How Students Should be Taught Mathematics:

Reflections from Research and Practice.

Jo Boaler, Professor of Mathematics Education, Stanford University.

Mathematics classrooms should be places where students:

Develop an inquiry relationship with mathematics, approaching math with curiosity, courage, confidence & intuition.

Talk to each other and the teacher about ideas – Why did I choose this method? Does it work in other cases? How is the method similar or different to methods other people used?

Work on mathematics tasks that can be solved in different ways and/or with different solutions.

Work on mathematics tasks with a low entry point but a very high ceiling – so that students are constantly challenged and working at the highest and most appropriate level for them.

Work on mathematics tasks that are complex, involve more than one method or area of mathematics, and that often, but not always, represent real world problems and applications.

Are given growth mindset messages at all times, through the ways they are grouped together, the tasks they work on, the messages they hear, and the assessment and grading.

Are assessed formatively – to inform learning - not summatively to give a rank with their peers. Students should regularly receive diagnostic feedback on their work, instead of grades or scores. Summative assessments are best used at the end of courses.

Mathematics classrooms should be places where students believe:

Everyone can do well in math.

Mathematics problems can be solved with many different insights and methods.

Mistakes are valuable, they encourage brain growth and learning.

Mathematics will help them in their lives, not because they will see the same types of problems in the real world but because they are learning to think quantitatively and abstractly and developing an inquiry relationship with math.

The Mathematical Task Analysis Guide

Lower-Level Demands

Memorization Tasks

- Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.

Procedures Without Connections Tasks

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Require no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demands

Procedures With Connections Tasks

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad, general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Doing Mathematics Tasks

- Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships.
- Demands self-monitoring or self-regulation of one's own cognitive processes.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Stein and Smith, 1998

Smarter Balance Assessment Consortium

CLAIMS

Claim #1: **Concepts and Procedures**
"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."

Claim #2: **Problem Solving**
"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."

- **understand (often in conjunction with one or more other relevant verbs), solve, apply, describe, illustrate, interpret, and analyze**.

Claim #3: **Communicating Reasoning**
"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."

- **understand, explain, justify, prove, derive, assess, illustrate, and analyze**

Claim #4: **Modeling and Data Analysis** "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

- **model, construct, compare, investigate, build, interpret, estimate, analyze, summarize, represent, solve, evaluate, extend, and apply**