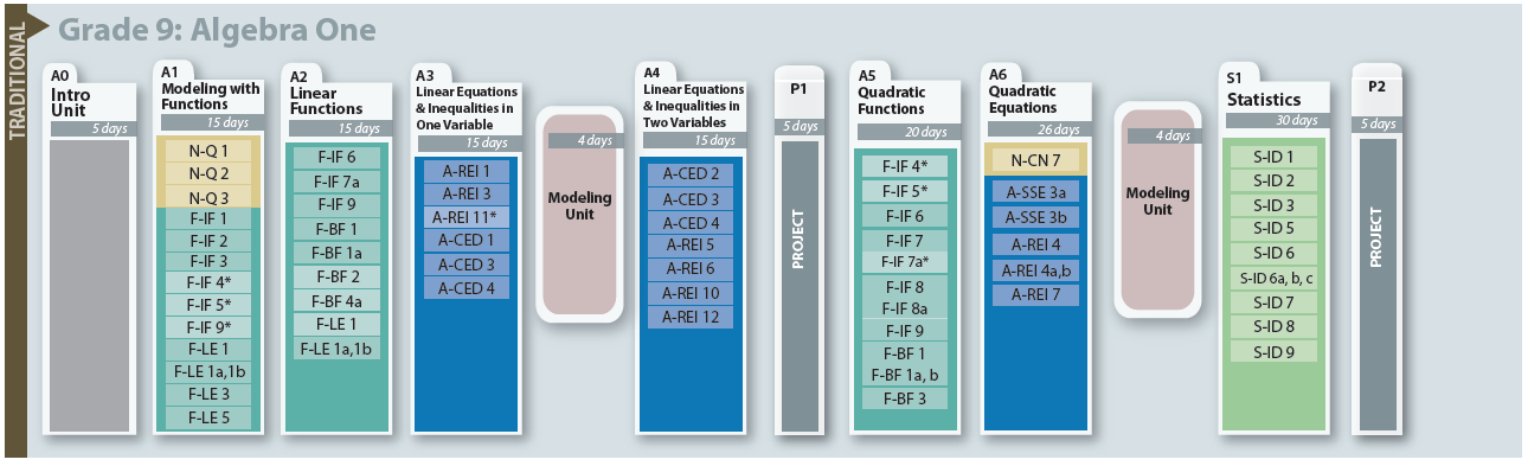

KINDERGARTEN
1ST GRADE
2ND GRADE
K.1 Playing With Shapes (Square, Triangle, Circle)
K.2 Numbers up to 10
K.3 Compose, Decompose Numbers up to 10
K.4 Shapes (two dimensional)
K.5 Addition & Addition Situations
K.6 Subtraction & Subtraction Situations
K.7 Figures (3-dimensional)
K.8 Numbers up to 20
K.9 Compose, Decompose Numbers up to 20
K.10 Which is Longer?
K.11 Numbers up to 100
K.12 Which is Heavier?
K.13 Mastering Addition & Subtraction
1.1 Time
1.2 Addition &/or Subtraction With Three Numbers
1.3 2D Figures
1.4 Addition within 20
1.5 Subtraction within 20
1.6 Comparing Length
1.7 Problems with unknown (I)
1.8 3D Figures
1.9 Problems with unknown (II)
1.10 2D Figures
1.11 Numbers greater than 20
1.12 Organizing Data
Picture Book Project
2.1 Time
2.2 Mastering Basic Numbers & Calculations
2.3 Measurement of Length
2.4 Addition within 100
2.5 Subtraction within 100
2.6 2D Figures
2.7 Problems with Unknown (I)
2.8 Data & Graphs
2.9 Problems with Unknown (II)
2.10 Numbers Greater Than 100
2.11 Calculation with Three Digit Numbers
2.12 3D Figures
Pattern Block Gallery Project
shape construction
symbolic representation
graphs
unit
properties of operations
data display

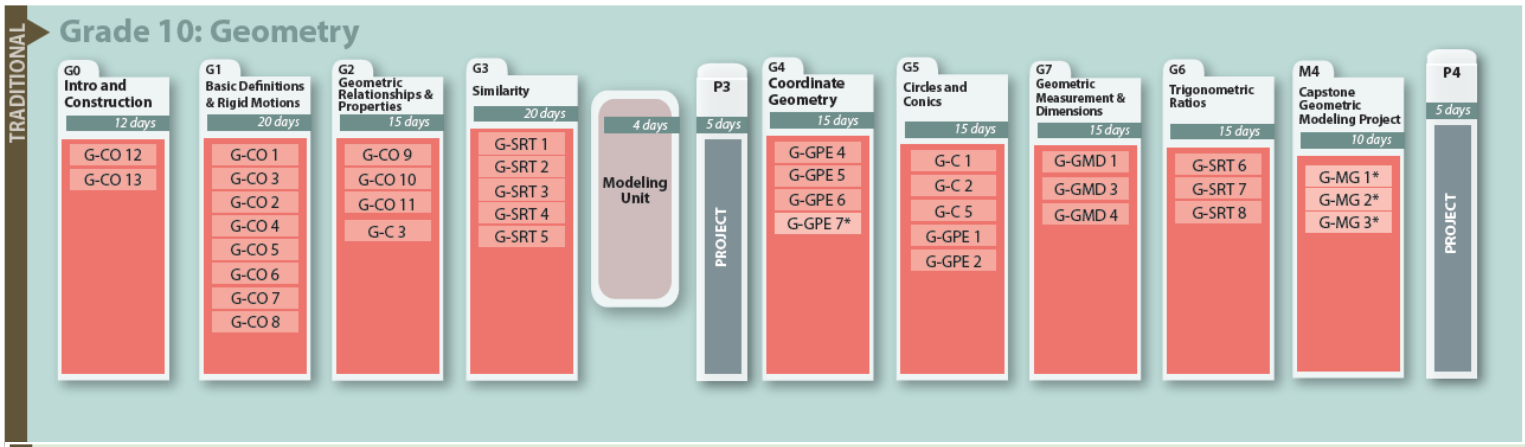

3RD GRADE
4TH GRADE
5TH GRADE

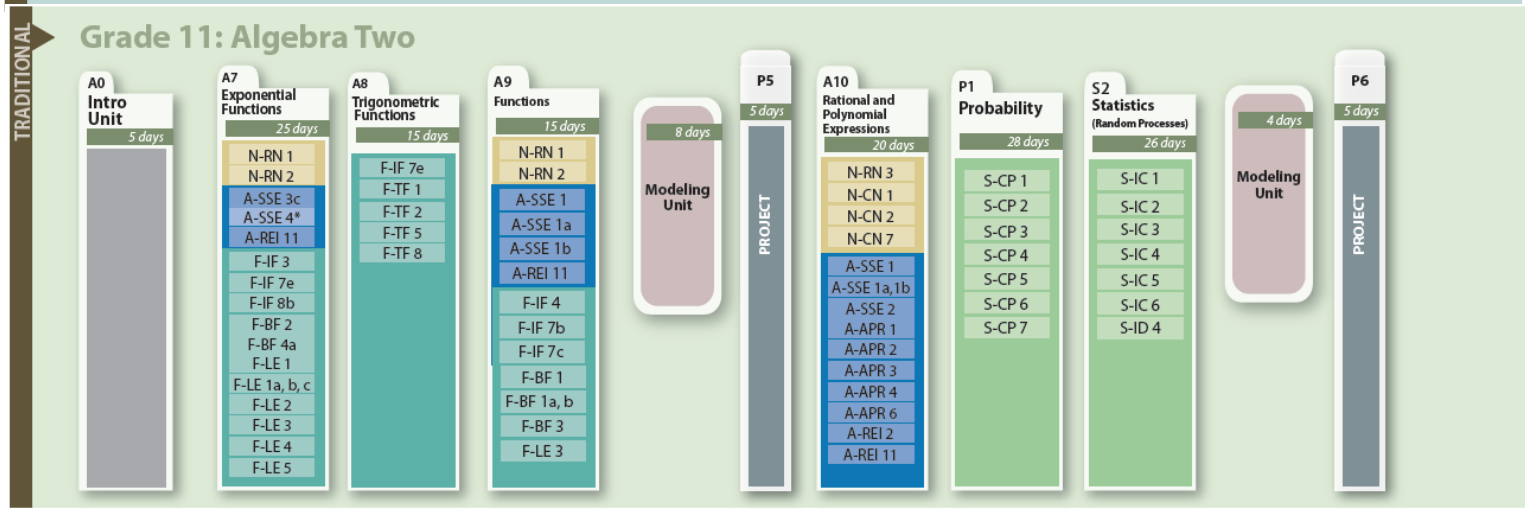
Middle School Curriculum



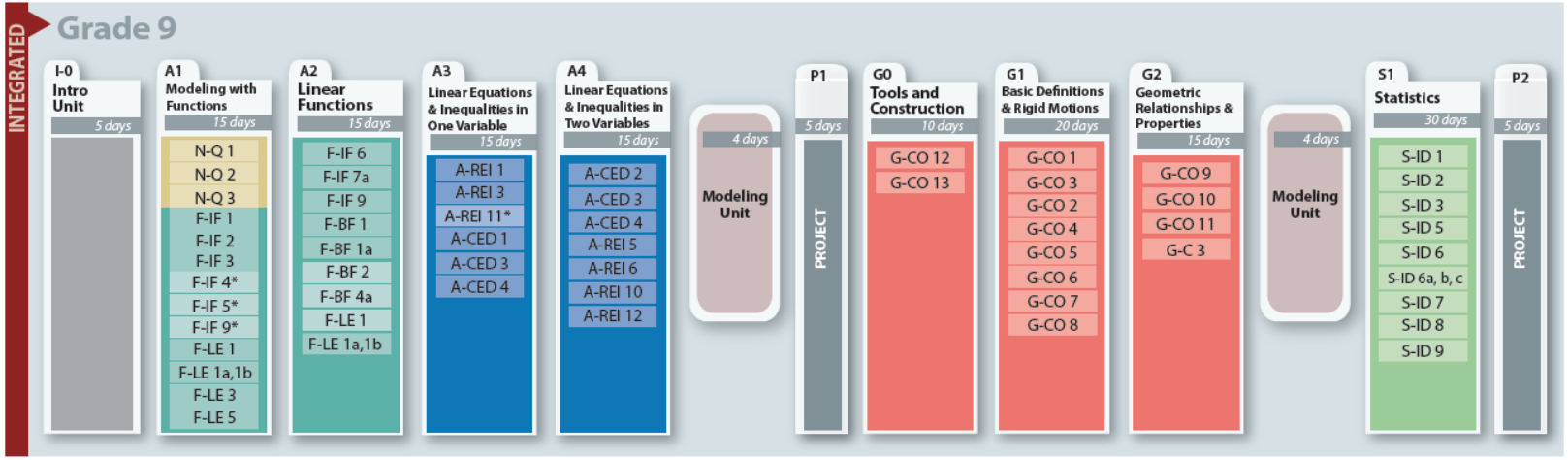
TRADITIONAL

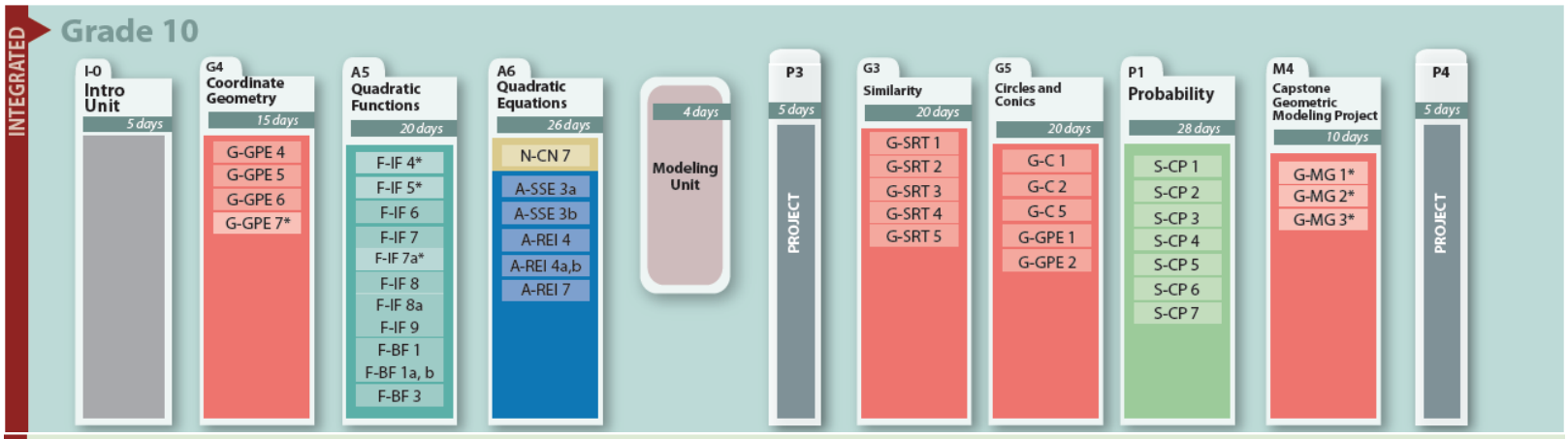


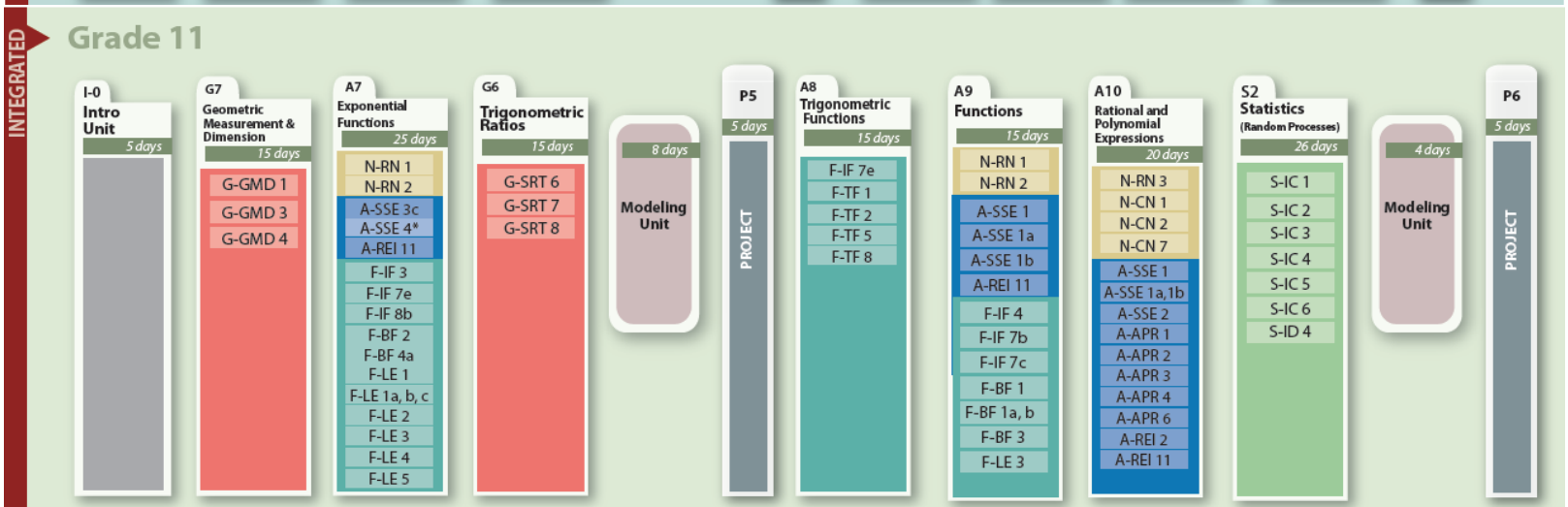




INTEGRATED

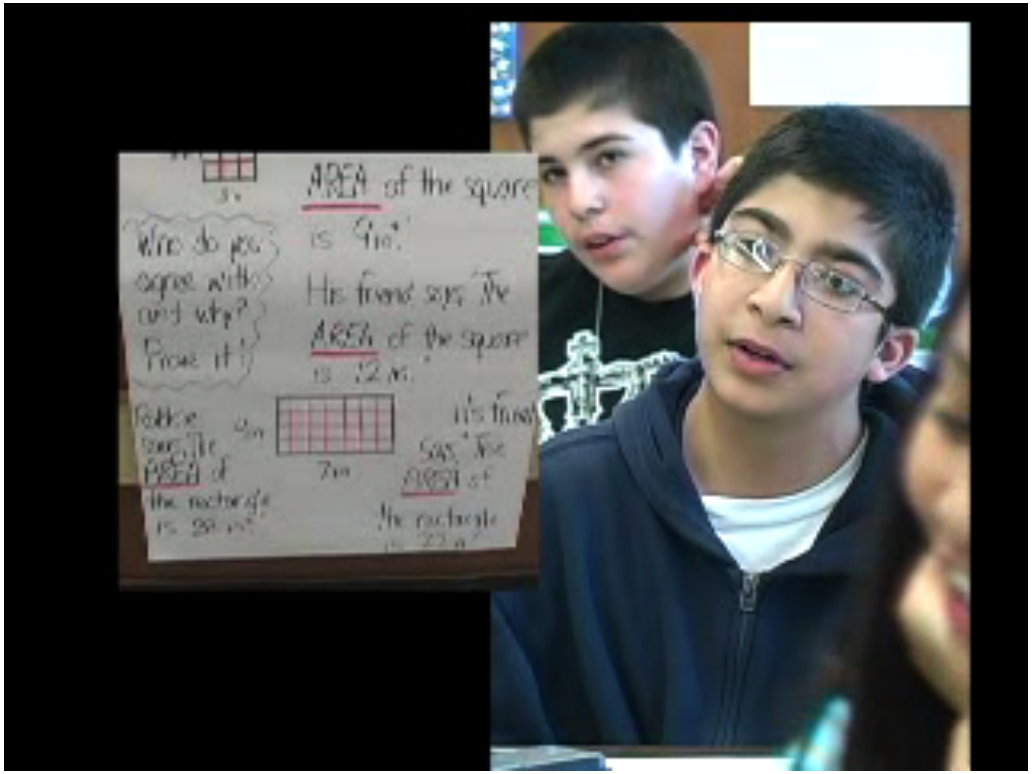

INTEGRATED
Grade 9
I-0
Intro Unit
5 days
A1
Modeling with Functions
15 days
N-Q 1
N-Q 2
N-Q 3
F-IF 1
F-IF 2
F-IF 3
F-IF 4*
F-IF 5*
F-IF 9*
F-LE 1
F-LE 1a,1b
F-LE 3
F-LE 5
A2
Linear Functions
15 days
F-IF 6
F-IF 7a
F-IF 9
F-BF 1
F-BF 1a
F-BF 2
F-BF 4a
F-LE 1
F-LE 1a,1b
A3
Linear Equations & Inequalities in One Variable
15 days
A-REI 1
A-REI 3
A-REI 11*
A-CED 1
A-CED 3
A-CED 4
A4
Linear Equations & Inequalities in Two Variables
15 days
A-CED 2
A-CED 3
A-CED 4
A-REI 5
A-REI 6
A-REI 10
A-REI 12
4 days
Modeling Unit
P1
5 days
PROJECT
G0
Tools and Construction
10 days
G-CO 12
G-CO 13
G1
Basic Definitions & Rigid Motions
20 days
G-CO 1
G-CO 3
G-CO 2
G-CO 4
G-CO 5
G-CO 6
G-CO 7
G-CO 8
G2
Geometric Relationships & Properties
15 days
G-CO 9
G-CO 10
G-CO 11
G-C 3
4 days
Modeling Unit
S1
Statistics
30 days
S-ID 1
S-ID 2
S-ID 3
S-ID 5
S-ID 6
S-ID 6a, b, c
S-ID 7
S-ID 8
S-ID 9
P2
5 days
PROJECT


INTEGRATED
Grade 10
I-0
Intro Unit
5 days
G4
Coordinate Geometry
15 days
G-GPE 4
G-GPE 5
G-GPE 6
G-GPE 7*
A5
Quadratic Functions
20 days
F-IF 4*
F-IF 5*
F-IF 6
F-IF 7
F-IF 7a*
F-IF 8
F-IF 8a
F-IF 9
F-BF 1
F-BF 1a, b
F-BF 3
A6
Quadratic Equations
26 days
N-CN 7
A-SSE 3a
A-SSE 3b
A-REI 4
A-REI 4a,b
A-REI 7
4 days
Modeling Unit
P3
5 days
PROJECT
G3
Similarity
20 days
G-SRT 1
G-SRT 2
G-SRT 3
G-SRT 4
G-SRT 5
G5
Circles and Conics
20 days
G-C 1
G-C 2
G-C 5
G-GPE 1
G-GPE 2
P1
Probability
28 days
S-CP 1
S-CP 2
S-CP 3
S-CP 4
S-CP 5
S-CP 6
S-CP 7
M4
Capstone Geometric Modeling Project
10 days
G-MG 1*
G-MG 2*
G-MG 3*
P4
5 days
PROJECT


INTEGRATED
Grade 11
I-0
Intro Unit
5 days
G7
Geometric Measurement & Dimension
15 days
G-GMD 1
G-GMD 3
G-GMD 4
A7
Exponential Functions
25 days
N-RN 1
N-RN 2
A-SSE 3c
A-SSE 4*
A-REI 11
F-IF 3
F-IF 7e
F-IF 8b
F-BF 2
F-BF 4a
F-LE 1
F-LE 1a, b, c
F-LE 2
F-LE 3
F-LE 4
F-LE 5
G6
Trigonometric Ratios
15 days
G-SRT 6
G-SRT 7
G-SRT 8
8 days
Modeling Unit
P5
5 days
PROJECT
A8
Trigonometric Functions
15 days
F-IF 7e
F-TF 1
F-TF 2
F-TF 5
F-TF 8
A9
Functions
15 days
N-RN 1
N-RN 2
A-SSE 1
A-SSE 1a
A-SSE 1b
A-REI 11
F-IF 4
F-IF 7b
F-IF 7c
F-BF 1
F-BF 1a, b
F-BF 3
F-LE 3
A10
Rational and Polynomial Expressions
20 days
N-RN 3
N-CN 1
N-CN 2
N-CN 7
A-SSE 1
A-SSE 1a,1b
A-SSE 2
A-APR 1
A-APR 2
A-APR 3
A-APR 4
A-APR 6
A-REI 2
A-REI 11
S2
Statistics
(Random Processes)
26 days
S-IC 1
S-IC 2
S-IC 3
S-IC 4
S-IC 5
S-IC 6
S-ID 4
4 days
Modeling Unit
P6
5 days
PROJECT

Formative Assessment

Navigating the Assessment Cycle to Inform Instruction



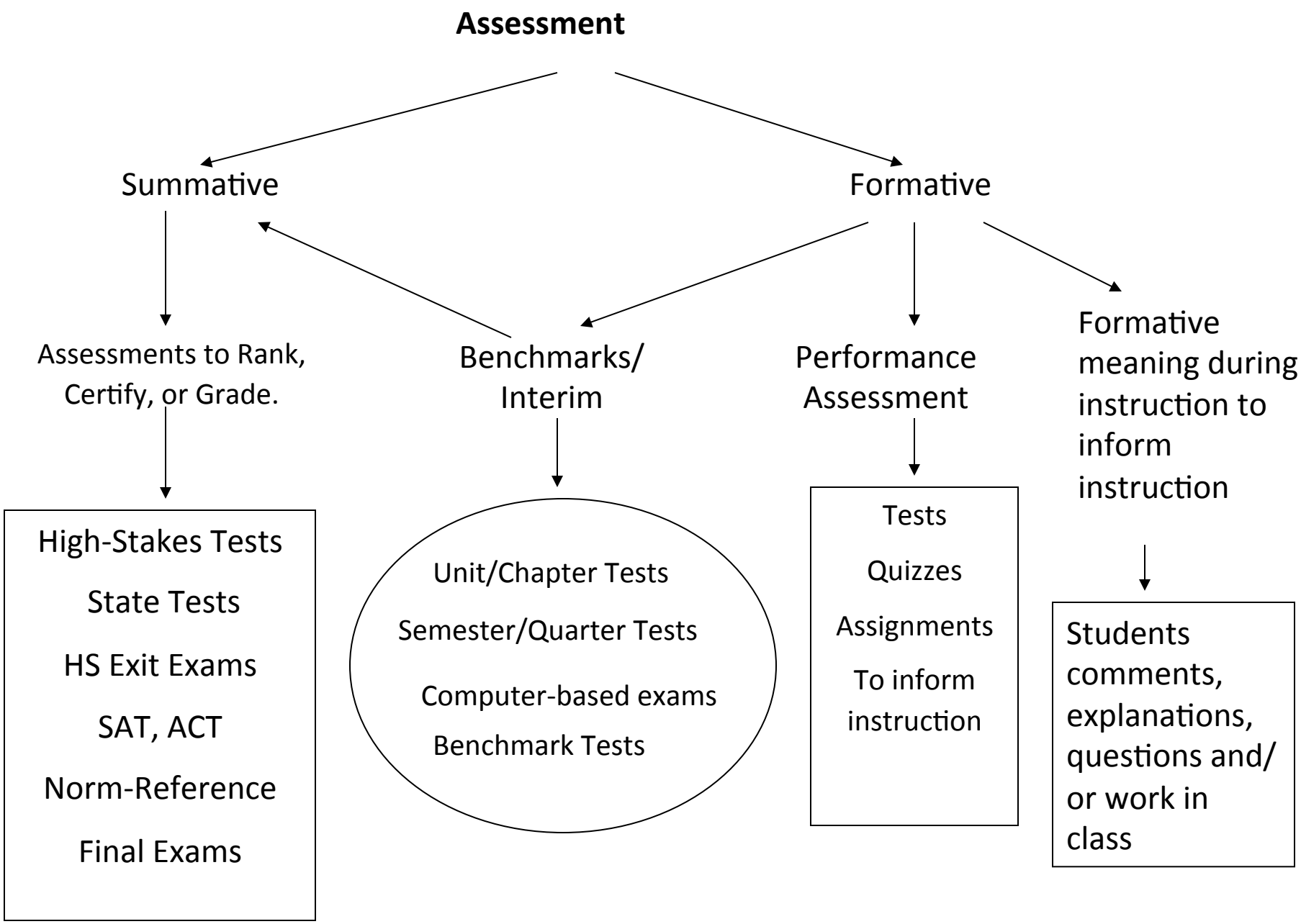
Looking at Student Work

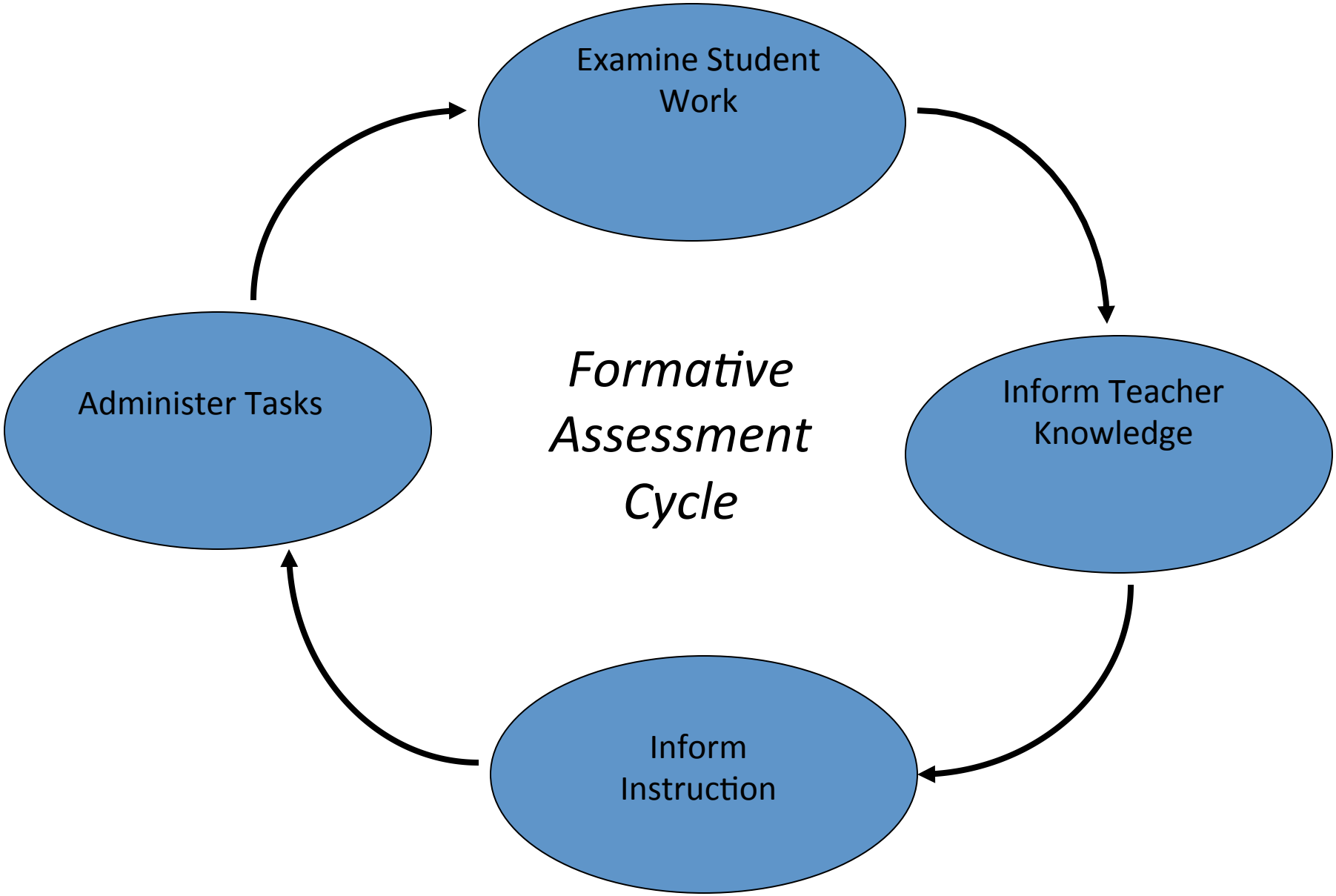
The process of studying student work is a meaningful and challenging way to be data-driven, to reflect critically on our instructional practices, and to identify the research we might study to help us think more deeply and carefully about the challenges our students provide us. Rich, complex work samples show us how students are thinking, the fullness of their factual knowledge, the connections they are making. Talking about them together in an accountable way helps us to learn how to adjust instruction to meet the needs of our students.

Annenberg Institute of School Reform

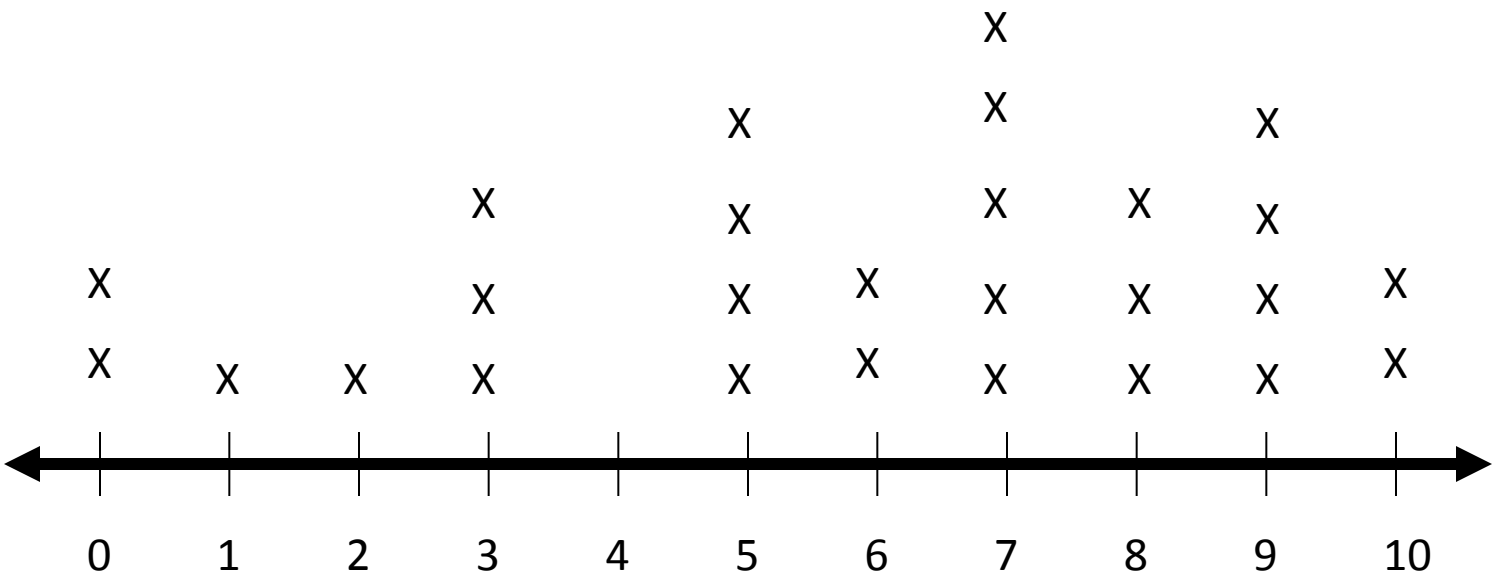
Educational Research:
Formative Assessment and Student Work to Inform Instruction

- *Assessing Student Outcomes*; Marzano, Pickering, McTighe
- *Inside the Black Box*; Black,Wiliams
- *Understanding by Design*; Wiggins, McTighe
- *Results Now*; Schmoker
- *Professional Learning Communities at Work*; Dufour, Eaker
- *Accountability for Learning*; Reeves
- *Math Talk Learning Community*; Fuson, et al
- *Normalizing Problems of Practice*; Little, Horn
- *Change the Terms for Teacher Learning*; Fullan
- *Working toward a continuum of professional development*; Loucks-Horsley, et al.


Assessment
Summative
Formative
Assessments to Rank, Certify, or Grade.
Benchmarks/ Interim
Performance Assessment
Formative meaning during instruction to inform instruction
High-Stakes Tests
State Tests
HS Exit Exams
SAT, ACT
Norm-Reference
Final Exams
Unit/Chapter Tests
Semester/Quarter Tests
Computer-based exams
Benchmark Tests
Tests
Quizzes
Assignments
To inform instruction
Students comments, explanations, questions and/ or work in class


Examine Student Work
Inform Teacher Knowledge
Inform Instruction
Administer Tasks
Formative Assessment Cycle

The Results from an Assessment



Students' performances are across the continuum

Traditionally Teachers Choose One of Three Options

- Go back and re-teach the topic with the entire class.
- Identify the students needing remediation and find some time/opportunity to re-teach the topic while the rest of the class continues on.
- Feeling the pressure of the over packed curriculum the teacher ventures on to the next topic.

Inside the Black Box

by Paul Black and Dylan Wiliam, *Phi Delta Kappan*, copyright 1998 http://blog.discoveryeducation.com/assessment/files/2009/02/blackbox_article.pdf

Follow up research:

Working Inside the Black Box

Inside the Black Box

Raising Standards Through Classroom Assessment

BY PAUL BLACK AND DYLAN WILIAM

Firm evidence shows that formative assessment is an essential component of classroom work and that its development can raise standards of achievement, Mr. Black and Mr. Wiliam point out. Indeed, they know of no other way of raising standards for which such a strong prima facie case can be made.

RAISING the standards of learning that are achieved through schooling is an important national priority. In recent years, governments throughout the world have been more and more vigorous in making changes in pursuit of this aim. National, state, and district standards; target setting; enhanced programs for the external testing of students' performance; surveys such as NAEP (National Assessment of Educational Progress) and TIMSS (Third International Mathematics and Science Study); initiatives to improve school plan-

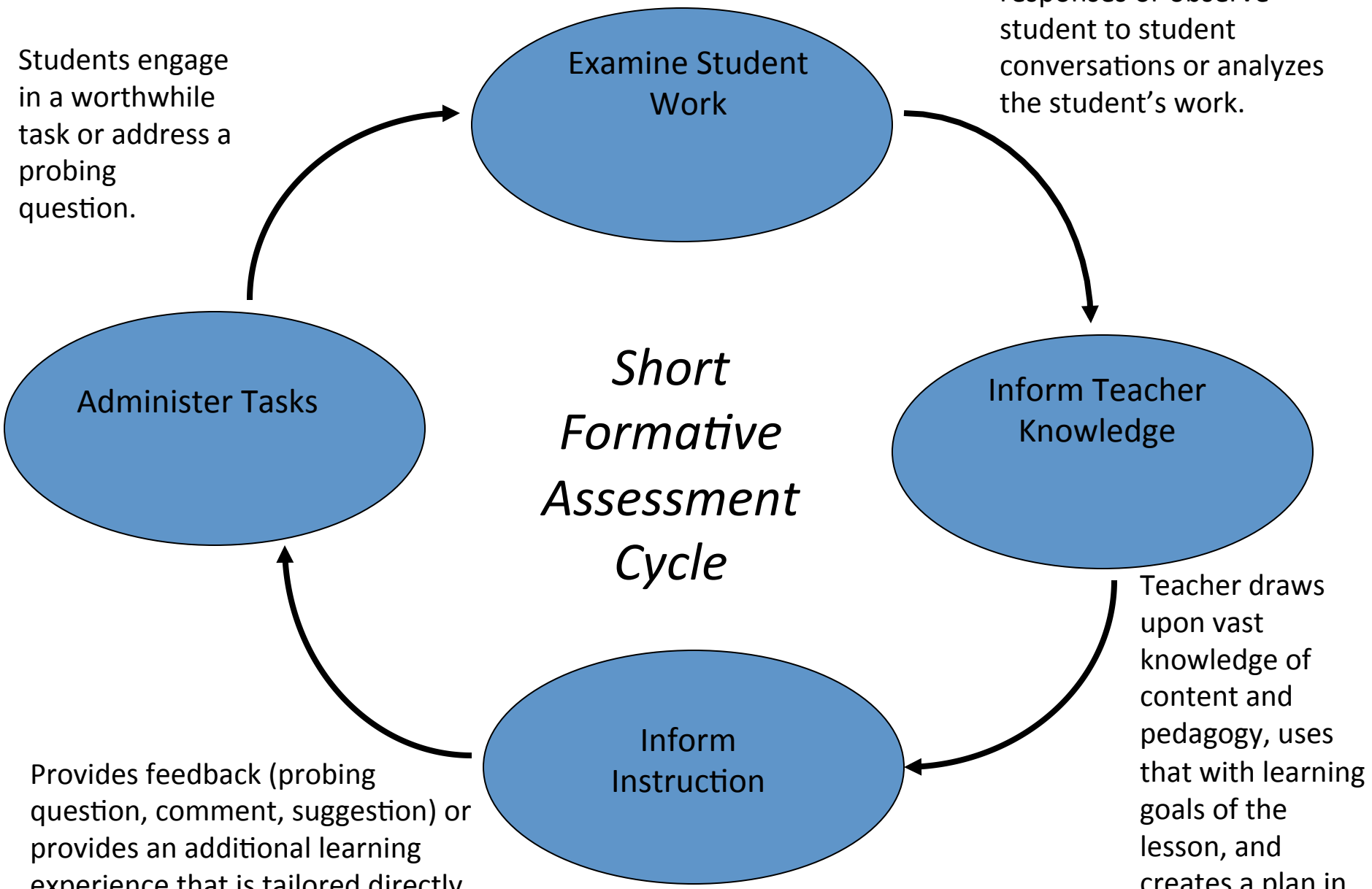
PAUL BLACK is professor emeritus in the School of Education, King's College, London, where DYLAN WILIAM is head of school and professor of educational assessment.

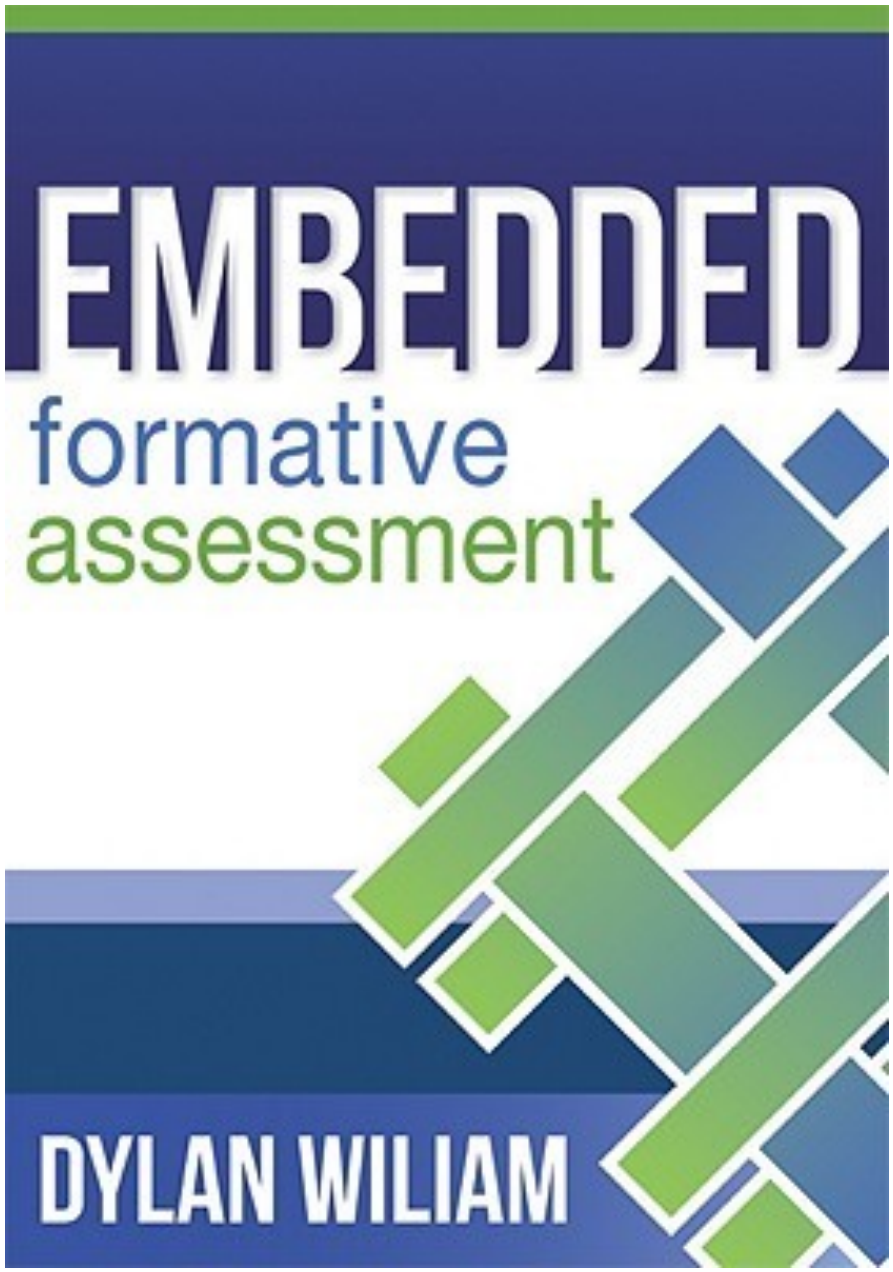
Illustration by A. J. Garces

Formative Assessment is:

Students and teachers
Using evidence of learning
To adapt teaching and learning
To meet immediate learning needs
Minute-to-minute and day to day

Dylan Wiliam, University of London


Examine Student Work
Teacher listens to student responses or observe student to student conversations or analyzes the student's work.
Inform Teacher Knowledge
Teacher draws upon vast knowledge of content and pedagogy, uses that with learning goals of the lesson, and creates a plan in real time.
Inform Instruction
Provides feedback (probing question, comment, suggestion) or provides an additional learning experience that is tailored directly to the students' learning needs.
Administer Tasks
Students engage in a worthwhile task or address a probing question.
Short Formative Assessment Cycle



STRATEGIES OF FORMATIVE ASSESSMENT

Clarifying, Sharing and Understanding Learning Intentions and Success Criteria

Eliciting Evidence of Learners' Achievement

Providing Feedback That Moves Learning Forward

Activating Students as Instructional Resources for One Another

Activating Students as owners of their own learning.

Using Performance Assessment for Formative Purposes

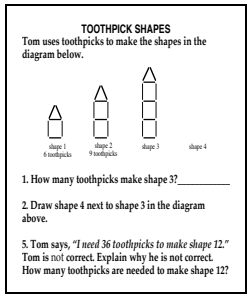
Re-Engagement Lessons

MARS Assessment Tasks

Developing Teacher Content Knowledge

Tools for Teachers

MARS Tasks



TOOTHPICK SHAPES

Tom uses toothpicks to make the shapes in the diagram below.

shape 1 6 toothpicks | shape 2 9 toothpicks | shape 3 | shape 4

1. How many toothpicks make shape 3?_____

2. Draw shape 4 next to shape 3 in the diagram above.

5. Tom says, *"I need 36 toothpicks to make shape 12."* Tom is not correct. Explain why he is not correct. How many toothpicks are needed to make shape 12?

Scoring and Student Works Protocols

Examine Student Work

Inform Teacher Knowledge

Inform Instruction

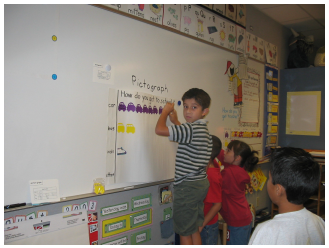
Administer Tasks

Formative Assessment Cycle

Common Core Standards

Tools for Teachers and PD Materials

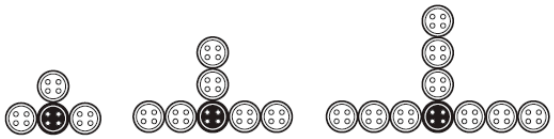
Re-engagement Lessons



Buttons

This problem gives you the chance to:
- describe, extend, and make generalizations about a numeric pattern

Gita plays with her grandmother's collection of black and white buttons.
She arranges them in patterns.
Her first 3 patterns are shown below.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

1. Draw Pattern 4 next to Pattern 3.

2. How many **white** buttons does Gita need for Pattern 5 and Pattern 6?

   Pattern 5 _____ Pattern 6 _____

Explain how you figured this out.

3. How many buttons in all does Gita need to make Pattern 11?

   _____

Explain how you figured this out.

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is **not** correct?

How many buttons does she need to make Pattern 24? _____

Buttons

Test 5 Form A Rubric

The core elements of performance required by this task are: • describe, extend, and make generalizations about a numeric pattern Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Draws correct Pattern 4.	1	1
2. Gives correct answers as: Pattern 5 = **15** white buttons Pattern 6 = **18** white buttons Gives explanation such as: Pattern 5 has 3 rows of 5 white buttons = 15 white buttons and Pattern 6 has 3 rows of 6 white buttons = 18 white buttons. *Special case:* Gives answers 16 and 19 with correct explanations including black buttons.	 1 1 1 2 s c	3
3. Gives correct answer as: **34** Gives explanation such as: Pattern 11 has 3 rows of 11 white buttons = 33 white buttons and 1 black button. *Special case:* Gives answer as 33 with a correct explanation for the white buttons.	 1 1 1 s c	2
4. Gives explanation such as: Pattern 24 needs 3 rows of 24 white buttons = 72 white buttons and 1 black button, 73 buttons in all. *Accept alternative correct explanations such as: 69 is divisible by 3, so it cannot be correct.* Gives correct answer as: **73**	 1 1	2
Total Points		**8**

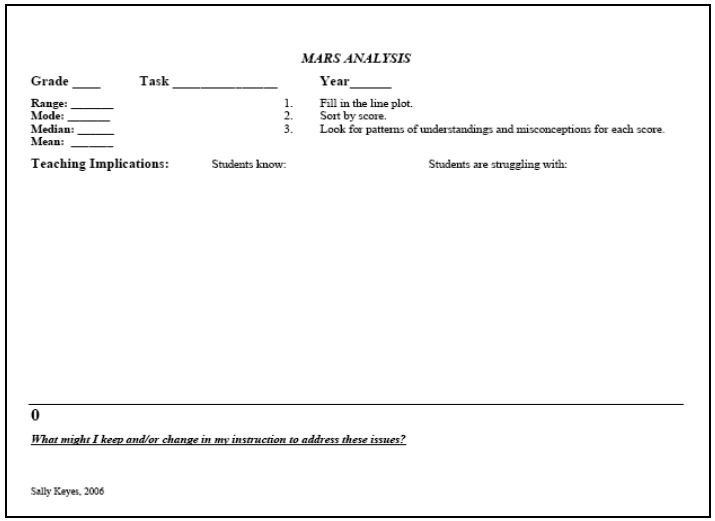
Analyzing Student Work

SVMI TOOLS

Analyzing Student Work

1. Sort the student work by score.
2. Flag interesting Work - unusual strategy, common misconceptions, good answer or "what were they thinking????"
3. From your student work complete MARS Analyzer (line plot and measures of center)
4. Discuss in group and complete *Score Analysis Chart*
5. Complete the MARS Task Analysis Sheet

MARS Analyzer - line plot



MARS ANALYSIS

Grade ____ **Task _____** **Year_____**

Range: _____
Mode: _____
Median: _____
Mean: _____

1. Fill in the line plot.
2. Sort by score.
3. Look for patterns of understandings and misconceptions for each score.

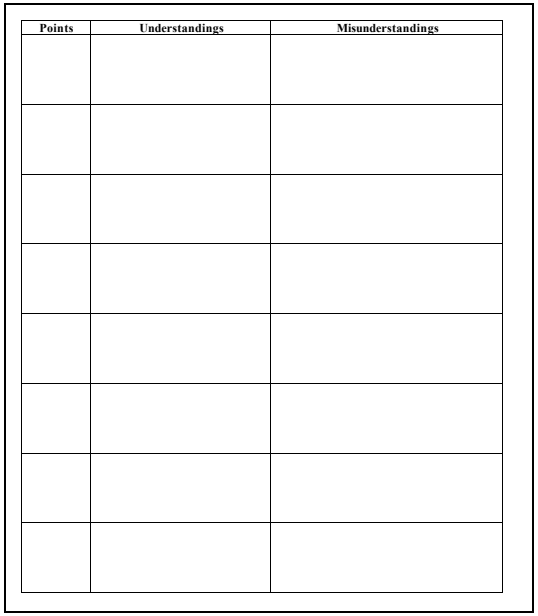
Teaching Implications: Students know: Students are struggling with:

0

What might I keep and/or change in my instruction to address these issues?

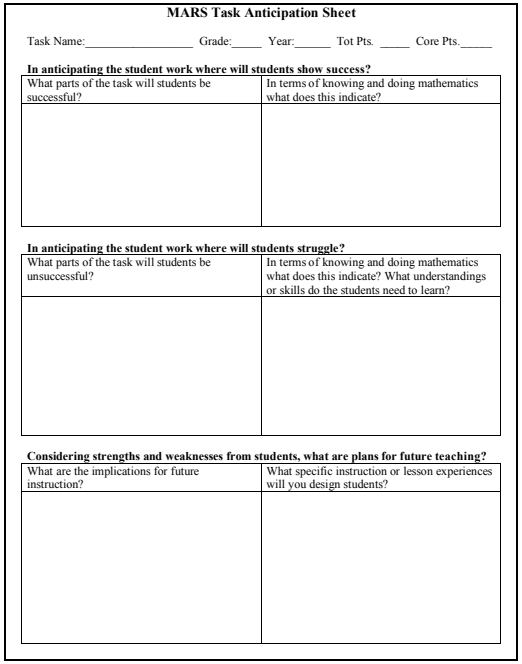
Sally Keyes, 2006

Score Analysis Chart



Points	Understandings	Misunderstandings

MARS Analysis Sheet



MARS Task Anticipation Sheet

Task Name:_____ Grade:_____ Year:_____ Tot Pts. _____ Core Pts._____

In anticipating the student work where will students show success?

What parts of the task will students be successful?	In terms of knowing and doing mathematics what does this indicate?

In anticipating the student work where will students struggle?

What parts of the task will students be unsuccessful?	In terms of knowing and doing mathematics what does this indicate? What understandings or skills do the students need to learn?

Considering strengths and weaknesses from students, what are plans for future teaching?

What are the implications for future instruction?	What specific instruction or lesson experiences will you design students?

Re-Engagement Protocol

Initial Activity

Describe the 'Story of the Task'

Examine the student work. What important mathematical ideas surfaced in the initial lesson and in students' written work?

Pick a few key examples from student work

They might represent common strategies, novel approaches, misconceptions, or lead to contradictory answers.

Design a Re-Engagement prompt

Use excerpts from the examples of student work to formulate a question to re-engage all students in the mathematics of the task.

Tools for Teachers
Linking Assessment and Learning

Questions for Reflection on Buttons:

- What are some rich problems that your students have done this year? What are some good resources for pattern problems?
- Do you ask questions like: "What stays the same?" and "What changes?" to help students develop the ability to form generalizations?
- Do students have opportunities to connect their number sentences to geometric patterns and share how they visualize the growth pattern?

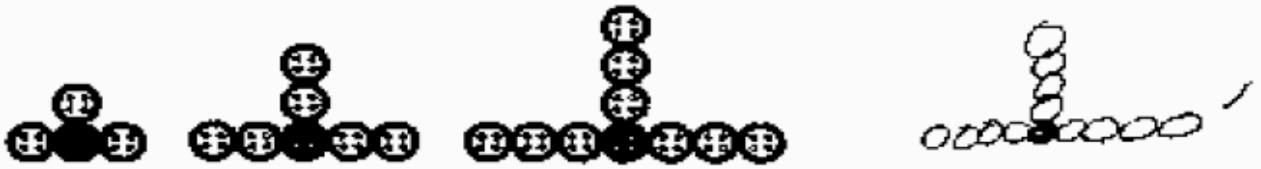
Look carefully at your student work. What strategies did they use in Part 4?

Draw a picture	24 x 3 or 24+24+24	24 x 3 + 1 or 24+24+24+1	Extend a table	Repeated addition of 3	Used a doubling strategy from a previous part of the problem

Teacher Notes:

Looking at Student Work – Buttons

Many students at this grade level are able to see the pattern and form a generalization in words or in number algorithms. These generalizations could easily be converted to algebraic symbols at later grade levels. Student A has a nice description of how the pattern grows and an algorithm for find the total number of buttons in part 4.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

1. Draw Pattern 4 next to Pattern 3.

2. How many white buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 15 buttons Pattern 6 18 buttons

Explain how you figured this out.

For each pattern there is the same number of white buttons on 3 sides of the black button, so you count the number of buttons on one side of the black buttons and multiply that by three.

3. How many buttons in all does Gita need to make Pattern 11?

34

Explain how you figured this out.

I did the same process in the problem above but I added 1 to my answer for the black button.

Student A, part 2

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is **not** correct?

Because if you follow the pattern of multiplying 3 to the number of buttons on one side and plus one for black button it doesn't equal 69.

How many buttons does she need to make Pattern 24? 73 buttons

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ +\ 1 \\ \hline 73 \end{array}$$

For further examples of making generalizations and algorithms, look at the work of Student B and C.

Student B

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is **not** correct?

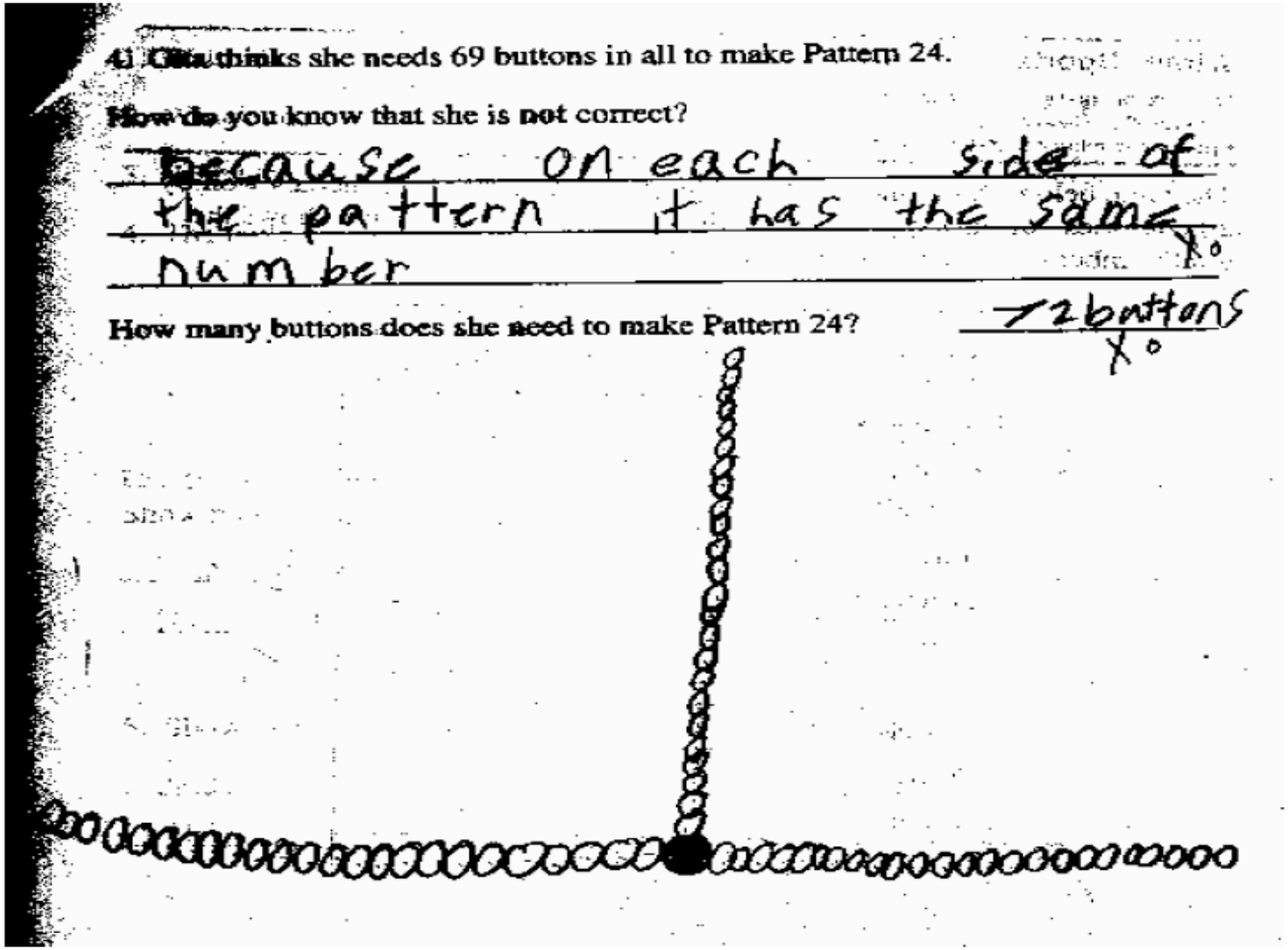
I know because (24 X 3) + 1 does NOT equal 69.

How many buttons does she need to make Pattern 24? 73

24
X3
72 + 1 = 73

Some students can find the answer or come close to a solution, but use strategies that are inefficient, tedious, and may lead to errors. Student F uses a drawing and counting strategy. Student G does not see the complete pattern and solves for the white buttons every time. Student G uses repeated addition, which is a correct process, but makes a calculation error. Student H uses graphing, but because of the inaccuracies of the graph, the student's answer is incorrect.

Student F



Buttons

Points	Understandings	Misunderstandings
0	Most students attempted this task.	
3	Students could make a drawing of the next pattern in a sequence. They could also find the number of white buttons for pattern 5 and 6.	Students had difficulty explaining how they figured out the white buttons in the pattern. They said things like, "I added", without specifying what or why.
6	Students could find and extend a pattern. To find a solution in part 4,	They did not convert from white buttons to finding the pattern for all buttons in part 3 and 4. 37% of all the students multiplied 24 by 3 in part 4, but did not add on the black button.
7	31% of all students used the expression of 24 times 3 plus 1 to solve for part 4. Another 7% used 24+24+24+1.	Some students used inefficient strategies, which may have led to errors. 13% drew a picture. 8% made a table and extended it, 3% just kept adding 3, and some students drew a graph.
8	Students could continue a pattern using a drawing. They could also find rules for extending it without drawing.	

5th grade Task 3 Buttons

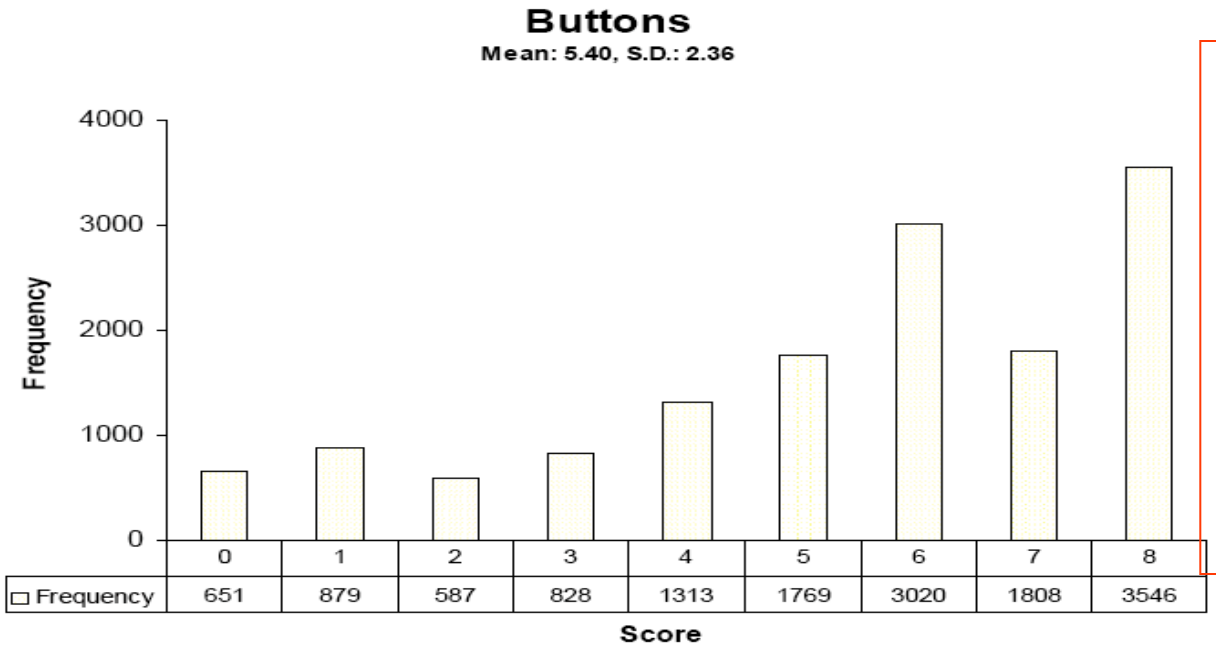
Student Task	Use a button arrangement pattern to describe, extend, and make generalization about its numeric pattern.
Core Idea 3 Patterns and Functions	**Understand patterns and use mathematical models such as algebraic symbols and graphs to represent and understand quantitative relationships.** • Describe and extend numeric patterns (3rd grade) • Represent and analyze patterns and functions using words (4th grade) • Investigate how a change in one variable relates to a change in a second variable

Based on teacher observations, this is what fifth grade students know and are able to do:

- Continue a pattern using pictures and numbers
- Explain how a pattern grows and use that algorithm to solve for larger numbers in the pattern

Areas of difficulty for fifth graders, fifth grade students struggled with:

- Distinguishing between part of a pattern and the whole pattern
- Explaining a pattern in words



70% were unable to explain how to find pattern 11

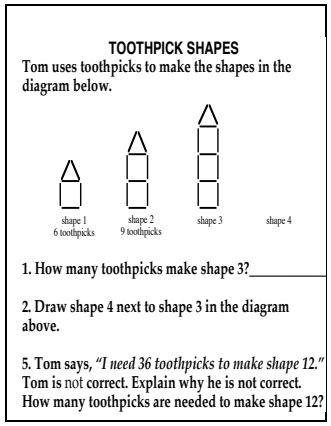
Score:	0	1	2	3	4	5	6	7	8
% < =	4.5%	10.6%	14.7%	20.4%	29.6%	41.9%	62.8%	75.4%	100.0%
% > =	100.0%	95.5%	89.4%	85.3%	79.6%	70.4%	58.1%	37.2%	37.2%

The maximum score available for this task is 8 points.
The cut score for a level 3 response is 4 points.

Most students (about 89%) could draw pattern 4 and give the correct number of buttons for pattern 5 and pattern 6. Many students (about 80%) could draw and extend the pattern and explain in words how the patterned worked. More than half the students (about 60%) could draw the pattern, extend the pattern for 5 and 6 and explain how it grew, find and explain the number of white buttons for pattern 11, and could find the total buttons for pattern number 24. Almost 26% of the students met all the demands of the task. About 5% of the students scored no points on this task.

Cycle of Formative Assessment to Inform and Improve Learning

Administer high-quality assessment tasks



TOOTHPICK SHAPES

Tom uses toothpicks to make the shapes in the diagram below.

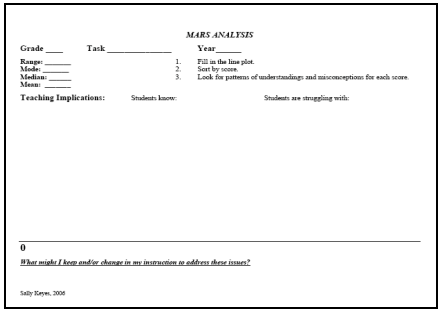
shape 1 — 6 toothpicks; shape 2 — 9 toothpicks; shape 3; shape 4

1. How many toothpicks make shape 3?_____

2. Draw shape 4 next to shape 3 in the diagram above.

5. Tom says, *"I need 36 toothpicks to make shape 12."* Tom is not correct. Explain why he is not correct. How many toothpicks are needed to make shape 12?

Collectively score and analyze student work



MARS ANALYSIS

Grade ____ Task _____ Year____

Range: _____
Mode: _____
Median: _____
Mean: _____

1. Fill in the line plot.
2. Sort by score.
3. Look for patterns of understandings and misconceptions for each score.

Teaching Implications: Students know: Students are struggling with:

0

What might I keep and/or change in my instruction to address these issues?

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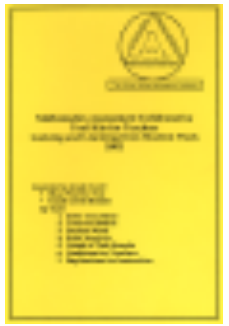
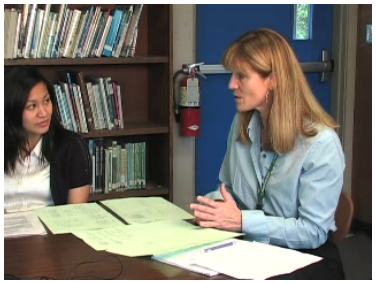


Points	Understandings	Misunderstandings

Document student thinking to inform instruction.



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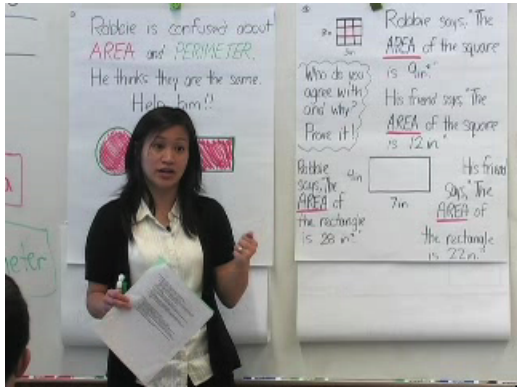
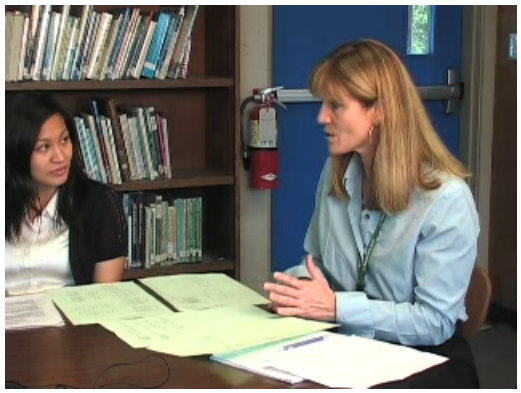
Drives the professional development experiences of the teachers to plan experiences focused on their students.

Leads to improved teaching and learning in the classroom (re-engagement)

Re-engagement





- Makes use of actual student work - including unique thinking, misconceptions and strategies.
- Has all students re-work a task from different perspectives.
- Confronts misconceptions, so that they can be dealt with and let go.
- Gives some students strategies for solving problem.
- Helps other students solidify, connect, and clarify their ideas.

Examine Learner A's Work

Learner A

3. How many buttons in all does Gita need to make Pattern 11?

34 Buttons

Explain how you figured this out.

I x (11 x 3) + 1 = 34 buttons
I added one for the black button in the middle

How is Learner A making sense of the mathematics?

Examine Learner B's Work

Learner B

3. How many buttons in all does Gita need to make Pattern 11?

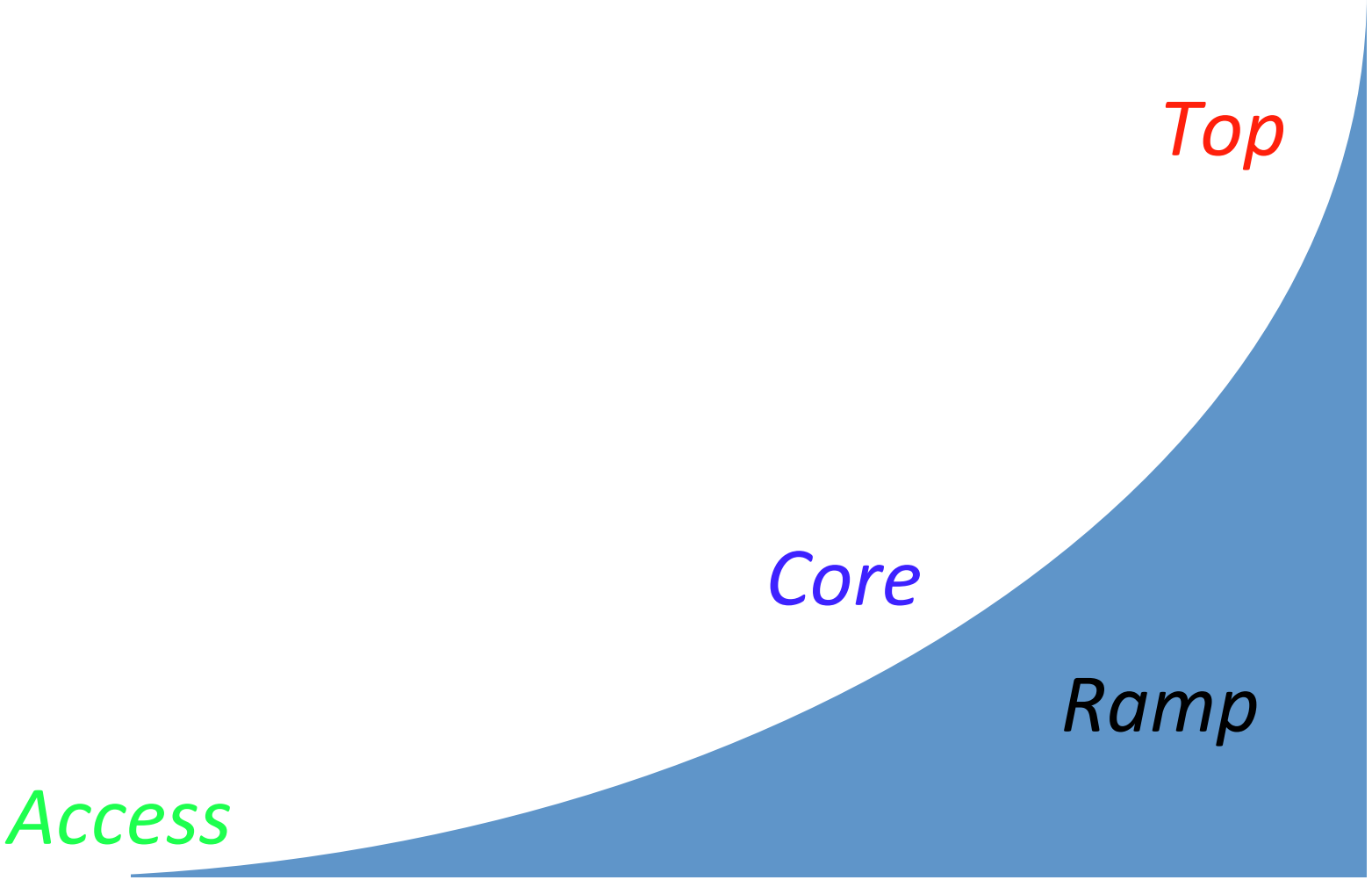
34

Explain how you figured this out.

I added 4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 34 which is the # of buttons.

How is Learner B making sense of the mathematics?

The design of *scaffolded* performance assessment tasks



San Carlos Charter School

Small Group Discussions

How was the re-engagement activity designed to provide access to all students in class?

What were the basic concepts the students needed to learn and understand?

What did the students communicate?

What did the teachers emphasize?

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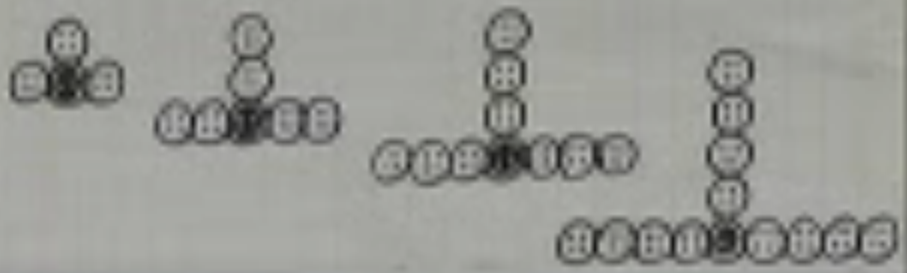
Weds 10/08/08

Grade 5 Task 3

Buttons



Pattern #

Small Group Discussions

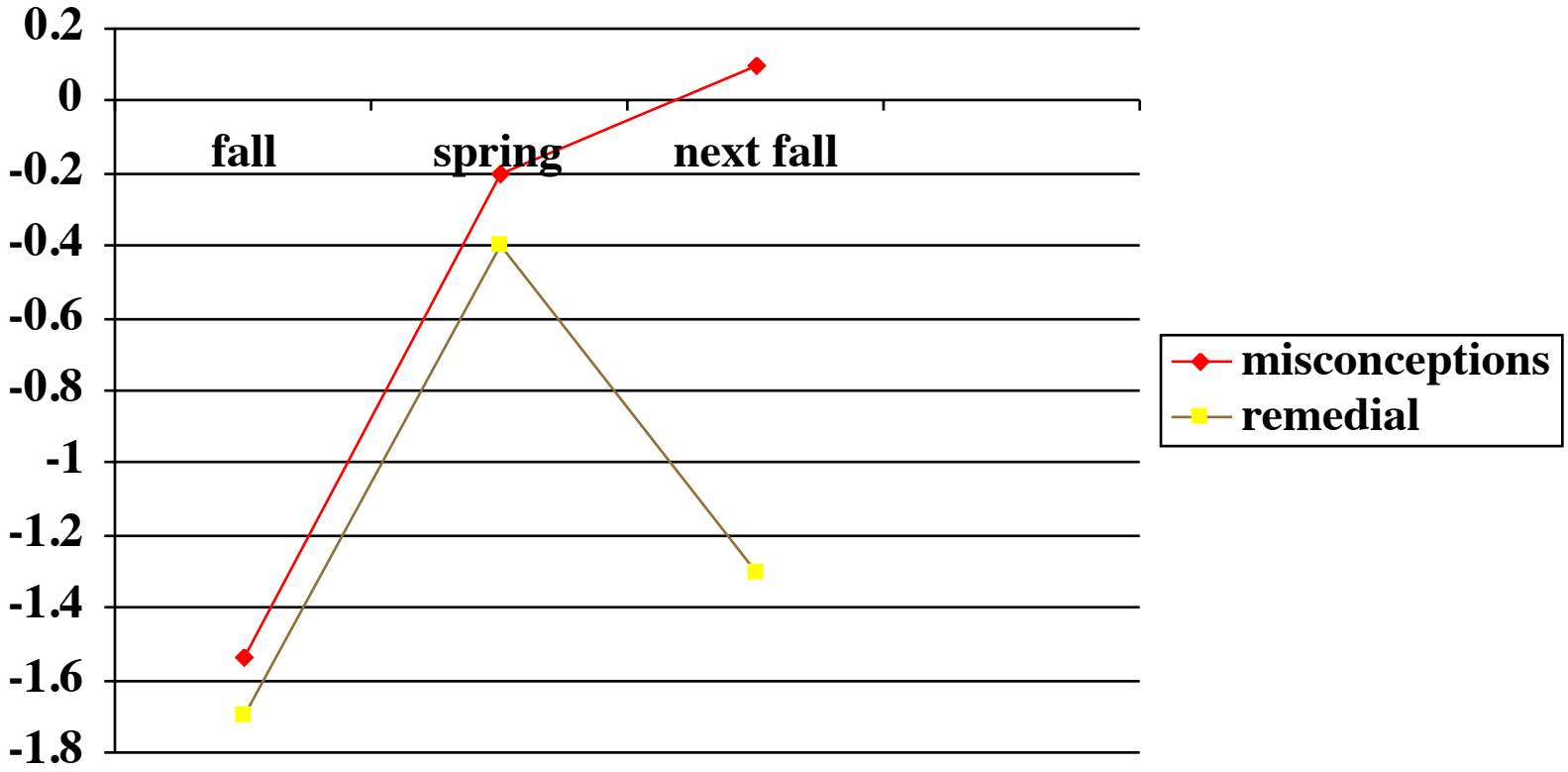
How was the re-engagement lesson designed to provide access, address the core, and create high cognitive demand?

How were the students making sense of the mathematics?

How did re-engagement promote class discussions?

Describe important teacher moves that facilitated the lesson.

Bell and Swan study



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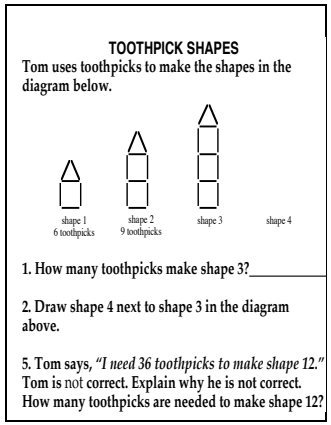

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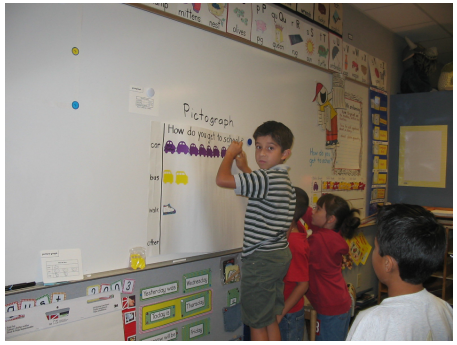


Leads to improved teaching and learning in the classroom (Re-engagement lessons)

Re-teaching vs. Re-engagement

- Teach the unit again.
- Address basic skills that are missing.
- Do the same or similar problems over.
- Practice more to make sure student learn the procedures.
- Focus mostly on underachievers.
- Cognitive level is usually lower.

- Revisit student thinking.
- Address conceptual understanding.
- Examine task from different perspective.
- Critique student approaches/ solutions to make connections.
- The entire class is engaged in the math.
- Cognitive level is usually higher.

1. Make sense of problems and persevere in solving them.

Traffic jam

Last Sunday an accident caused a traffic jam 12 miles long on a freeway

When the accident was cleared, the cars drove away from the front, one car every two seconds. Estimate how long it took before the last car moved.

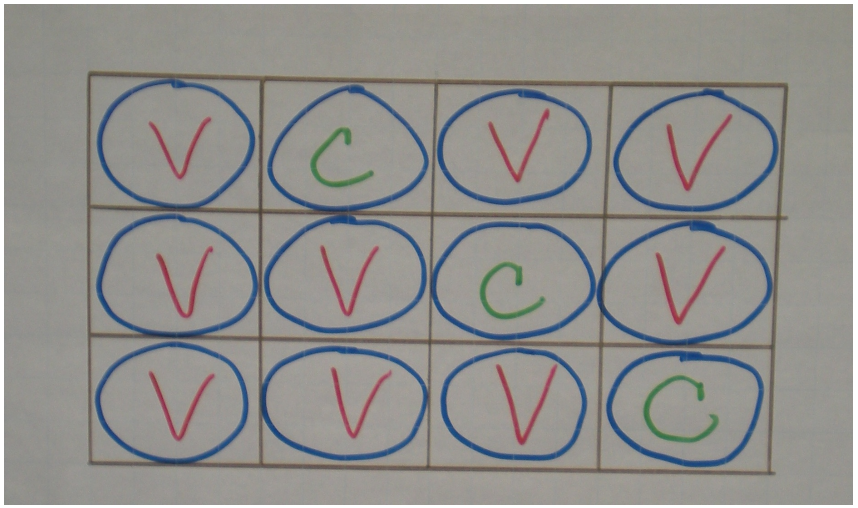
2. Reason Abstractly and Quantitatively

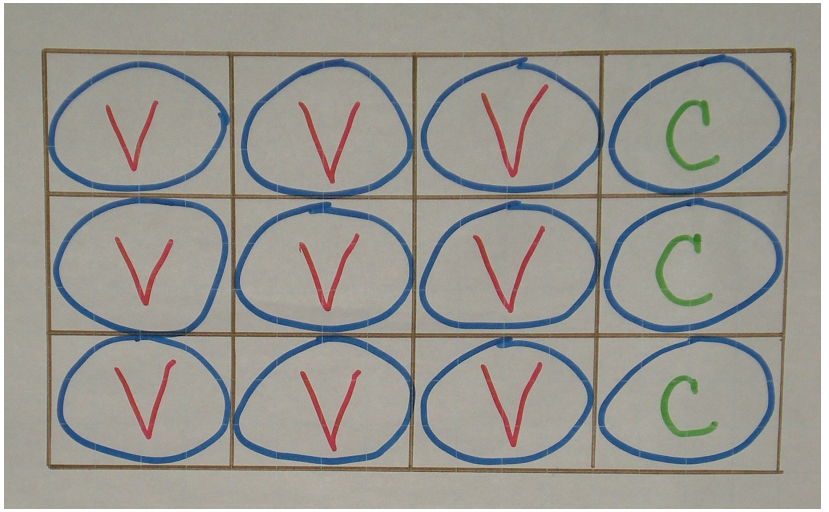
Where are a+b, b-a and a-b?

What can you say about where a/b is?

3. Construct viable arguments and critique the reasoning of others.

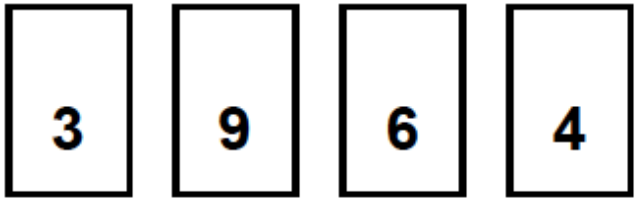
Valerie shares some of the 12 candies. She gives Cindy 1 candy for every 3 candies she eats herself. How many does she give Cindy?



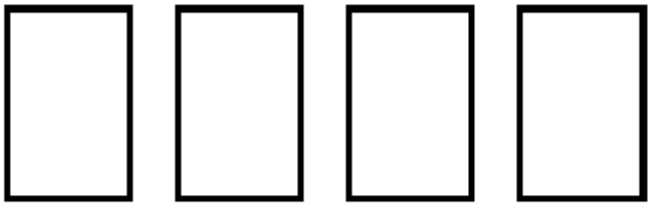


3. Construct viable arguments and critique the reasoning of others.

Jane and Tom are playing number card games. They have the four cards shown below.



5. Show how they arrange the four cards to make the number that is nearest to **5000**.



Explain how you figured it out.

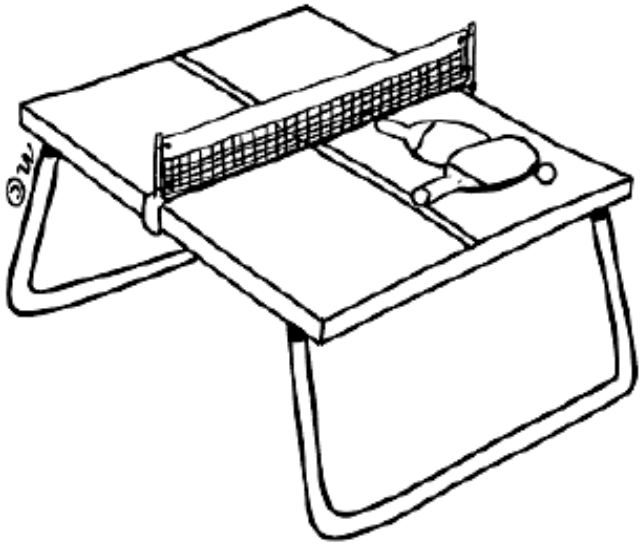
_____

_____

_____

4. Model with mathematics

PLANNING AND ORGANIZING: A TABLE TENNIS TOURNAMENT

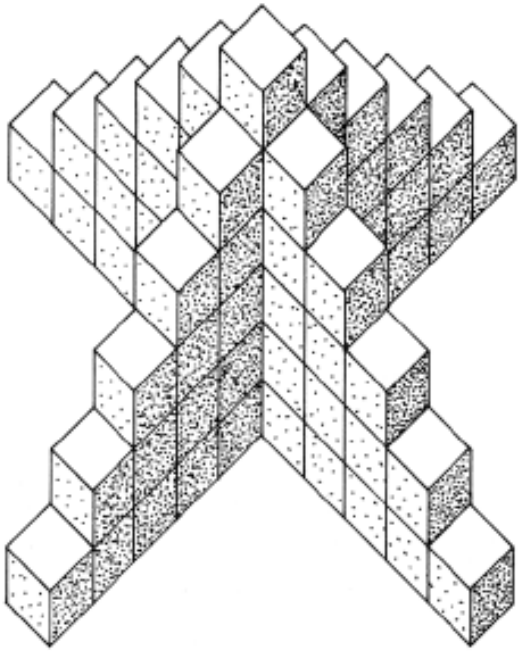


You have the job of organising a table tennis league.

- 7 players will take part
- All matches are singles.
- Every player has to play each of the other players once.
- There are four tables at the club.
- Games will take up to half an hour.
- The first match will start at 1.00pm.

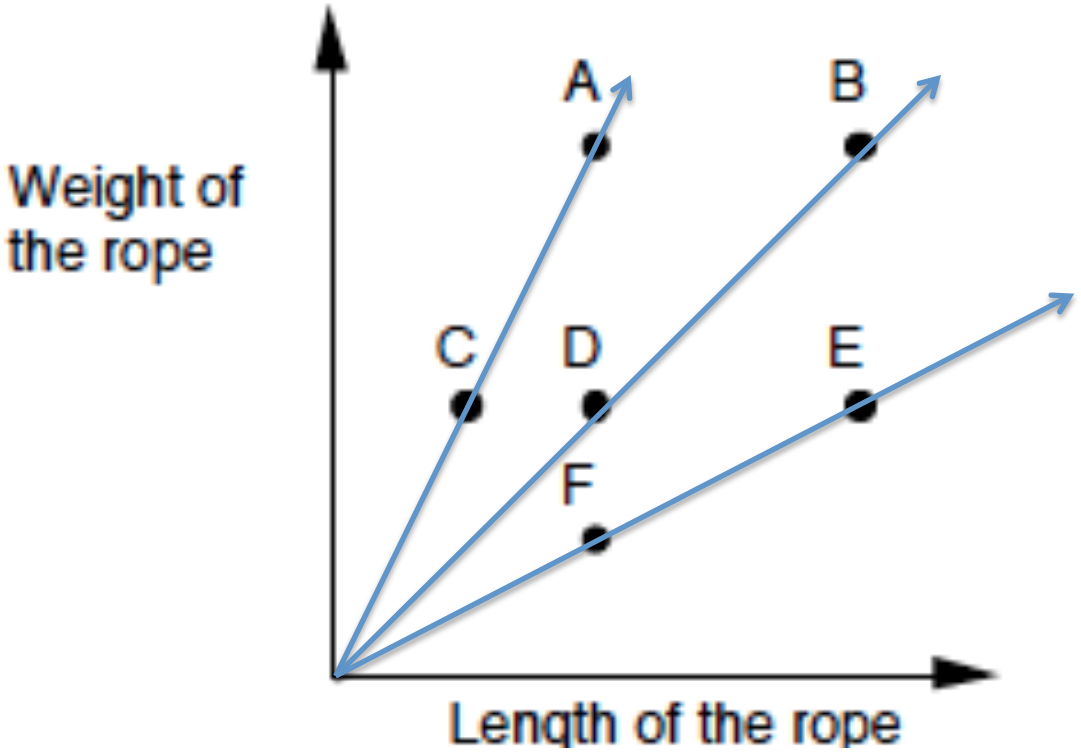
4. Model with mathematics

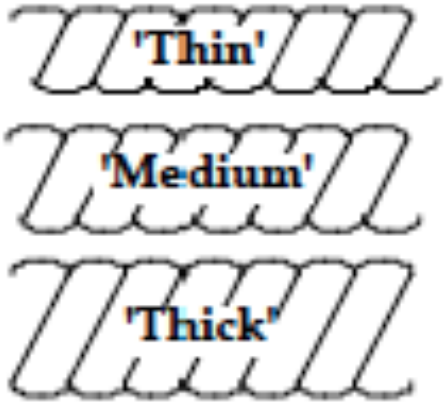
SKELETON TOWER



1. How many cubes are needed to build this tower?
2. How many cubes are needed to build a tower like this, but 12 cubes high? Explain how you work out your answer.
3. How would you calculate the number of cubes needed for a tower n cubes high?

5. Use appropriate tools strategically





Which ropes are ‘Thin’?
Which ropes are ‘Medium’?
Which ropes ‘Thick’?

Explain your reasoning.

6. Attend to Precision

- Precision in Calculations
- Precision in Vocabulary
- Precision in Argumentation

6. Attend to Precision

Imagine that you have just discovered this ancient floor tiling pattern in Syria.

You telephone New York to tell them about this exciting discovery.

Describe the pattern as accurately as you can, so that someone else can draw it without seeing it.

Describe the shapes as completely as you can.

_____

_____

_____

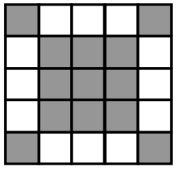
_____

7. Look for and make use of structure

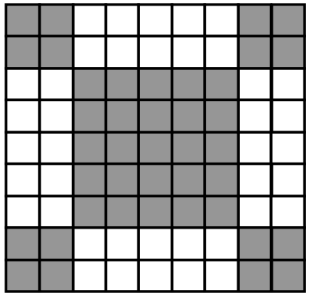
Sidewalk Patterns

In Prague some sidewalks are made of small square blocks of stone.

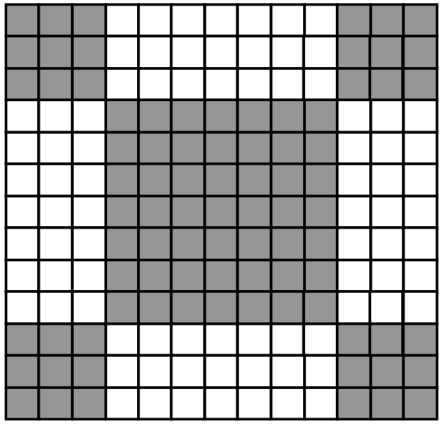
The blocks are in different shades to make patterns that are in various sizes.



Pattern



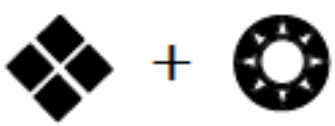
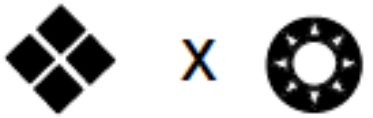
Pattern number



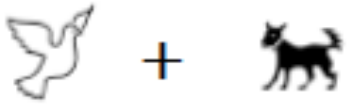
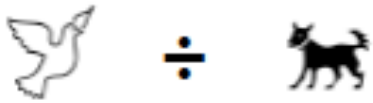
Pattern number 3

Write formulas for the number W of white tiles, and G of grey tiles, in terms of the pattern number n.

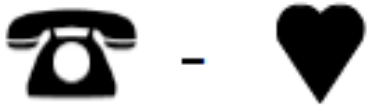
7. Look for and make use of structure

 and 

____ + ____ = 7 and ____ x ____ = 12

 and 

____ + ____ = 10 and ____ ÷ ____ = 4

 and 

____ ÷ ____ = 3 and ____ - ____ = 6

8. Look for and express regularity in repeated reasoning

Explain why the answer to:

$$4/5 \div 2/3$$

Makes sense knowing why 45 ÷ 8 makes sense.

8. Look for and express regularity in repeated reasoning

When Aaron plays the numbers game, he needs to decide which numbers belong in each set. Here is another game for you to play.

2. All of these numbers are Grogs.

123	789	456	345

None of these numbers are Grogs.

121	81	246	5678

Which of the numbers below are Grogs? Draw a circle around each Grog.

234 56 678 989

Explain how you know which numbers are Grogs and which are not.

_____

_____

Inside Mathematics Website

http://www.insidemathematics.org

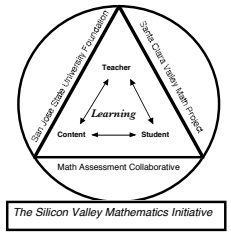
Mathematics Assessment Pro

UC Berkeley & Shell Centre for Mathematical Education

MARS Team
Mathematics Assessment Resource Service

http://map.mathshell.org/materials/lessons.php

Silicon Valley Mathematics Initiative



http://www.svmimac.org

"Don't be encumbered by history-- go off and do something wonderful."

Dr. Robert N. Noyce

Inventor of the Silicon Chip

Co-founder of Intel

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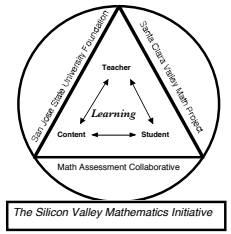

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