MATH | Standards for Mathematical Practice in Action

Practice	Sample Student Evidence	Sample Teacher Actions
1. Make sense of problems and persevere in solving them.	☐ Display sense-making behaviors. ☐ Show patience and listen to others. ☐ Turn and talk for first steps or generate a solution plan. ☐ Analyze information in problems. ☐ Use and recall multiple strategies. ☐ Self-evaluate and redirect. ☐ Assess the reasonableness of process and answer.	☐ Provide open-ended problems. ☐ Ask probing questions. ☐ Probe student responses. ☐ Promote and value discourse. ☐ Promote collaboration. ☐ Model and accept multiple approaches.
2. Reason abstractly and quantitatively.	☐ Represent abstract and contextual situations symbolically. ☐ Interpret problems logically in context. ☐ Estimate for reasonableness. ☐ Make connections, including real-life situations. ☐ Create and use multiple representations. ☐ Visualize problems. ☐ Put symbolic problems into context.	☐ Model context to symbol and symbol to context. ☐ Create problems such as, "What word problem will this equation solve?" ☐ Give real-world situations. ☐ Offer authentic performance tasks. ☐ Place less emphasis on the answer. ☐ Value invented strategies. ☐ Think aloud.
3. Construct viable arguments and critique the reasoning of others.	☐ Question others. ☐ Use examples and nonexamples. ☐ Support beliefs and challenges with mathematical evidence. ☐ Form logical arguments with conjectures and counterexamples. ☐ Use multiple representations for evidence. ☐ Listen and respond to others well. ☐ Use precise mathematical vocabulary.	☐ Create a safe and collaborative environment. ☐ Model respectful discourse behaviors. ☐ Provide find-the-error problems. ☐ Promote student-to-student discourse (do not mediate discussion). ☐ Plan effective questions or Socratic formats. ☐ Provide time and value discourse.
4. Model with mathematics.	☐ Connect math (numbers and symbols) to real-life situations. ☐ Symbolize real-world problems with math. ☐ Make sense of mathematics. ☐ Apply prior knowledge to solve problems. ☐ Choose and apply representations, manipulatives, and other models to solve problems. ☐ Use strategies to make problems simpler. ☐ Use estimation and logic to check the reasonableness of an answer.	☐ Model reasoning skills. ☐ Provide meaningful, real-world, authentic, performance-based tasks. ☐ Make appropriate tools available. ☐ Model various modeling techniques. ☐ Accept and value multiple approaches and representations.
5. Use appropriate tools strategically.	☐ Choose appropriate tool(s) for a given problem. ☐ Use technology to deepen understanding. ☐ Identify and locate resources. ☐ Defend mathematically the choice of a tool.	☐ Provide a toolbox at all times with all available tools; students then choose as needed. ☐ Model tool use, especially technology for understanding.

MATH | Standards for Mathematical Practice in Action

Practice	Sample Student Evidence	Sample Teacher Actions
6. Attend to precision.	◻ Communicate (orally and in writing) with precise vocabulary. ◻ Carefully formulate questions and explanations (not retelling steps). ◻ Decode and interpret the meaning of symbols. ◻ Pay attention to units, labeling, scale, and so forth. ◻ Calculate accurately and effectively. ◻ Express answers within context when appropriate.	◻ Model problem-solving strategies. ◻ Give explicit and precise instruction. ◻ Ask probing questions. ◻ Use English language arts strategies of decoding, comprehending, and text-to-self connections for interpreting symbolic and contextual math problems. ◻ Guided inquiry.
7. Look for and make use of structure.	◻ Look for, identify, and interpret patterns and structures. ◻ Make connections to skills and strategies previously learned to solve new problems and tasks. ◻ Breakdown complex problems into simpler and more manageable chunks. ◻ Use multiple representations for quantities. ◻ View complicated quantities as both a single object and a composition of objects.	◻ Let students explore and explain patterns. ◻ Use open-ended questioning. ◻ Prompt students to make connections and choose problems that foster connections. ◻ Ask for multiple interpretations of quantities.
8. Look for and express regularity in repeated reasoning.	◻ Design and state shortcuts. ◻ Generate rules from repeated reasoning or practice (e.g., integer operations). ◻ Evaluate the reasonableness of intermediate steps. ◻ Make generalizations.	◻ Provide tasks that allow students to generalize. ◻ Don't teach steps or rules, but allow students to explore and generalize to discover and formalize. ◻ Ask deliberate questions. ◻ Create strategic and purposeful check-in points.

Source: Adapted from "Common Core Look Fors (CCL4s)" (iPad App). Adapted from NCSM Summer Leadership Academy, June, 2011, Atlanta, Ga.

Depth of Knowledge (DOK) Overview Chart

<table>
<tr><th>Level of Complexity (measures a student's Depth of Knowledge)</th><th colspan="2">Key Verbs That May Clue Level</th><th>Evidence of Depth of Knowledge</th></tr>
<tr><td>Level 1
Recall/Reproduction
Recall a fact, information, or procedure. Process information on a low level.

<u>Bloom</u>
Know/Remember
"The recall of specifics and universals, involving little more than bringing to mind the appropriate material."

Comprehend/Understand
"Ability to process knowledge on a low level such that the knowledge can be reproduced or communicated without a verbatim repetition."</td><td>Arrange
Calculate
Cite
Define
Describe
Draw
Explain
Give examples
Identify
Illustrate
Label
Locate
List
Match</td><td>Measure
Name
Perform
Quote
Recall
Recite
Record
Repeat
Report
Select
State
Summarize
Tabulate</td><td>• Explain simple concepts or routine procedures
• Recall elements and details
• Recall a fact, term or property
• Conduct basic calculations
• Order rational numbers
• Identify a standard scientific representation for simple phenomenon
• Label locations
• Describe the features of a place or people
• Identify figurative language in a reading passage</td></tr>
<tr><td>Level 2
Skill/Concept
Use information or conceptual knowledge, two or more steps

<u>Bloom</u>
Apply
"Uses information in another familiar situation."
(Executes - Carries out a procedures in a familiar task)
(Implements - Uses a procedure in an unfamiliar task)</td><td>Apply
Calculate
Categorize
Classify
Compare
Compute
Construct
Convert
Describe
Determine
Distinguish
Estimate
Explain
Extend
Extrapolate
Find
Formulate</td><td>Generalize
Graph
Identify patterns
Infer
Interpolate
Interpret
Modify
Observe
Organize
Predict
Relate
Represent
Show
Simplify
Solve
Sort
Use</td><td>• Solve routine multiple-step problems
• Describe non-trivial patterns
• Interpret information from a simple graph
• Formulate a routine problem, given data and conditions
• Sort objects
• Show relationships
• Apply a concept
• Organize, represent and interpret data
• Use context clues to identify the meaning of unfamiliar words
• Describe the cause/effect of a particular event.
• Predict a logical outcome
• Identify patterns in events or behavior</td></tr>
</table>

Level of Complexity (measures a student's Depth of Knowledge)	**Key Verbs That May Clue Level**		**Evidence of Depth of Knowledge**
Level 3 **Strategic Thinking** Requires reasoning, developing a plan or a sequence of steps, some complexity Bloom *Analyze* "Breaking information into parts to explore understanding and relationship." *Evaluate* "Checks/Critiques – makes judgments based on criteria and standards."	Appraise Assess Cite evidence Check Compare Compile Conclude Contrast Critique Decide Defend Describe Develop Differentiate Distinguish	Examine Explain how Formulate Hypothesize Identify Infer Interpret Investigate Judge Justify Reorganize Solve Support	• Solve non-routine problems • Interpret information from a complex graph • Explain phenomena in terms of concepts • Support ideas with details and examples • Develop a scientific model for a complex situation • Formulate conclusions from experimental data • Compile information from multiple sources to address a specific topic • Develop a logical argument • Identify and then justify a solution • Identify the author's purpose and explain how it affects the interpretation of a reading selection
Level 4 **Extended Thinking** Requires an investigation, time to think and process multiple conditions of the problem. Most on-demand assessments will not include Level 4 activities. Bloom *Synthesize* "Putting together elements and parts to form a whole *Evaluate* Making value judgments about the method."	Appraise Connect Create Critique Design Judge Justify Prove Report Synthesize		• Design and conduct an experiment that requires specifying a problem; report results/solutions • Synthesize ideas into new concepts • Critique experimental designs • Design a mathematical model to inform and solve a practical or abstract situation. • Connect common themes across texts from different cultures • Synthesize information from multiple sources

Levels of Complexity

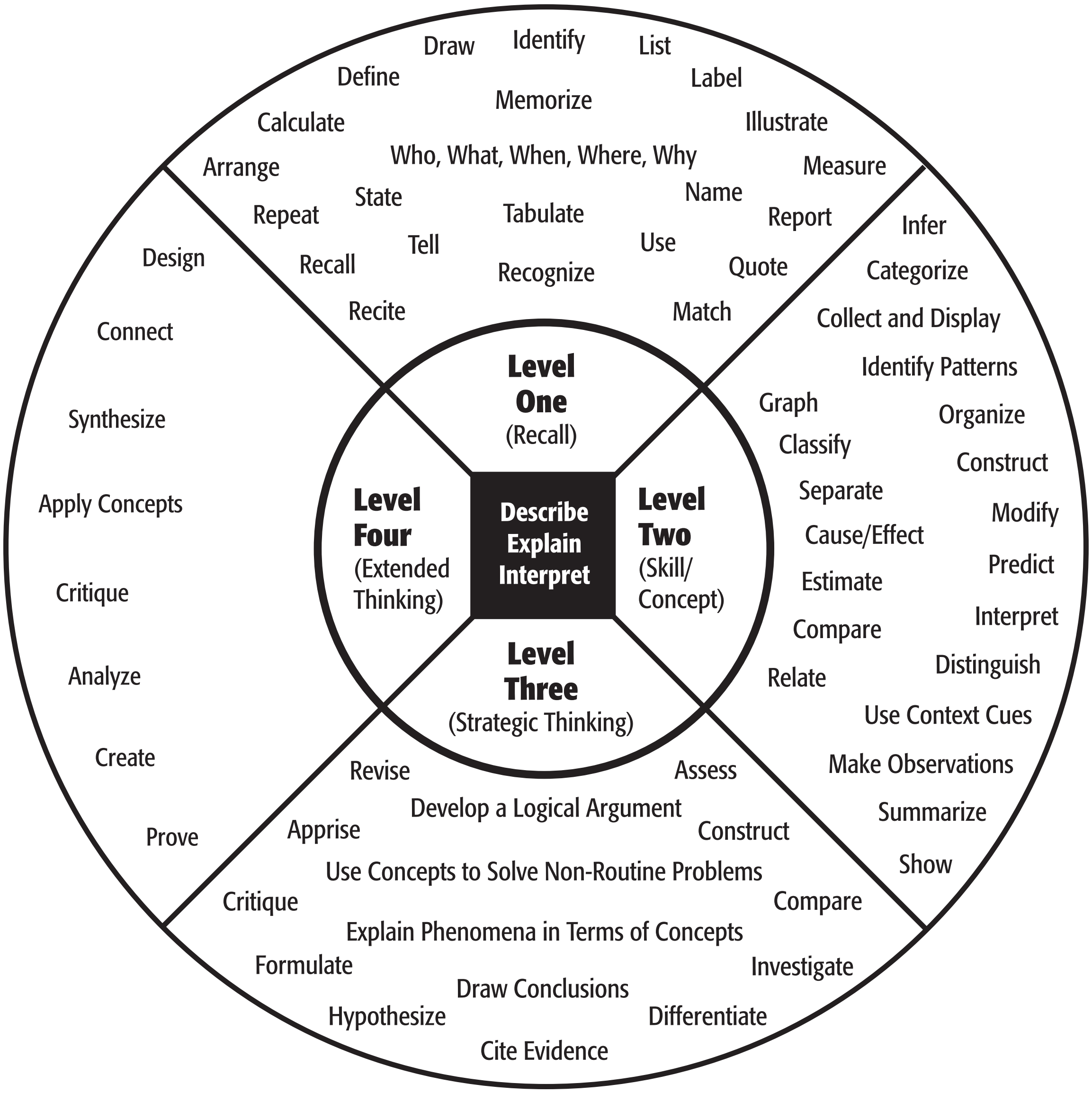
- Recall/Reproduction – Recall a fact, information, or procedure; process information on a low level

- Skill/Concept – Use information or conceptual knowledge, two or more steps

- Strategic Thinking – Requires reasoning, developing a plan or a sequence of steps, more than one reasonable approach

- Extended Thinking – Requires connections and extensions, high cognitive demands and complex reasoning

Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting.	Identify and summarize the major events in a narrative.	Support ideas with details and examples.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.
Conduct basic mathematical calculations.	Use context cues to identify the meaning of unfamiliar words.	Use voice appropriate to the purpose and audience.	Apply mathematical model to illuminate a problem or situation.
Label locations on a map.	Solve routine multiple-step problems.	Identify research questions and design investigations for a scientific problem.	Analyze and synthesize information from multiple sources.
Represent in words or diagrams a scientific concept or relationship.	Describe the cause/effect of a particular event.	Develop a scientific model for a complex situation.	Describe and illustrate how common themes are found across texts from different cultures.
Perform routine procedures like measuring length or using punctuation marks correctly.	Identify patterns in events or behavior.	Determine the author's purpose and describe how it affects the interpretation of a reading selection.	Design a mathematical model to inform and solve a practical or abstract situation.
Describe the features of a place or people.	Formulate a routine problem given data and conditions.	Apply a concept in other contexts.	
	Organize, represent and interpret data.		

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>.

Odd or Even

7th Grade—Teacher Notes

Overview	
Students are asked to settle an argument between two people about the probability of either one winning a contest.	**Prerequisite Understandings** • Ability to calculate basic probabilities. • Definitions of even and odd numbers.

Curriculum Content	
CCSSM Content Standards	7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
CCSSM Mathematical Practices	3. **Construct viable arguments and critique the reasoning of others**: Students explain who is correct and explain the results based on the mathematics. 4. **Model with mathematics**: Students use probability to simulate the situation.

Task	
Supplies • Spinners (manual and/or electronic)	**Core Activity** In pairs, students will explore the experimental probabilities and calculate the theoretical probabilities of even and odd sums of random numbers.
Launch Briefly review the definitions of even numbers and odd numbers and probability. Specifically discuss that 0 is an even number. Consider patterns from the number line and the definition of even.	**Extension(s)** Ask students to work in groups and create a "fair" game where each player would have an equal chance of winning. Make sure they can explain why it is fair.

Odd or Even

Launch

Spinner A will be the three-part spinner and Spinner B will be the four-part spinner. Spin both spinners, and record the results in the chart below. Create a fraction in the next column, writing the result as a rational number $\frac{A}{B}$. Then, in the remaining columns, simplify your fraction, if possible, and find the decimal and percent equivalents. Repeat this 5 times (or for 5 "trials").

Trial #	Spinner A	Spinner B	Fraction $\frac{A}{B}$	Simplified Fraction	Decimal	Percent
1						
2						
3						
4						
5						

In the following tables, list all possible outcomes of the rational number $\frac{A}{B}$. Then simplify and find their decimal and percent equivalents. (Hint: There are 12 possible outcomes.) (Hint #2: You already have some of these done in the first table!)

$\frac{A}{B}$	Simplified Fraction	Decimal	Percent

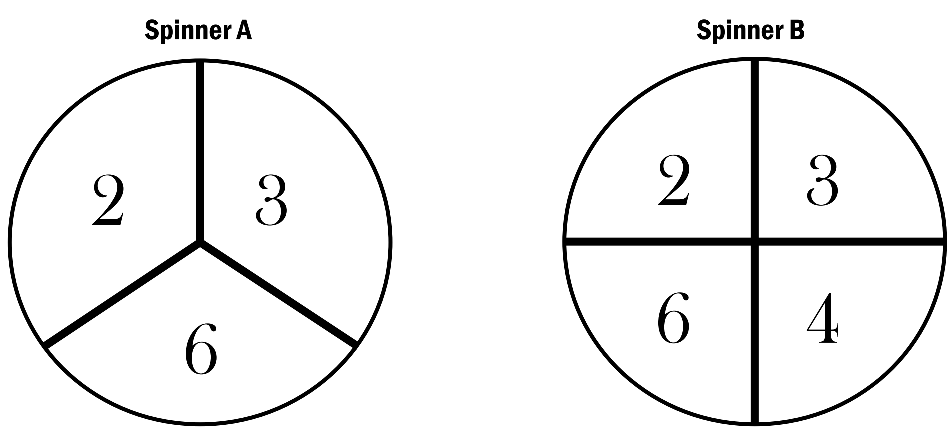
$\frac{A}{B}$	Simplified Fraction	Decimal	Percent

Organize your distinct **outcomes** $\frac{A}{B}$ (as simplified fractions) from least to greatest in the first column of the table shown to the right.

Find the **frequency** and the **probability** of each outcome and record each in the remaining columns.

Outcomes	Frequency	Probability

If we were to play a game where Player 1 gets a point when the outcome is greater than or equal to 1 and Player 2 gets a point for an outcome less than 1, which player would you want to be and why?



Additional Resources

Appendices A & B contain instructions for **Generating Random Integers** on both the TI-Nspire™ handhelds and the TI-84 Plus graphing calculators.

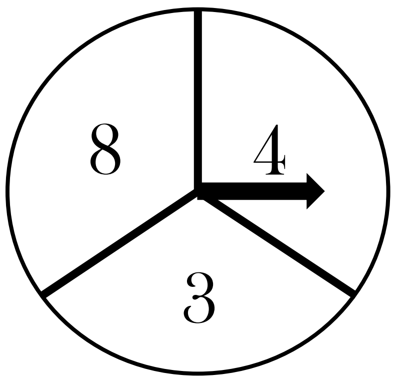
Odd or Even

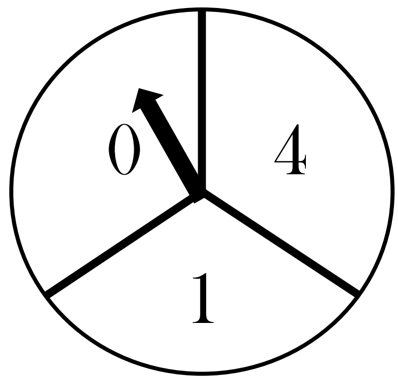
Activity

Leo and Tarra are playing a spinner game with the following rules:

> When it is a player's turn, the player spins both spinners. They then find the sum of the two numbers. If the sum is EVEN, player 1 wins (Leo). If the sum is ODD, player 2 wins (Tarra).

Leo takes a test spin first. Here is what he spins:





The sum from the first spin is EVEN because 4 + 0 is even. Leo wins. Leo says, "I like this game. I have a better chance to win it than you do."

Tarra says, "No, I have a better chance to win it than you do."

1. Use mathematics to decide which player is correct.

2. Write a note to the players explaining how you know who has the better chance of winning.

Odd or Even

Results from the Classroom

Olivia

Olivia understood and was careful not hurt Leo's feelings by saying he was partially right. She was able to answer the probabilities with precision.

Dear Leo- I belive you are wrong and right. If you are talking about theoretical probability you are correct. Because you have 5/9 possibilities as opposed to Tara who has a 4/9 chance ~~and~~

Latisha

Latisha has a clear and complete understanding of problem. Her explanation distinguishes between theoretical and experimental probabilities. Her argument is correct.

Leo.

You are correct, theoretically speaking. The chances that an even sum will be spun is 5/9. Tarra could win. however. You could do many test trials to find the experimental probability of even or odd outcomes. which might be different than the theoretical probability. Yes, the chances say you will win. but that can change.

Sincerely.

Dear Leo + Tarra,

Whoever has the even numbers has a $\frac{5}{9}$ chance of winning with theore probability. With experimental though, I d know who would win, I would have to look at the data for previous spins. Leo is probably going to win not for sure, just probably! So good luck to you guys with your game, and, go Leo!

Juan

Juan talks about looking at the data and uses the term probably to describe Leo's chance of winning. He is more specific by mentioning that the even-number player will win. All of the students have been motivated by the situation and have written specifically to the learners.

Isabella

Isabella is specific with her information and lists the possibilities for each person to win. She extends the information without being asked to explain how she thinks that the players could make it a fair game. She demonstrates both of the highlighted mathematical practices. She has modeled the situation with mathematics and was able to present a clear mathematical argument.

Dear Leo and Tarra,

I'm sorry but Leo has a better theoretical possibility to win the game. While Tarra only has 4 possibilites, Leo has five. You would have to make it fair, though. You could take turns on whoever gets to be player one.

P.S. Tarra, here are your possibilities:
3+8
8+1
4+1
3+4

Leo, here are your possibilities:
4+8
8+8
3+1
8+4
4+4

See, there are more for player 1!
(Sorry player 2)

Problem of the Month: *Circular Reasoning*

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:

In the Problem of the Month *Circular Reasoning,* students use geometric reasoning to solve problems involving circles and the number π. The mathematical topics that underlie this POM are the attributes of polygons, circles, the irrational number π, spatial visualization, and angle measurement.

In the first level of the POM, students are presented with the task of examining the relationship between the measure of the diameter of a circle and its circumference. Their task involves determining the number of times bigger the circumference is than the diameter (approximation of π). Level B requires students to continue to investigate the number π by investigating the relationship between the measurements of a tennis ball can. They find the height of the can and the circumference of the base of the can. They reason why the two measurements are close to the same size and search for a pattern to determine a rationale. In level C, students investigate different size pizzas made in the form of a circle. The pizzas are divided into fractional slices shaped like a sector of a circle. The students investigate the relationships between the sizes of the pizzas, including the dimensions of the slices (radii and degrees), the costs, and whether the costs are proportional to their sizes. In level D, the students investigate the rings that support wooden barrels. In this problem, the rings are made a foot longer than they were supposed to be. Since the barrels come in a range of sizes from very large to very small, the foot long error must be explored. The goal is to determine the relationship between error in rings to the original size and how the extra length affects a redesigned barrel. In level E, students investigate Archimedes' approach to calculating an approximation for π. Students are asked to duplicate the drawings and calculations of Archimedes and determine the accuracy of his approximation. They are also asked to justify the formula for the area of a circle.

Mathematical Concepts:

Geometry and measurement are important real-world experiences. From the most complex structures created by designers, architects, and construction workers to arranging the furniture in a room, geometry and measurement are essential. In this POM, students explore various aspects of circular geometry. This includes understanding the attributes of circles. In particular, students study regular polygons, diameters, circumference, and angles. The attributes studied include the diagonals of polygons as well as angles and their measurements. Students explore the irrational number π. In level E, students use an historical approach to calculate an approximation of π. In addition to the geometric and measurement aspects of this POM, the students are seeking to find patterns, develop functional relationships, make generalizations, and justify their conclusions. The mathematics involves higher-level cognitive skills.

π Problem of the Month Circular Reasoning π

Level A

Janet and Lydia want to learn more about circles. They decide to measure different size circles that they can find. They measure the circles in two ways. One way is across a centerline of the circle. The centerline is called the diameter. Another way is around the outside of the circle. The distance around the outside of a circle is called the circumference.

Measure various size circles using either physical objects or the included page of circles. Measure both the circumference and diameter of each circle with a tape measure (paper or cloth tape). Create a table that compares the length of the diameter with the circumference of the circle. Measure at least five different size circles.

Diameter	Circumference

Compare the numbers between the two columns. What patterns do you see?

Explain any relationship you see between the length of the diameter and the circumference.

Level B

Measure a tennis ball can. First measure the height of the can, from the top of the can to the bottom. Record the measurement.

Next, measure the circumference around the can. Be accurate with your measurements.

Compare the measurements.

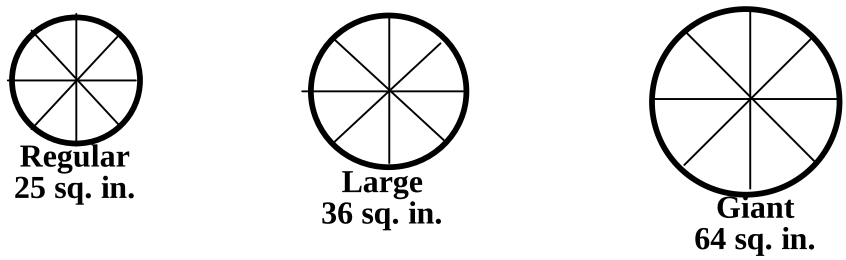
Explain the relationship between the two measurements.

How does the number of balls in the can relate to the measurements and their relationship?

What other parts of the can or tennis balls can you measure and compare?

Describe the relationship of those measurements? How are they related?

Level C
You work for a pizza parlor. You currently have three sizes of pizza: regular (area 25 square inches), large (area 36 square inches) and giant (area 64 square inches).



The storeowner has asked you to determine some information about the amount of pizza that is made.

How much bigger is the giant pizza than the large pizza in terms of area? How much bigger is the large pizza than the regular pizza in terms of area?

The owner sells a regular cheese pizza for $8.25. He wants to know what to charge for the large and giant sized pizzas. He wants to cover the cost of the ingredients and make a proportional amount of profit on the other two pizzas. What should he charge for a large and giant cheese pizza?

The owner sells pizza by the slice. He has been selling a large pizza by the slice for $1.35. Each pizza is cut into eight equal slices. The owner wants to know whether he has priced it right to make more money selling by the slice than the whole pizza. Explain to him, using mathematics, how his prices for the slices and the whole pizza compare.

Now the owner wants to cut each pizza into nine slices, instead of eight. How much smaller will the slices be? He is interested in comparing both the width (angle in numbers of degrees) and the area of each slice.

The owner wants to sell the nine slices at the same price as he did when the pizza was cut into eight slices. He wants to know how much more he will make cutting the pizzas into smaller slices. He wants to compare the prices of selling the nine slices individually with the price of selling an entire large pizza. He asks you to calculate and explain the difference in prices and amounts.

The owner also thinks he can save more money if he sells slices from regular sized pizzas instead of large sized pizzas. You cut both the regular and large pizzas into nine equal slices. How much larger is a single slice from a large pizza than that of a slice from a regular pizza? Explain your answer in terms of degrees of the angle and area of the slice.

Level D

You are a quality controller for a barrel manufacturing company. Your company makes barrels of all different sizes. The company makes very large storage vats for vineyards. They even make very small barrels as earrings that are sold in the gift shop next to the factory. Without exception, the company makes every barrel the same way. They use wood supported with metal rings that go around the barrels. A computer controls the ring-making machine. The computer malfunctioned and made every ring, from the smallest for the earrings to the largest for the wine vats, larger than normal by exactly one foot. In other words, every ring your factory has turned out recently has a circumference that is twelve inches longer than the right size. Your boss has asked you to investigate this matter.

How does the altered rings compare to the original rings? How much bigger do the altered rings look in comparison to the rings on the barrels that they were designed to fit? How will the altered rings compare in size to the rings on the small barrels, like the earrings? How will the altered rings compare to the rings on the large barrels, like the wine vats?

What should the company do with all those altered rings? It would be very costly to dispose of them. Your boss has asked you to tell him what size barrels need to be made to use those rings. Are the barrels usable? What size would you have to make the new barrels in relationship to the original size barrels?

What conclusion can you draw from your findings? Can you explain your findings using mathematics? Why are the results what they are?

Level E

Archimedes is considered one of the greatest mathematicians of all time. He lived in the Greek city of Syracuse, Sicily between 287 BC and 212 BC and is known for his work in many areas of mathematics and science.

Archimedes mathematical work includes inventing methods of calculating the area and volume of geometric objects, such as the circles, parabolas, cylinders, cones, spheres, hyperbolas and ellipses. He also spent time investigating a rational number approximation of π. The method he used to approximate π involved examining the size of many-sided polygons. Archimedes, as the story is told, drew a large circle on the floor of his apartment. He measured the diameter of the circle and called that distance one unit in length. Next he began to inscribe and circumscribe different size polygons, tangent to the circle. Archimedes would measure the perimeters of the polygons and average the two to approximate π. As the numbers of sides of the polygons were increased, the shaped of the polygon looked more like the circle. The polygons got closer and closer to being the size of the circle and perimeters became ever closer to the value of π. Using that method, he was able to calculate the approximation of π, correct to a value between 3 1/7 and 3 10/71.

How precise was his approximation of π? Give your answer in terms of the number of correct decimal places.

Estimate how many sides his final polygon needed to be to get a number that accurate? Explain your estimate.

Illustrate the method Archimedes used by drawing an interior regular polygon inscribed in a circle and an exterior regular polygon circumscribed about the same circle. Determine the perimeter of both polygons and show how π might be approximated.

Find the area of both polygons and compare the areas with that of the circle. Explain how the area formula is derived from the area formula of the polygon.

π Problem of the Month π
Circular Reasoning

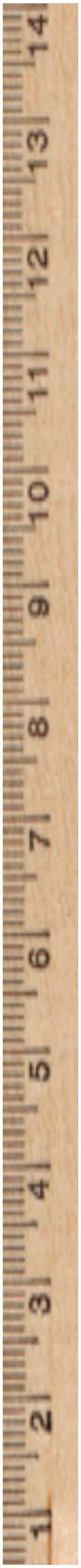
Primary Version Level A

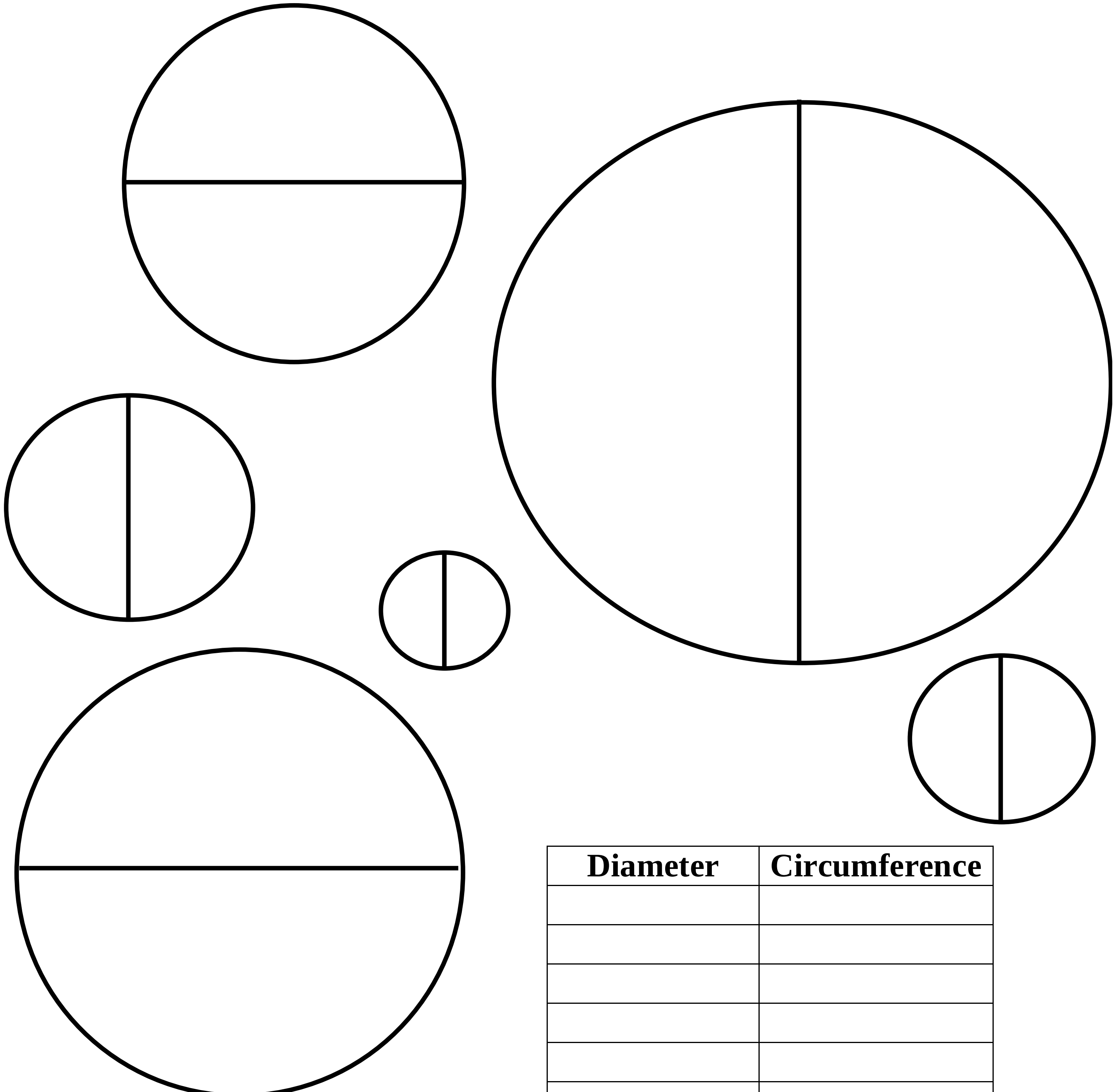
Materials: A picture of a circle, circular objects (i.e. counters, coins), the π-day picture, paper and pencil.

Discussion on the rug: (Teacher holds up a picture of a circle) "What is the name of this shape?" (Students respond to the question). (Teacher hands out circular objects) "These objects are shaped like a circle. Tell us about the shapes?" (Students explore the objects and describe the circles). (Teacher holds up a circle) "A circle is a special shape. Long ago people discovered the circle and tried to measure it. They found a special number called pi. Some people who like math celebrate circles on a special day, March 14. It is called pi day. Here is a picture of people having a picnic on pi day." (Teacher hands-out the picture of the π-day picnic).

In small groups: (Each student has the picture of the π-day picnic). "Look at the picture of the picnic. Find all the things in the picture that are shaped like a circle. Circle each shape you find." (Students answer the following questions).
How many things did you find that are shaped like a circle?
Name the things in the picture that are shaped like a circle.
(At the end of the investigation, have students either discuss or dictate a response to this summary question below.)
Explain how you decided which ones to circle.


π day picnic
STOP





Diameter	Circumference

Problem of the Month

Circular Reasoning

Task Description – Level A

This task challenges a student to explore various aspects of circular geometry. Students are asked to examine the relationship between the measure of the diameter of a circle and its circumference. Their task involves determining the number of times bigger the circumference is than the diameter (approximation of π).

Common Core State Standards Math - Content Standards

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.)

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Measurement and Data

Describe and compare measurable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe measurable attributes of a single object.

Measure and estimate lengths in standard units.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Operations and Algebraic Thinking

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. …

Expressions and Equations

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Functions

Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. …

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. …

High School – Functions - Building Functions

Build a function that models a relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Circular Reasoning

Task Description – Level B

This task challenges a student to explore various aspects of circular geometry. A student is required to continue to investigate the number π by investigating the relationship between the measurements of a tennis ball can, such as the height of the can and the circumference of the base of the can. They are to reason why the two measurements are close to the same size and to search for a pattern to determine a rationale for this relationship. Additionally, a student is asked to explore and describe the relationship between the number of balls in a can and the height of a can.

Common Core State Standards Math - Content Standards

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Measurement and Data

Describe and compare measurable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe measurable attributes of a single object.

Measure and estimate lengths in standard units.

2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Operations and Algebraic Thinking

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. …

Expressions and Equations

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Functions

Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. …

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. …

High School – Functions - Building Functions

Build a function that models a relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month
Circular Reasoning
Task Description – Level C
This task challenges a student to explore various aspects of circular geometry. A student is asked to investigate different size pizzas made in the form of a circle. The pizzas are divided into fractional slices shaped like a sector of a circle. The students investigate the relationships between the sizes of the pizzas, including the dimensions of the slices (radii and degrees), the costs, and whether the costs are proportional to their sizes.
Common Core State Standards Math - Content Standards
<u>Operations and Algebraic Thinking</u> **Represent and solve problems involving addition and subtraction.** 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, … **<u>Ratio and Proportional Relationships</u>** **Understand ratio concepts and use ratio reasoning to solve problems.** 6.RP.3 c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. **Analyze proportional relationships and use them to solve real-world and mathematical problems.** 7.RP.1 Recognize and represent proportional relationships between quantities. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. … **<u>Geometry</u>** **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems… 7.G.5 Use facts about …angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are

comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month:

Circular Reasoning

Task Description – Level D

This task challenges a student to explore various aspects of circular geometry. A student is asked to investigate the rings that support wooden barrels. In this problem, the rings are made a foot longer than they were supposed to be. Since the barrels come in a range of sizes from very large to very small, the foot long error must be explored. The goal is to determine the relationship between error in rings to the original size and how the extra length affects a redesigned barrel.

Common Core State Standards Math - Content Standards

Geometry

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems…

7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects…

Ratio and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.2 Recognize and represent proportional relationships between quantities.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. …

Expressions and Equations

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px+q=r$ and $p(x+q)=r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month
Circular Reasoning
Task Description – Level E
This task challenges a student to explore various aspects of circular geometry. A student is asked to investigate Archimedes' approach to calculating an approximation for π. Students are asked to duplicate the drawings and calculations of Archimedes and determine the accuracy of his approximation. They are also asked to justify the formula for the area of a circle.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** 7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems… 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three- dimensional objects… **<u>High School – Geometry - Congruence</u>** **Make geometric constructions.** G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. **<u>High School – Geometry - Similarity, Right Triangles, and Trigonometry</u>** **Apply trigonometry to general triangles.** G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical

situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month
Circular Reasoning
Task Description – Primary Level
This task challenges a student to explore various aspects of circular geometry. This level asks students to describe and identify circular shapes in real world objects such as coins, counters, containers, etc. Then students are asked to ring and identify all shapes in a given picture which are shaped like a circle.
Common Core State Standards Math - Content Standards
<u>Geometry</u> **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.* K.G.2 Correctly name shapes regardless of their orientations or overall size.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Common Core Standards for Mathematical Practice Look-for Tool

	Mathematics Practices	Student dispositions:	Teacher actions to engage students in Practices:
Overarching habits of mind of a productive math thinker	**1. Make sense of problems and persevere in solving them**	☐ Have an understanding of the situation ☐ Use patience and persistence to solve problem ☐ Be able to use strategies ☐ Use self-evaluation and redirections ☐ Communicate both verbally and written ☐ Be able to deduce what is a reasonable solution Comments:	☐ Provide open-ended and rich problems ☐ Ask probing questions ☐ Model multiple problem-solving strategies through *Think- Aloud* ☐ Promotes and values discourse ☐ Cross-curricular integrations ☐ Promotes collaboration ☐ Probe student responses (correct or incorrect) for understanding and multiple approaches ☐ Provide scaffolding appropriately ☐ Provide a safe environment for learning from mistakes Comments:
	6. Attend to precision	☐ Communicate with precision-orally and written ☐ Use mathematics concepts and vocabulary appropriately ☐ State meaning of symbols and use appropriately ☐ Attend to units/labeling/tools accurately ☐ Carefully formulate explanations and defend answers ☐ Calculate accurately and efficiently ☐ Formulate and make use of definitions with others and their own reasoning ☐ Ensure reasonableness of answers ☐ Perseverance through multiple-step problems Comments:	☐ Encourage students to think aloud/talk aloud ☐ Explicit instruction/teacher model of think aloud/talk aloud ☐ Guided Inquiry including teacher gives problem, students work together to solve problems, and debriefing time for sharing and comparing strategies ☐ Probing questions targeting content of study ☐ Promote mathematical lingo ☐ Give room to discuss why wrong answers are wrong Comments:
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	☐ Create multiple representations ☐ Interpret problems in contexts ☐ Estimate first/answer reasonable ☐ Make connections ☐ Represent symbolically ☐ Visualize problems ☐ Talk about problems, real life situations ☐ Attending to units ☐ Using context to think about a problem Comments:	☐ Develop opportunities for and model problem solving strategies ☐ Give time for processing and discussing ☐ Tie content areas together to help make connections ☐ Give real world situations ☐ Think aloud for student benefit ☐ Value invented strategies and representations ☐ Less emphasis on the answer Comments:
	3. Construct viable arguments and critique the reasoning of others	☐ Ask questions ☐ Use examples and counter examples ☐ Reason inductively and make plausible arguments ☐ Use objects, drawings, diagrams, and actions ☐ Students develop ideas about mathematics and support their reasoning ☐ Analyze others arguments ☐ Encourage the use of mathematics vocabulary Comments:	☐ Create a safe environment for risk-taking and critiquing with respect ☐ Model each key student disposition ☐ Provide complex, rigorous tasks that foster deep thinking ☐ Provide time for student discourse ☐ Plan effective questions and student grouping ☐ Probe students Comments:

Mathematics Practices		Students:	Teacher(s) promote(s) by:
Modeling and Using Tools	**4. Model with mathematics**	☐ Realize they use mathematics (numbers and symbols) to solve/work out real-life situations ☐ Analyze relationships to draw conclusions ☐ Interpret mathematical results in context ☐ Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable. If not, go back and look for more information ☐ Make sense of the mathematics Comments:	☐ Allow time for the process to take place (model, make graphs, etc.) ☐ Model desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) ☐ Make appropriate tools available ☐ Create an emotionally safe environment where risk taking is valued ☐ Provide meaningful, real world, authentic, performance-based tasks (non traditional work problems) ☐ Discourse ☐ Investigations Comments:
	5. Use appropriate tools strategically	☐ Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base 10 blocks, compass, protractor) ☐ Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) ☐ Compare the efficiency of different tools ☐ Recognize the usefulness and limitations of different tools Comments:	☐ Maintain knowledge of appropriate tools ☐ Effective modeling of the tools available, their benefits and limitations ☐ Model a situation where the decision needs to be made as to which tool should be used ☐ Compare/contrast effectiveness of tools ☐ Make available and encourage use of a variety of tools Comments:
Seeing structure and generalizing	**7. Look for and make use of structure**	☐ Look for, interpret, and identify patterns and structures ☐ Make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers ☐ Reflect and recognize various structures in mathematics ☐ Breakdown complex problems into simpler, more manageable chunks ☐ Be able to “step back” / shift perspective ☐ Value multiple perspectives Comments:	☐ Be quiet and structure opportunities for students to think aloud ☐ Facilitate learning by using open-ended questioning to assist students in exploration ☐ Careful selection of tasks that allow for students to discern structures or patterns to make connections ☐ Allow time for student discussion and processing in place of fixed rules or definitions ☐ Foster persistence/stamina in problem solving ☐ Through practice and modeling time for students Comments:
	8. Look for and express regularity in repeated reasoning	☐ Identify patterns and make generalizations ☐ Continually evaluate reasonableness of intermediate results ☐ Maintain oversight of the process ☐ Search for and identify and use short-cuts Comments:	☐ Provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge ☐ Provide adequate time for exploration ☐ Provide time for dialogue and reflection, peer collaboration ☐ Ask deliberate questions that enable students to reflect on their own thinking ☐ Create strategic and intentional check in points during student work time Comments:

- All indicators are not necessary for providing full evidence of practice(s). Each practice may not be evident during every lesson.
- Document originally created by NCSM Summer Leadership Academy then edited by Region 2 Algebra Forum