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Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades

Educator's Practice Guide

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Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades

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Introduction to Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades

National Assessment of Educational Progress (NAEP) scores indicate that students who struggle do not experience the same growth in mathematics performance as higher-achieving students. In 2019, students at the 10th percentile showed significantly worse performance than a decade ago while the highest performing students demonstrated significant growth.[1] The growth of students at the 25th percentile remained stagnant. These data suggest that students with difficulties learning mathematics are falling even further behind their peers.

Recent intervention research has demonstrated success in raising the achievement level of students who are struggling with mathematics. This practice guide, developed by the What Works Clearinghouse™ (WWC) in conjunction with an expert panel, distills this contemporary mathematics intervention research into easily comprehensible and practical recommendations for teachers to use when teaching elementary students in intervention settings. Two federal laws, the Elementary and Secondary Education Act (ESEA)[2] and Individuals with Disabilities Education Act (IDEA), require use of instructional practices supported by evidence. The recommendations presented in the guide address these laws by translating the body of high-quality evidence into actionable practices for teachers to use with their students. Although this guide is an update of the 2009 guide, *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle School*,[3] it is narrower in scope, focusing only on practices and principles underlying effective small-group interventions in grades K-6. In contrast, the earlier guide was a broad overview of multi-tiered systems of support (MTSS) in mathematics—then typically referred to as RtI—a data-driven, systemic, problem-solving framework that helps educators provide core instruction, screening, intervention, progress monitoring, and support for students with various need. It also included grades 7 and 8.

See the **Glossary** for a full list of key terms used in this guide and their definitions. These terms are underlined and hyperlinked to the glossary when first introduced in the guide.

We narrowed the focus for this update for three reasons. First, MTSS is more widely used than when the original guide was published,[4] and a broad overview to facilitate implementation of MTSS was no longer necessary. Therefore, topics such as progress monitoring, screening, and motivation, though important, are not included in this guide.

Second, the field experienced a dramatic rise in research on mathematics interventions, so we are now able to focus on evidence-based instruction in intervention settings for students at risk for or with disabilities. Historically, mathematics intervention research has received far less energy and funding than reading intervention research. Yet, over the past 10 to 15 years, this discrepancy has begun to change. More experimental studies have been conducted to build the evidence base on effective intervention practices. Additionally, interventions have focused on more challenging topics, sometimes including critical mathematics topics from grade-level standards to improve student access to the same content as classroom peers and glean greater meaning from their core classes.

Third, we limited the scope to grades K–6. This was done because the updated research focused primarily on grades K–6. The panel felt limiting the guide's scope to grade levels where the bulk of the research existed provided stronger support for the recommendations.

The practices that appear in this guide's six recommendations highlight effective approaches to mathematics intervention that meet the needs of the students in small-group or one-on-one settings. Each of these practices move students toward more fluent performance of mathematics. **Recommendation 1** (Systematic Instruction) provides specific instructional and intervention design features that represent the backbone of effective interventions. Recommendations 2 through 6 focus on more specific practices. Recommendations 2 and 4 (Mathematical Language and Number Lines) reflect evidence from recent advances and provide support for some changes in contemporary state standards since the original guide, *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle School*,[5] was published in 2009. **Recommendation 3** (Representations), **Recommendation 5** (Word Problems), and **Recommendation 6** (Timed Activities), were included in the 2009 guide. In contrast to how they were presented a decade ago, this updated guide frames them differently and highlights new evidence and ideas for intervention.

Using Evidence to Develop the Recommendations

This practice guide grounds the six recommendations in high-quality evidence-based research studies focused on mathematics intervention. Each recommendation includes features of intervention and/or instructional practices, with guidance on how to implement them, advice on how to overcome potential obstacles, and a short summary of the research evidence that supports the recommendation.

After considering the evidence, the expert panel and the WWC drafted recommendations and assigned a level of evidence to each.

The recommendations and the panel's strength of evidence assessment are shown in **Table 1**.

Box 1. Levels of evidence

Strong: There is consistent evidence that meets WWC standards and indicates that the practices improve outcomes for a diverse student population.

Moderate: There is some evidence meeting WWC standards that the practices improve student outcomes, but there may be ambiguity about whether that improvement is the direct result of the practices or whether the findings can be replicated with a diverse population of students.

Minimal: Evidence may not meet standards or may exhibit inconsistencies, but the panel determined that the recommendation must be included because the intervention is based on strong theory, is new and has not yet been studied, or is difficult to study with a rigorous research design.

More detailed information can be found in **Appendix A** and **Appendix C**.

Table 1. Recommendations and corresponding levels of evidence

Practice Recommendation	Level of Evidence: Minimal	Moderate	Strong
1. Systematic Instruction: Provide systematic instruction during intervention to develop student understanding of mathematical ideas.			✓
2. Mathematical Language: Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.			✓
3. Representations: Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.			✓
4. Number Lines: Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.			✓
5. Word Problems: Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.			✓
6. Timed Activities: Regularly include timed activities as one way to build fluency in mathematics.			✓

Who Might Find This Guide Useful

This guide is designed to be used by teachers providing mathematics intervention to students who are struggling. This professional group includes special educators, mathematics general education teachers, mathematics specialists, and mathematics coaches. The recommendations outline evidence-based practices that can help teachers tailor their instructional approaches and/or their mathematics intervention programs to meet the needs of their students—that is, students with or at risk for mathematics disabilities who may also have reading, language, attention, behavior, working memory, or processing-speed difficulties. The guidance provided in these evidence-based recommendations may also be useful to school, district, or state personnel involved in adopting intervention curricula for their schools, and to parents to understand what mathematics assistance might be helpful for their children.

How to Use This Practice Guide

The panel suggests that the practices recommended in this guide be used in combination to help students achieve the strongest outcomes. Users of this guide are encouraged to use the advice provided in ways that best fit the varied lessons and contexts in which they work.

For each of the six recommendations, we include the following:

- **Recommendation:** Each recommendation includes details about the recommended practice and a summary of the evidence. **Appendix C** contains a detailed rationale for the Level of Evidence with supporting details from individual studies.
- **How to carry out the recommendation:** The "how-to steps" include the bulk of the guidance for teachers and other educators on

how to implement the recommended practice. This guidance is informed by the body of studies contributing to that recommendation in concert with the panel's expertise and knowledge of mathematics instruction and intervention. Examples are included to give the reader ideas for implementing the recommendation. Examples are not intended to endorse specific products for purchase.

- **Potential obstacles/roadblocks:** The panel offers guidance for addressing potential challenges to implementation.

Recommendation 1: Systematic Instruction

Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

Introduction

Effective interventions for improving the mathematics achievement of students with mathematics difficulties share one key feature: the design of the curricular materials and the instruction provided are systematic.[6] The term systematic indicates that instructional elements intentionally build students' knowledge over time toward an identified learning outcome(s).[7] Systematic intervention materials are designed to develop topics in an incremental and intentional way, and the instruction provided supports student learning.[8] This approach specifically addresses the needs of students who are struggling.[9]

Systematically designed interventions most often include a "bundle" of practices used to build and support student learning strategically.[10] These practices and design features appear in other recommendations. For example, reviewing and integrating previously and newly learned content is highlighted in **Recommendation 5** (Word Problems); incrementally building knowledge is illustrated in **Recommendation 3** (Representations), **Recommendation 4** (Number Lines), and **Recommendation 5** (Word Problems); and providing adequate supports for students to learn and understand new concepts and procedures is highlighted in **Recommendation 2** (Mathematical Language) and **Recommendation 3** (Representations). Regardless of the intervention's focus, these aspects of instructional design are critical for supporting student learning.

The WWC and the expert panel assigned a **strong level** of evidence to this recommendation based on 43 studies of the effectiveness of systematic intervention design features and systematic instruction.[11] Thirty-two of the studies meet WWC group design standards without reservations,[12] and 11 studies meet WWC group design standards with reservations.[13] See **Appendix C** for a detailed rationale for the Level of Evidence for **Recommendation 1**.

This recommendation highlights an overarching set of instructional features that form the backbone of effective, systematic interventions. This section describes strategies, examples, and tools that instructors can use to implement effective systematic interventions.

How to carry out the recommendation

1. Review and integrate previously learned content throughout intervention to ensure that students maintain understanding of concepts and procedures.

The panel recommends that interventions include systematic review of content by including a mix of *previously* and *newly learned* material within and across lessons. Review relevant previously taught material before introducing new, related content within and across lessons. Help students understand the link between the previous content and the new content they are learning.[14] Present students with opportunities to think about or solve familiar types of problems. During this review, students can explain what they know about a topic, or they can solve problems. Review can be completed individually or while working with a small group or a partner (see **Recommendation 2** on Mathematical Language).

Regularly present a variety of problems that require students to discriminate among problem types (see **Recommendation 5** on Word Problems and **Recommendation 6** on Timed Activities). This practice avoids having students overgeneralize new concepts or procedures to previously learned material.[15] For example, once regrouping has been introduced with double-digit subtraction, include some problems that do not require regrouping, so students do not overgeneralize regrouping to all problems. When fraction multiplication and division are introduced, continue to include previously covered fraction addition and subtraction problems throughout intervention. In doing this, students learn to discriminate when they do not need to find an equivalent fraction as in the case of fraction multiplication, from when they do, as in the case of addition, subtraction, or division of fractions when the denominators are not the same.

Mathematical ideas are complex, and virtually all learners need to use, discuss, and explain the ideas multiple times over a long period of time to understand them.[16] Provide students with opportunities to use and explain previous or newly learned mathematics concepts or procedures throughout the allotted time for intervention. So that students remain actively engaged during all parts of intervention, avoid saving these opportunities for discussion until the end of each lesson.

2. When introducing new concepts and procedures, use accessible numbers to support learning.

Use numbers that are easy for students to understand and work with during initial instruction.[17] When teaching a new concept or procedure, use single-digit or easy-to-understand numbers so that students can focus on the new concept or procedure rather than on difficult calculations. For example, when teaching students to find equivalent fractions, first work on equivalencies to unit fractions. Start with fractions equivalent to one-half, one-third, and one-fourth that are familiar and accessible to students. Use a concrete representation or a number line (see **Recommendation 3** on Representations and **Recommendation 4** on Number Lines for more details). A concrete representation or a number line can help students visualize the equivalence as they compare two-fourths to one-half, two-sixths to one-third, and two-eighths to one-fourth. When students have a grasp of the concept, systematically add other fractions to prevent students from overgeneralizing that equivalencies are only applicable to unit fractions.

3. Sequence instruction so that the mathematics students are learning builds incrementally.

Present mathematics concepts in a cohesive and logical way. Introduce concepts strategically so that the new learning relates to concepts taught previously. In the opinion of the panel, lessons taught on topics in isolation should be avoided during intervention. Instead, lessons should be connected day-to-day and across units of study to build toward specific learning outcomes. This carefully planned, intentional sequence of instruction capitalizes on prior learning and ensures that students have the knowledge necessary to learn new content effectively.[18]

Focus lessons on smaller tasks needed to solve complex problems before pulling it all together. This may apply to highly procedural multi-digit computation problems, or when teaching students to understand and solve word problems. In the view of the panel, the key to building knowledge in this incremental way is to help students become comfortable with smaller subtasks of problem solving so they can eventually connect them to solve complex problems.

One way to do this is to use worked-out examples as a way to focus on smaller tasks. Strategically exclude steps in a worked-out example and ask students to provide those steps until they become more comfortable with the procedures in solving problems. Additional, specific ways to build knowledge incrementally in word problem solving is provided in **Recommendation 5** (Word Problems).

4. Provide visual and verbal supports.

At the heart of all interventions for students who struggle with mathematics are efforts to support student learning.[19] This can be done using visual and verbal supports.

Verbal supports may include teacher prompting or questioning to help students attend to and remember the connections between prior learning and the new mathematics they are doing. These verbal supports may be accompanied by a visual which could include a gesture or a concrete or semi-concrete representation. A visual may also include a picture or diagram to be used as a "hint" for a next step or as a reminder to think about a certain concept.

Each recommendation in this practice guide offers specific approaches for supporting student learning visually and verbally. For example, the word wall offered in **Recommendation 2** on Mathematical Language supports students in providing explanations of the mathematics. **Recommendation 3** (Representations) and **Recommendation 4** (Number Lines) offer detailed explanations of how to help students visualize the mathematics concepts they are learning. **Recommendation 5** (Word Problems) describes the use of strategic prompt cards that support students in completing complex problems.

5. Provide immediate, supportive feedback to students to address any misunderstandings.

If students are not able to explain their understanding of key mathematics concepts or do not execute procedures correctly, provide them with immediate feedback. When students are solving problems, encourage them to articulate their thinking so that you can identify their strengths. Ask probing questions to identify any misconceptions and build on their strengths to correct those misunderstandings. Structure questions in such a way as to help students self-identify where their thinking went wrong. It might be helpful for students to use representations (see **Recommendation 3** on Representations) to help them articulate what they are thinking. Correcting misunderstandings early can prevent the confusion from becoming an enduring problem.[20] Tailor feedback to individual students, unless more than one student in a small group setting is struggling with the same misunderstanding.

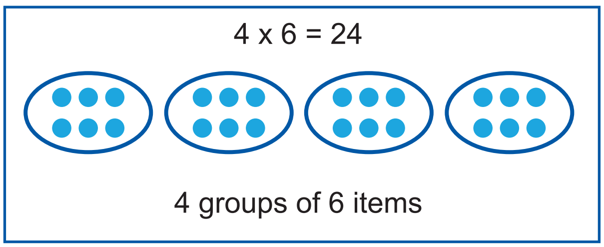
Example 1.1 depicts one way to address all the steps in this recommendation. Terms in the example that demonstrate steps in this recommendation are bolded.

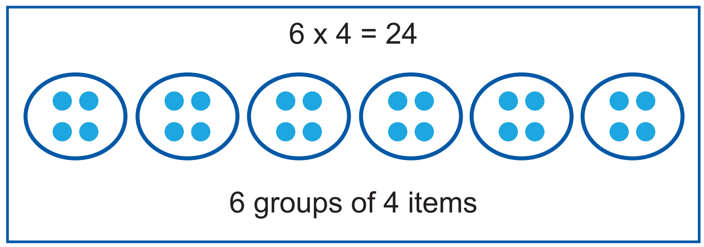
In this example, the learning outcome that has been identified for the small group of 3–6 students is *multi-digit division*. The equal-sized groups model of multiplication and division has been built incrementally and intentionally over multiple lessons using correct mathematical language and visual representations. Students have demonstrated understanding of the equal-sized groups model and how it relates to both multiplication and division. Now, the students are ready to learn how to apply this model to solve multi-digit division problems. The teacher plans to launch her lesson by reviewing students' prior knowledge of multiplication and division concepts to lead into the lesson on multi-digit division.

Example 1.1. Putting together the steps of Recommendation 1.

The teacher **reviews what the students have previously learned** about the equal-sized groups model (also known as the "groups of" or repeated subtraction model) and how it can be used to solve multiplication and simple division problems. The teacher reminds students of fact families and the inverse relationship between multiplication and division.

After **explicitly reviewing** what students know, the teacher **asks a student to explain** how the equal-sized group model can be used to solve the problem 4 × 6. Because the student has **recently practiced** solving multiplication and division problems **with a visual representation**, the student draws 4 circles with 6 dots in each and explains how she created 4 groups of 6 and skip counted to solve the multiplication problem. If needed, the teacher is poised to **prompt the student** if she misses a key point and provide **corrective feedback**. The teacher asks another student to solve and explain the problem 6 × 4.





The teacher asks another student to explain how the equal-sized group model can be used to solve the division problem 24 divided by 6. The student draws 24 dots and puts them in equal groups of 6 dots. The teacher **helps the student explain how the problem** can be solved by repeatedly subtracting groups of 6 from 24 to find out how many equal groups of 6 are in 24. Notice that they record the number of groups subtracted on the right side.

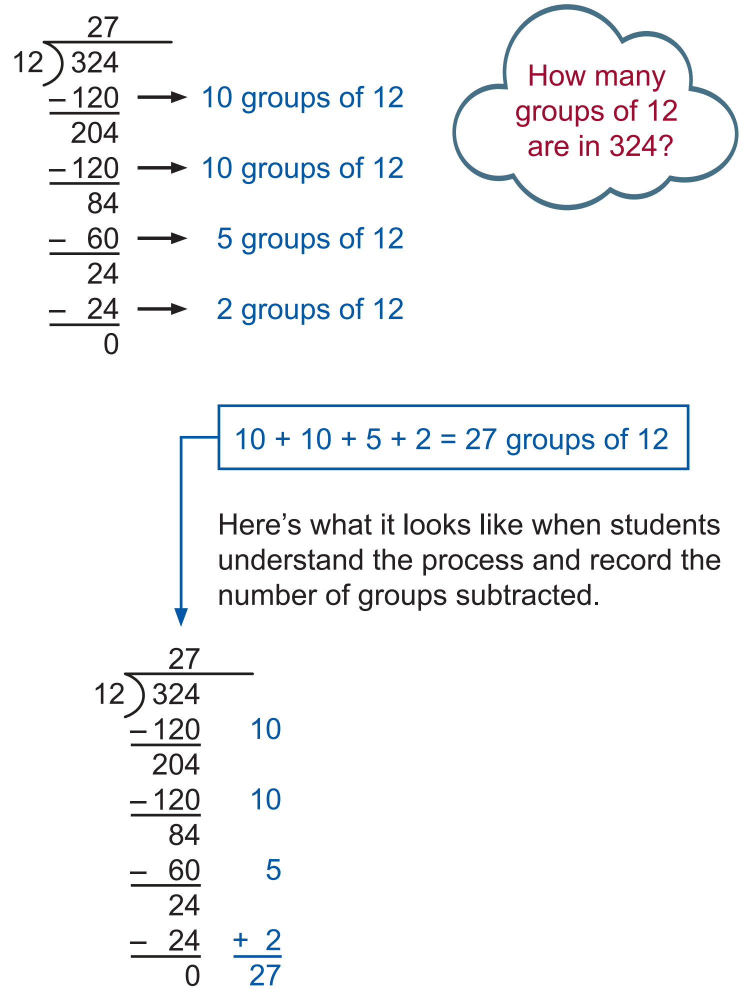
$24 \div 6 = 4$

$$\begin{array}{r|l}
 & 4 \\
6\,\big)\; & 24 \\
 & \underline{-\,6} \quad 1 \\
 & 18 \\
 & \underline{-\,6} \quad 1 \\
 & 12 \\
 & \underline{-\,6} \quad 1 \\
 & 6 \\
 & \underline{-\,6} \quad 1 \\
 & 0
\end{array}
\quad \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{4 groups of 6}$$

The teacher **presents students with a variety of problems to solve** with multiplication and division fact families. The students practice solving the problems individually or with a partner. As students share their solutions with the group, the teacher **provides corrective feedback**. When the teacher hears or observes a student missing a key point, the teacher **asks guiding questions to support the students with their explanations and support students' use of mathematically accurate language.**

The teacher **builds on students' prior knowledge** by explaining that the same equal-sized group model used to solve multiplication and related division problems can be used to solve division problems with two-digit divisors. The teacher **reminds the students that they are already familiar** with the equal-sized group model that they use to solve division problems with single-digit divisors. The teacher

presents a worked-out example of a multi-digit division problem that has been solved using the equal-sized group model using repeated subtraction. The problem uses groups of 12 in the solution strategy that are chosen according to known multiplication facts, **numbers that are familiar and accessible to the students**. The teacher explains each step of the solution strategy and the reasoning behind the steps taken.



For the next **worked example**, the teacher **asks students to help explain each step of the solution strategy**. The teacher leads several examples, **asking students to help solve the problem** and discuss their thinking process with the rest of the group. The teacher **prompts students or asks guiding questions** to **help the students engage** in the problem solving for each problem they solve together. **Students are asked to share the reasoning** for the strategies they are using and the answers they give.

The teacher asks students to solve multi-digit division problems **using accessible numbers** with a partner. The teacher listens to each discussion and observes the recording of the process, **providing corrective feedback and prompts as necessary**.

The teacher includes **more difficult problems in subsequent lessons as the students gain confidence and competence** in the equal-sized group model to solve multi-digit division problems through repeated subtraction.

Potential Obstacles and the Panel's Advice

OBSTACLE: *"I don't have access to an intervention curriculum in my school. Are you saying I should create my own materials or locate free materials? How do I know if the resources I create or find are systematic?"*

PANEL'S ADVICE: The panel is not suggesting teachers create materials that align with the steps in this recommendation. Instead, the panel suggests using these steps as guidelines for evaluating curricula to adopt. Finding materials on your own may be difficult. Work with a team (such as a mathematics coach and special educator) to look for materials that come with a scope and sequence of instruction which build from one lesson to the next to a learning outcome. Evaluate the lesson scope and sequence to determine if there are clear procedures for introducing new content, ample opportunities for students to respond, and built-in feedback procedures.

OBSTACLE: *"I feel like there is so much to cover at every grade level that choosing topics for more intensive instruction and/or slowing down instruction means I cannot cover all the grade-level material. This feels like I am doing my students a disservice."*

PANEL'S ADVICE: Intervention is an opportunity for students to build understanding in the most critical grade-level topics. Students are receiving intervention because they need more time and more frequent work with an adult to learn grade-level mathematics. Structure the pace and topics in intervention in such a way that promotes learning the mathematics more deeply; this often means taking more time.[21] By collaborating, intervention teachers and general mathematics teachers can ensure that the intervention complements grade-level mathematics instruction. In particular, teachers can identify together what the students in intervention need to work on and understand in order to access grade-level content. Fractions in grades 3 and 4, for example, can be difficult for students and are critical for students to understand for virtually all new mathematics learning through middle and high school. For students with Individualized Education Programs (IEPs), the panel recommends that teachers make sure to look at students' specific goals to guide instructional decisions.

Recommendation 2: Mathematical Language

Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

Introduction

Mathematical language is academic language that conveys mathematical ideas.[22] This includes vocabulary, terminology, and language structures used when thinking about, talking about, and writing about mathematics. Mathematical language conveys a more precise understanding of mathematics than the conversational or informal language used every day to communicate with others.[23]

Understanding mathematical language is critical to students' learning because it is used in textbooks, curricular and assessment materials, and teachers' instruction.[24] By providing instruction on mathematical language, teachers support students' learning of subtle and complex mathematical ideas.[25] Focusing on mathematical language during intervention also helps students access the language used during core instruction.[26] Therefore, developing students' mathematical language is critical for their success in mathematics,[27] especially as the material gets more complex.

Teachers and students can communicate more clearly during class when they are both using mathematical language.[28] As teachers use and model correct mathematical language, their students hear how the words fit with the mathematics they are learning and begin to integrate this language into their own explanations of the mathematics.[29]

When teachers use conversational or informal language instead of mathematical language, students may get confused. For example, some teachers may refer to the commutative property (a + b = b + a) as the "flip-flop property." Although this creative property name may be viewed as a fun memory device, replacing accurate terms with informal language such as this can cause serious confusion later in the students' schooling when other teachers do not use the "flip-flop property" or when learners don't know the connection to the correct formal term *commutative property*. Using and practicing correct terminology from the start can eliminate later challenges.

Contemporary standards emphasize the need for students to use precise mathematical language when providing explanations of the mathematics.[30] By learning mathematical language, students will be able to provide explanations of the decisions they make while solving problems and will be better equipped to construct logical arguments when clarifying a solution strategy.[31]

The WWC and the expert panel assigned a **strong level** of evidence to this recommendation based on 16 studies of the effectiveness of embedding instruction in mathematical language throughout interventions.[32] Twelve of the studies meet WWC group design standards without reservations,[33] and four studies meet WWC group design standards with reservations.[34] See **Appendix C** for a detailed rationale for the Level of Evidence for **Recommendation 2**.

This recommendation presents steps for teaching mathematical language and supporting students in using mathematical language as they communicate about mathematics. Include these steps throughout the intervention curriculum.[35] Students in intervention settings will need multiple exposures to mathematical language to understand the terminology and begin to integrate it into their vernacular.[36] This section describes strategies, examples, and tools that can support instructors in effectively embedding mathematical language instruction in their interventions.

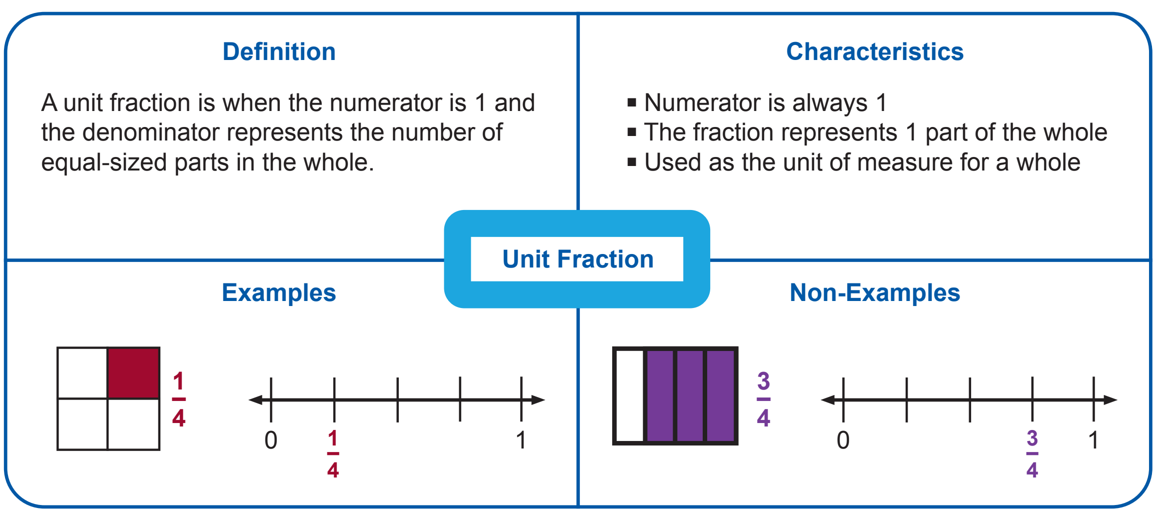
How to carry out the recommendation

1. Routinely teach mathematical vocabulary to build students' understanding of the mathematics they are learning.

Introduce new mathematical vocabulary during instruction to provide context and meaning to the words.[37] Use student-friendly definitions with simple and familiar mathematical words.[38] Link new vocabulary to a variety of examples when possible, including concrete or semi-concrete representations. In **Example 2.1**, a graphic organizer pairs a new vocabulary word, *unit fraction*, with its student-friendly definition, characteristics, examples, and non-examples. Use this type of graphic organizer with any vocabulary word to visually and symbolically depict the word's meaning.[39]

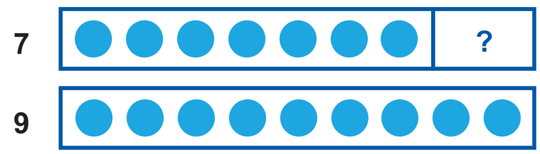
Simply providing a definition of a term is not sufficient for developing students' understanding of mathematical vocabulary and concepts.[40] Deepen students' understanding of the words by connecting them to concrete and semi-concrete representations. Hand gestures and role-playing can also provide context and meaning to mathematical vocabulary. The context provided by representations, hand gestures, and role-playing will help students better understand what they are learning.[41] **Example 2.2** demonstrates a hands-on activity with concrete representations (counters), to help students learn and understand the meaning of the equal sign and the word "equal." **Example 2.3** demonstrates a role-playing activity to help students learn and understand the meaning of "divide."

Example 2.1. Graphic organizer that depicts a student-friendly definition, characteristics, examples, and non-examples for the term *unit fraction*.



Example 2.2. Concrete representation used to build students' understanding of the meaning of *equal* and the *equal sign* symbol in early elementary school (grades K–2).

The teacher counts out 9 counters and 7 counters and lays them on the table in rows. She explains that the two rows need to be **equal** and explains that **equal means the same amount**.

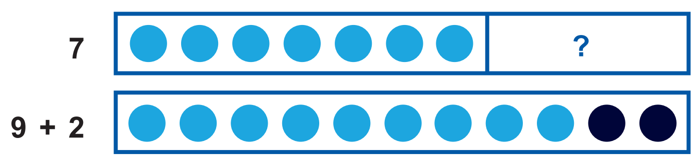


Next, she writes out the mathematical notation that is represented, explaining that the 7 counters are recorded as 7, the addition sign is recorded as +, the blank line represents an unknown value, the **equal sign** shows that the quantities represent the same value, and the 9 represents the total amount.

$$7 + \underline{\quad} = 9$$

The teacher asks, "How many counters do you need to add to 7 to equal 9?" Students add 2 counters to the top row. Then, the students write 2 on the blank line.

Next, the teacher builds the idea of the **equal sign** by changing the number of counters in the two rows. The teacher removes the two new counters from the top row and adds two counters to the group of 9 in the bottom row.



Then she writes the following equation.

$$7 + \underline{\quad} = 9 + 2$$

She asks the students to point out where the 7, 9, and 2 counters are on the table. Then she asks students how they might figure out how to make the two quantities in the two rows and on either side of the **equal sign** the same value.

Students add 4 counters to the top row and write a 4 on the blank line. The teacher and students discuss how the two rows have the same number of counters and that now both sides of the equation (on either side of the **equal sign**) show the same amount.

Example 2.3. Role-playing with hand gestures that teach the meaning of mathematical ideas or vocabulary.

12 ÷ 3 = __

Teacher: *Look at this problem, twelve divided by three: 12 ÷ 3. This problem asks us to* ***divide*** *12 into 3 groups. To help us understand what it means to* ***divide*** *an amount of something into equal groups, let's pretend we are* ***dividing*** *a group of 12 apples among three families. We want to find out how many apples each family will receive.*

If we pretend that each paper plate is a basket for each family and each counter is an apple, how many plates will we need? How many counters will we need?

Based on the problem, students should select 3 plates and 12 counters. The teacher asks guiding questions if students need help.

Teacher: *Now I'd like to you* ***divide*** *the counters among the 3 families.*

The teacher shows students the action of divvying up the counters into the plates. Her gesture looks like a dealer dealing out cards or more generally a hand motion that "gives" one item per basket until the "apples" are all distributed equally. The teacher wraps up by counting the number of apples per family.

To support learning across grade levels and settings, schools should consider creating a shared list of mathematical terminology that strategically becomes more sophisticated with each grade.[42] A shared list can ensure that teachers use consistent vocabulary and language across intervention and core classrooms, further supporting the learning of students who receive instruction in both settings.[43] **Table 2.1** depicts examples of precise mathematical language that teachers may use across grades and settings in a school.

Table 2.1. Example word list that can be used across settings in grades K–6 by all teachers in the school.[44]

Rather than using this term…	*Consider using this term…*
Reduce	Simplify
Borrowing or Carrying	Regrouping
Flat Shape or Fat Shape	Two-Dimensional or Three-Dimensional Shapes
Bigger, Smaller	Greater Than, Less Than
Flip-Flop Property	Commutative Property

Note: This list is not comprehensive. It only contains a sample of words that might appear on a more comprehensive shared list used in a school.

2. Use clear, concise, and correct mathematical language throughout lessons to reinforce students' understanding of important mathematical vocabulary words.

Use and emphasize clear, concise, and correct mathematical language throughout instruction: when referring to a new or previously learned topic, when discussing homework, and when responding to questions. It may take several lessons for students to understand new mathematical vocabulary and develop a deep understanding of the mathematics connected to the words.[45] Consistent use of mathematical language helps students learn how the terms should be used and develop a deeper understanding of the terms.[46]

Model precise mathematical language when explaining your thought process and demonstrating how to solve a problem. In **Example 2.4**, the teacher uses the term ratio while saying out loud her reasoning for solving a word problem. Her ongoing and repetitive use of the term helps reinforce the meaning of a ratio and how the ratio in the problem provided critical information for solving the problem.

Example 2.4. Teacher using mathematical vocabulary when thinking aloud during mathematics intervention in upper elementary (grades 3–6).

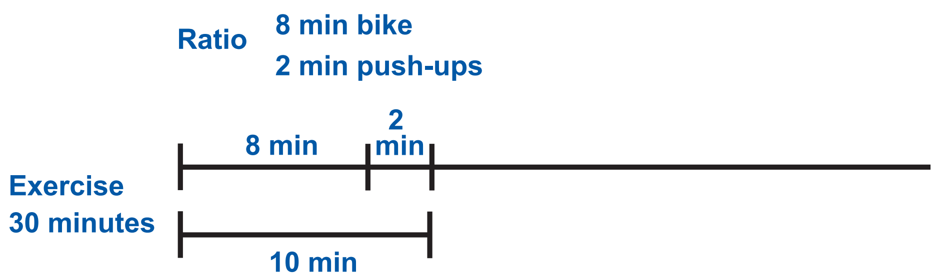
First, the teacher reads the problem out loud. During her think-aloud below, the teacher points to the work she is doing as she talks so that students attend to the correct parts of each solution.

Vocabulary: ratio

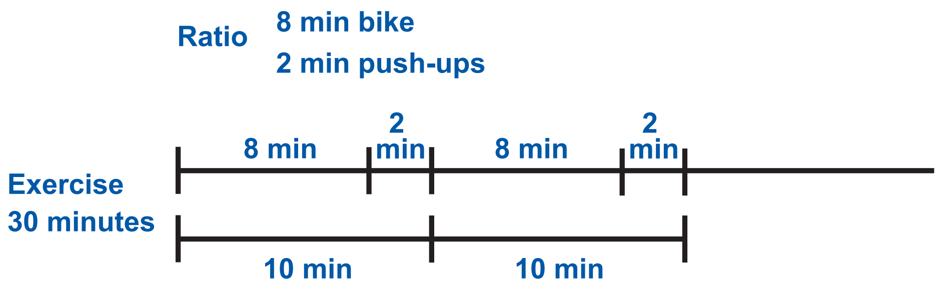
Problem: Kesha likes to exercise. For every 8 minutes that she uses the exercise bike, she does push-ups for 2 minutes. If she exercises for 30 minutes, for how many minutes does she do push-ups?

Teacher: *I see that the question wants me to figure out how many minutes Kesha spends doing push-ups in 30 minutes of exercise. I use the* ***ratio*** *for minutes spent using the bike and minutes spent doing push-ups. Remember, a* ***ratio*** *is a statement of how two numbers compare or relate to one another. The problem gives a* ***ratio*** *for minutes spent using the bike and minutes doing push-ups. I am going to write down the* ***ratio*** *for minutes spent exercising on the bike and minutes spent doing push-ups here. I create a strip diagram that will include all 30 minutes of exercise.*

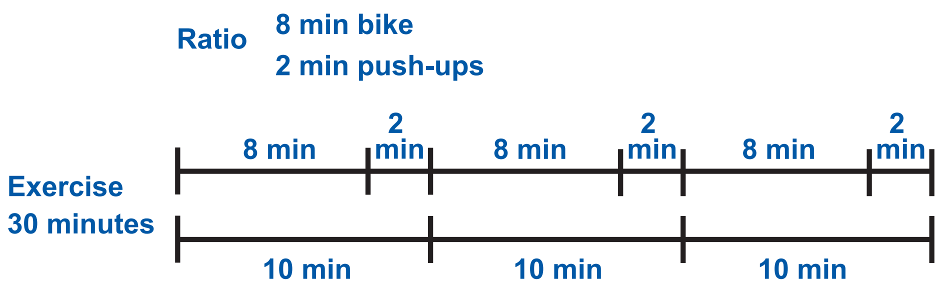
Now I use the ***ratio****. Because I am using a* ***ratio****, every time she spends 8 minutes exercising on the bike, she spends 2 minutes doing push-ups. I write the* ***ratio*** *once on the strip to see how many minutes she spent doing both activities. The first time she is on the bike for 8 minutes she then does 2 minutes of push-ups. Together that equals 10 minutes of exercise.*



Now I keep using the ***ratio*** *until I get to 30 minutes of exercise. I add another 8 minutes of biking and 2 minutes of pushups. I check to see if I've reached 30 minutes yet. I have not.*



I use the ***ratio*** *again and Kesha exercises 10 more minutes. Now I have 10 + 10 + 10. That is 30 minutes. Now I can use my recordings on the strip diagram to figure out how many minutes Kesha spent doing pushups during 30 minutes of exercise. I can skip-count by 2s: 2, 4, 6. Now I write the answer; Kesha did push-ups for 6 minutes.*



Some words may have more than one meaning or are used in more than one context in mathematics.[47] Provide instruction in the various ways words are used, using examples. In **Example 2.5**, the teacher leads an activity to expand students' understanding of the terms *factor* and *product*. The students have already learned that factors are the two numbers multiplied to get a product in a multiplication equation. In this lesson, students learn that a product can have multiple factors, not just two factors. Understanding that a number can have multiple factors leads students to being able to find a common denominator for adding or subtracting fractions. This new understanding of "factor" also prepares students for finding the greatest common factor between two numbers.

Example 2.5. Teacher leads an instructional activity to broaden students' understanding of the terms *factor* and *product*.

The lesson below focuses on the vocabulary words **factor** and **product**. The teacher also uses other correct mathematical language like *multiplication* and *equal*.

Teacher: *Some multiplication problems ask us to multiply two numbers to find the **product**. The two numbers we multiply are called **factors**. Today, I am going to give you a **product** and you need to find the missing **factor**.*

Product:

24

*First, let's try finding a missing factor with 24 as the **product**. I will give you the first factor. I know 24 is an even number, so I know that 2 can be multiplied by another number to get 24. That means 2 is a **factor** of 24. Does anyone know what number times 2 would equal 24?*

The teacher gives students time to respond and arrive at 12 as another ***factor*** of 24. If students are struggling, the teacher may have students use 24 counters and group them into equal groups of 2 to show the students that 2 times 12 equals 24. Teacher should write on the board as each ***factor*** is given by students.

Product:

24

Factors:

2 × 12

Teacher: *Now I want you to work with a partner to find other factor pairs that can be multiplied to get a **product** of 24.*

Allow students time to work with partners.

Teacher: *I'm going to ask each set of partners to give me two **factors** that when multiplied equal the **product** of 24. We will continue until we have listed all the possible **factors** that equal the **product** of 24.*

Students share their responses. As students respond, the teacher records the **factors** on the whiteboard with 24 at the top, labeled **product**. If students need to use the counters in arrays to find **factors** for 24, give them access to this support. Showing the different arrays on a document camera or in a small group can help students see the patterns of the **factors**. If using counters to create these arrays, emphasize that there are always 24 counters, and counters are not being added or taken away when constructing the different arrays for the **product** 24. Written on the whiteboard is:

Product:

24

Factors:

2 × 12   6 × 4   8 × 3   12 × 2   4 × 6   24 × 1   3 × 8   1 × 24

Teacher: *Remember, the two numbers that are multiplied to get a **product** are called **factors**. Let's list each **factor** for 24 one at time in numerical order so we know which numbers are factors of 24. The **factors** are:* 1, 2, 3, 4, 6, 8, 12, and 24.

Factors for 24:

1, 2, 3, 4, 6, 8, 12, 24

3. Support students in using mathematically precise language during their verbal and written explanations of their problem solving.

Have students provide verbal and written explanations of mathematics concepts during intervention to help them develop key mathematical insights. Explaining their work provides students with opportunities to communicate their mathematical understanding using newly learned vocabulary and also allows teachers to check for understanding and provide immediate corrective feedback.[48]

Students will likely need support to explain their thinking using mathematical language.[49] Offer students a framework for providing explanations, such as sentence starters or a set of guiding questions. It is also helpful for teachers to restate the students' explanations using correct language when students do not.[50] In **Example 2.6**, the teacher asks a student guiding questions using precise mathematical terminology while the student explains his own thinking. During the discussion of their solution methods, one student, Kerry, explained his thinking to the other students in the intervention group. The teacher provided clarifying questions and restated Kerry's comments to support his use of more precise mathematical language. The goal is for Kerry to adopt the vocabulary the teacher uses when she clarifies Kerry's statements about his approach for solving the problem.

Example 2.6. Teacher prompts students to use mathematical terminology in their explanations.

$$\begin{array}{r} 327 \\ -\ 148 \\ \hline \end{array}$$

Kerry: *I noticed that 8 is bigger than 7.*

Teacher: *You first **compared** the **digits** in the **ones place**? Did you mean 8 is **greater than** 7? Remember, when a number is "bigger" or "larger" than another number we say **greater than**.*

Kerry: *Yes, I started with the **ones place**, 7 and 8. 8 was larger, so I needed to cross out the 2 and make it 1. That made 17.*

Teacher: *Because 8 was **greater than** 7, you **regrouped**. You took 10 from the 20 in 327 and added it to the 7 ones to get 17. Then you changed the 2 in the **tens place** to a 1 in the **tens place**.*

The teacher points to the numeral 2 in the ***tens place***.

Kerry: *Yes, because there were 2 tens, I used one of them to make 17. Then I subtracted the 8 and got 9 **ones**. Next, I looked over and the 4 was **greater than** the 1. So, I had to change it again.*

Teacher: *You are describing how you knew to **regroup** the **ones** and then needed to use that approach again for the **tens place**. When you **regrouped**, you used what you knew about **place value**.*

Kerry: *So then, I **regrouped** from the **hundreds** because there were 3 **hundreds** in 327. I could break apart 300 into 200 and 100. I added 100 to the 1 ten in the **tens** column, and now I can **subtract** 40 from 110! That gave me a 7 in the **tens place** for the answer. That's 7 **tens**.*

Teacher: *So, your answer for the **difference** is 79 so far?*

Kerry: *Yes, and then I just had to **subtract** the **hundreds**. I did not need to **regroup**. 200 minus 100 equals 100. The **difference** is 179!*

Remind students to include the mathematical language modeled and taught during instruction by displaying mathematical vocabulary on the classroom wall. These kinds of supports can be useful for both verbal and written explanations.[51] **Tables 2.2** and **2.3** are visual displays that could be used in early elementary (grades K-2) and upper elementary (grades 3-6), respectively, to support students' use of mathematical language when providing both written and verbal explanations.

Table 2.2. A mathematical language chart that supports early elementary (grade K–2) students as they use mathematical language to present their thinking.

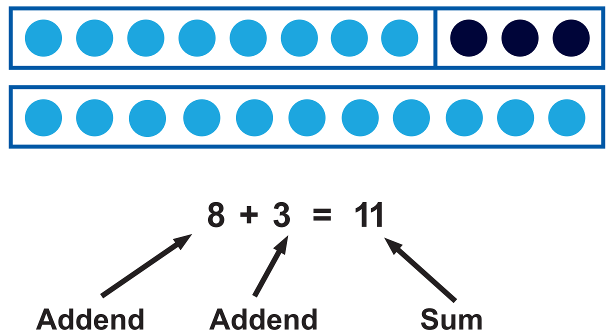
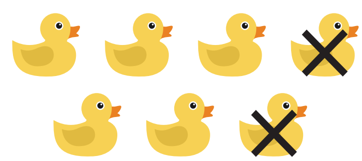
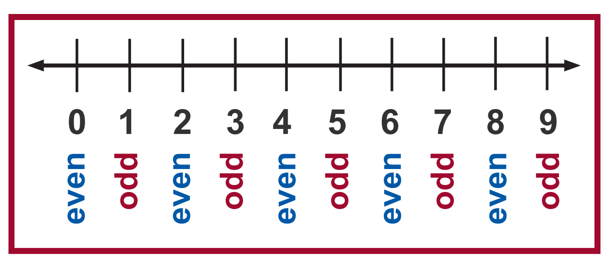
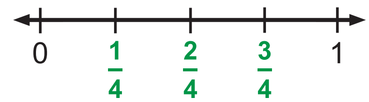
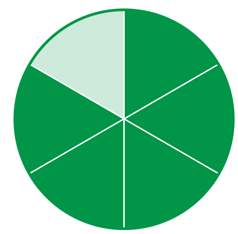
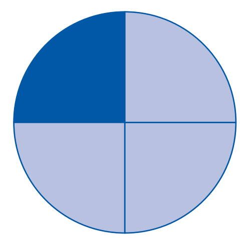
Term	Definition	Example/Representation
Addition	Joining or combining two sets together. Addition is represented with the symbol +.	Example: 8 + 3 = 11 is an addition equation. 
Subtraction	Taking away an amount or comparing two quantities to find the difference. Subtraction is represented with the minus sign, –.	Example of decreasing: 7 − 2 = 5 If we have 7 rubber ducks and then subtract 2, we are left with 5 rubber ducks.  Example of comparing the difference: Rosie is 11 years old. Eric is 9 years old. How much older is Rosie than Eric?
Even Number	Any integer that can be divided by 2 with a remainder of zero. The digit in the ones place of an even number is 0, 2, 4, 6, or 8.	0, 2, 4, 6, and 8 are all even numbers. 

Table 2.3. A mathematical language chart that supports upper elementary (grade 3–6) students as they use mathematical language to present their thinking.

Term	Definition	Example/Representation
Fraction	Fractions have different meanings: Part-whole (part of a whole) Measurement (a unit of measure)	$\frac{1}{8}, \frac{5}{8}, \frac{3}{4}, \frac{5}{5}, \frac{3}{2}$ $\frac{3}{4}$ of the area of the garden is planted in carrots. 
Numerator	The number of equal-sized parts being considered or used. It is the number of times the unit fraction is repeated. In this example 5 is the numerator.	$\frac{5}{6}$ 
Denominator	The number of equal-sized parts that make up the whole. In this example 4 is the denominator.	$\frac{1}{4}$ 
Unit Fraction	Used as the unit of measure for a whole (e.g., $\frac{1}{8}$ is copied 8 times to create a whole or 1). A unit fraction is a fraction with 1 in the numerator.	$\frac{1}{8}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}$

Potential Obstacles and the Panel's Advice

OBSTACLE: *"I don't know what words I'm supposed to use. Everyone seems to use different terminology."*

PANEL'S ADVICE: Review your state's mathematics standards to identify the important language for students to learn. Also consider state assessment guidelines and the curriculum materials used in the school. Consult with your colleagues to draft a list of accurate and precise vocabulary that the school can agree to use in mathematics classes across grade levels and settings. This could be a shared list of mathematical language on which teachers across the school agree.[52]

OBSTACLE: *"Teaching vocabulary takes time that we don't have."*

PANEL'S ADVICE: Integrate language instruction throughout mathematics intervention. Introduce and use mathematical words intentionally and throughout lessons, to reinforce their meaning.[53] Taking this approach does not require adding an activity that might take up additional instructional time. There does not need to be an entire mathematics intervention lesson focused on vocabulary.

Recommendation 3: Representations

Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

Introduction

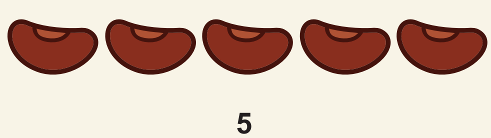
Concrete and semi-concrete representations are included in core instructional programs. However, students who struggle to learn mathematics need additional, focused instruction using representations to model mathematical ideas.[54] Choose representations carefully and connect them explicitly to the abstract representations (mathematical notation). If teachers do this, students can conceptualize the connection between the representations and the mathematics. It is also important to provide students with many opportunities to use representations and to help students understand the abstract nature of mathematics over time.

The WWC and the expert panel assigned a **strong level** of evidence to this recommendation based on 28 studies of the effectiveness of using a well-chosen set of representations to support student learning.[55] Nineteen of the studies meet WWC group design standards without reservations,[56] and nine studies meet WWC group design standards with reservations.[57] See **Appendix C** for a detailed rationale for the Level of Evidence for **Recommendation 3**.

What are concrete, semi-concrete, and abstract representations?

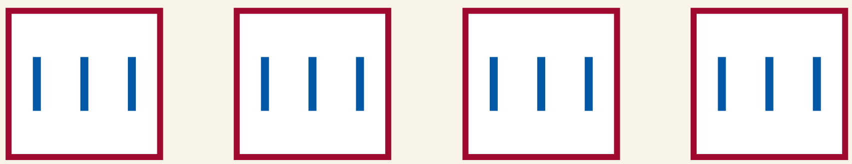
Representations illustrate the value of numbers and the relationship between quantities. Concrete and semi-concrete representations are powerful ways to make mathematics visible and more accessible for students. Creating visual models with concrete or semi-concrete representations may help students think through and solve problems more successfully because they help students understand the logic behind mathematical concepts and procedures.[a]

- **Concrete representations** are three-dimensional (3D), physical materials or actions that students can organize, act upon, or manipulate to better understand mathematics content (e.g., regrouping with base 10 blocks, using fraction tiles to compare two fractions, role playing a problem situation). Concrete representations may help students better understand mathematical concepts when they physically model with the manipulatives or act out problem scenarios to make the underlying mathematical concepts visible and less challenging to interpret.[b] For example, by counting out beans or connecting cubes, students learn one-to-one correspondence.



- **Semi-concrete representations** are two-dimensional (2D) visual depictions such as strip diagrams, simple drawings, tables, arrays, graphs, and number lines that may help students organize information. They can be used in conjunction with concrete representations to transition to more abstract representations. For example, students who struggle with addition, subtraction, multiplication, or division may find it useful to represent quantities with tick marks or by drawing dots. Because the number line is an essential semi-concrete representation in many interventions and standards, we devote a separate recommendation to using the number line effectively (see **Recommendation 4**).

$$4 \times 3 = 12$$



Pictures of concrete and semi-concrete representations described above are sometimes presented virtually on a computer or tablet screen.

- **Abstract representations** are mathematical notations that can include numbers, equations, operations, relational symbols, and expressions (such as 4, 16, multiplication and equal signs, greater than or less than signs, as well as equations such as $4 \times 4 = 16$).

a. Fuchs et al., 2005; Fuchs & Fuchs, 2001; Jitendra et al., 2016.
b. Fuchs et al., 2005; Fuchs & Fuchs, 2001; Jitendra et al., 2016.

This recommendation outlines four steps teachers can use to help students understand mathematics using concrete and semi-concrete representations. This section describes strategies, examples, and tools that can assist instructors in effectively using representations to support student learning.

How to carry out the recommendation

1. Provide students with the concrete and semi-concrete representations that effectively represent the concept or procedure being covered.

Not all representations work for every mathematical concept.[58] Choosing representations must be intentional and selective to be effective. It is therefore critical to provide students with the representations that most accurately model the concept or procedure being addressed. **Table 3.1** provides some examples of representations that work well for a sample of concepts and procedures.

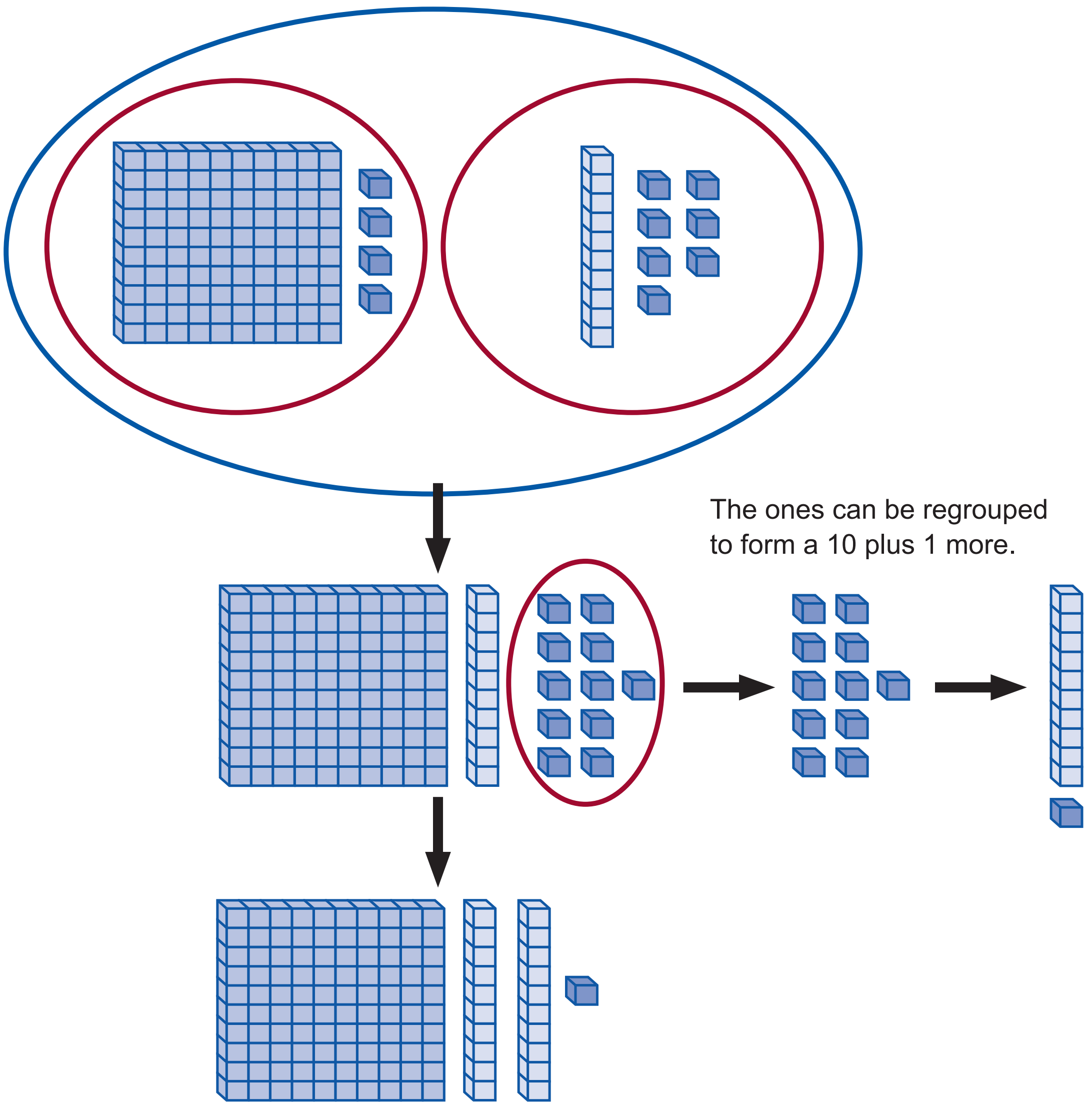
Table 3.1. Examples of common concrete and semi-concrete representations that can be used for a sample of mathematics concepts and procedures.

Mathematics concepts and procedures	Concrete	Semi-concrete
Counting/skip counting Addition Subtraction Multiplication Division Equality	• base 10 blocks • connecting cubes • Cuisenaire rods® • beads • two-colored counters • beans and cup • 1-inch tiles • balances	• hundreds chart • 5 frames, 10 frames, double 10 frames • strip diagrams • arrays
Place value Decimals/operations with decimals	• base 10 blocks • 1-inch tiles • connecting cubes • decimal squares	• place value chart • base 10 pictures • hundreds chart • rational number wheels
Fractions/operations with fractions Data Ratios and proportions	• Cuisenaire rods® • connecting cubes • pattern blocks • fraction bars • fraction tiles • fraction circles • two-sided counters • 1-inch tiles	• table • number line • strip diagram • bar graph • line plot
Patterns Geometry Graphing Area/perimeter Volume/capacity Line symmetry Length measurement 2D shapes 3D shapes	• pattern blocks • 1-inch tiles • connecting cubes • rulers, tape measures • geometric solids • protractors, angle rulers • containers	• pictures of shapes • graph paper grids • number line • isometric paper
Note: This table is not a complete listing of all representations, nor does it list all matching mathematics concepts and procedures.		

When appropriate, use representations that are proportional. For example, when teaching place value, the representation for ones should be one-tenth the size of the representation for tens, and the tens should be one-tenth the size of the representation for hundreds. In **Example 3.1**, the problem is depicted with base 10 blocks, which represent the place value for ones, tens, and hundreds proportionally. Notice how the representations for place value depicts regrouping accurately. The single-unit cubes, when grouped into a set of ten, match the size, shape, and length of the tens unit. Ten of the tens units match the size and shape of the hundreds unit. If the representations were not proportional, the concept of place value would be harder for students to grasp. The proportionality of the representation is also important when representing fractions concepts and operations.

Example 3.1. Teacher represents the addition problem with base 10 blocks, which are proportional for showing place value and regrouping concepts.

104 + 17 = _______



121

104 + 17 = 121

2. When teaching concepts and procedures, connect concrete and semi-concrete representations to abstract representations.

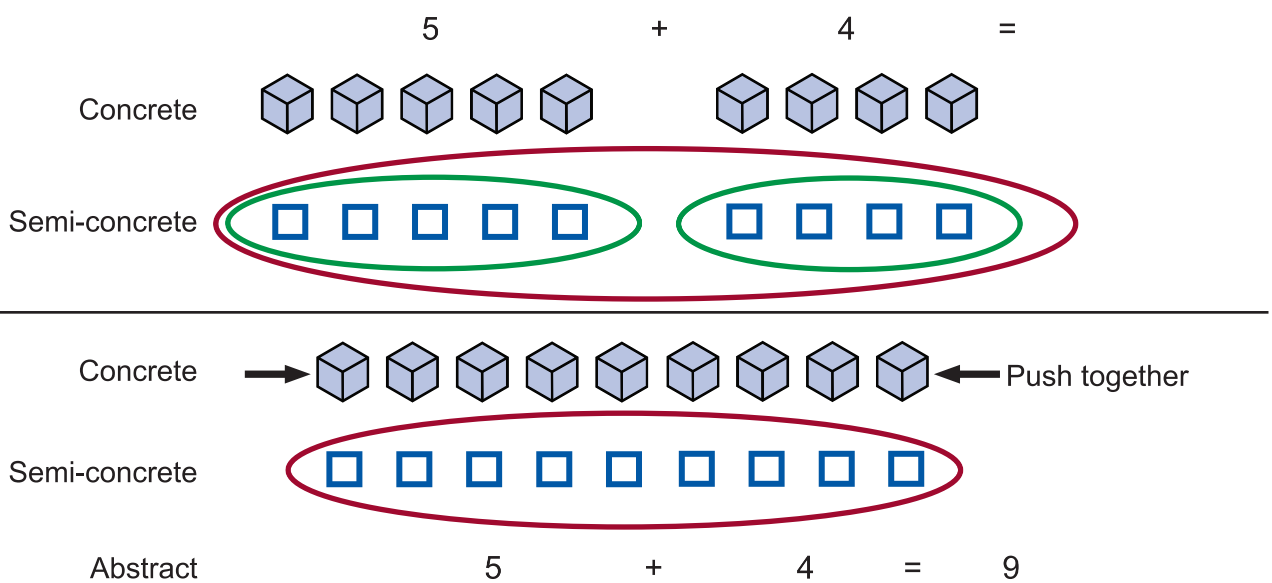
Although, in some circumstances, choosing either a concrete or semi-concrete representation may be appropriate for teaching and representing a particular concept, most concepts and procedures can be effectively represented by connecting both a concrete and a semi-concrete representation to the abstract representation (mathematical notation). When demonstrating concepts and procedures with concrete and semi-concrete representations, present the mathematical notation simultaneously so that students can conceptualize the connection between the representations and the mathematics. It is also important to connect concrete and semi-concrete representations to each other when teaching the same concept.[59] It is helpful to make these connections when introducing new material and when reviewing previously learned content. It is also important to make these connections when using familiar representations in a new way.

Example 3.2 demonstrates how the representations can be aligned vertically to demonstrate their connection. When teaching early addition concepts, a teacher may use counters as a concrete representation while also using pictures or sketches as a semi-concrete representation. Linking the counters to the pictorial representations and also to the mathematical notation can help students solidify the concept of addition.

Example 3.2. Teacher shows how combining two groups (a group of 4 and a group of 5) relates to concrete and semi-concrete representations and to an equation.

5 + 4 = _______

Teacher: *When looking at this problem we see that we need to add or combine the 4 and the 5. I can use counters to count out a group of 4 and another group of 5. Then I can combine them and count how many I have. I can also draw 4 squares to represent the 4 and 5 squares for the 5, and then count how many squares I drew in all. I find that I drew 9 squares, so the answer to the problem 4 + 5 equals 9.*



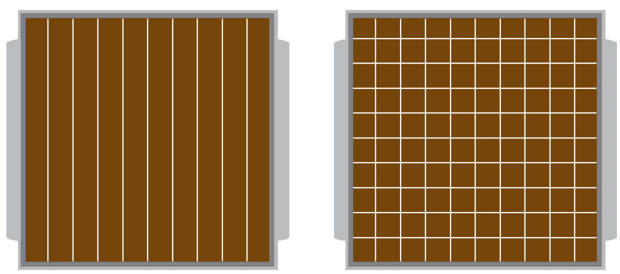
Rather than counting the entire combined number of cubes or squares starting with 1, students can be encouraged to count on from one of the addends. Students start with 5 and count on 4 more until they reach 9. Additionally students could also use their fingers to solve this addition problem.

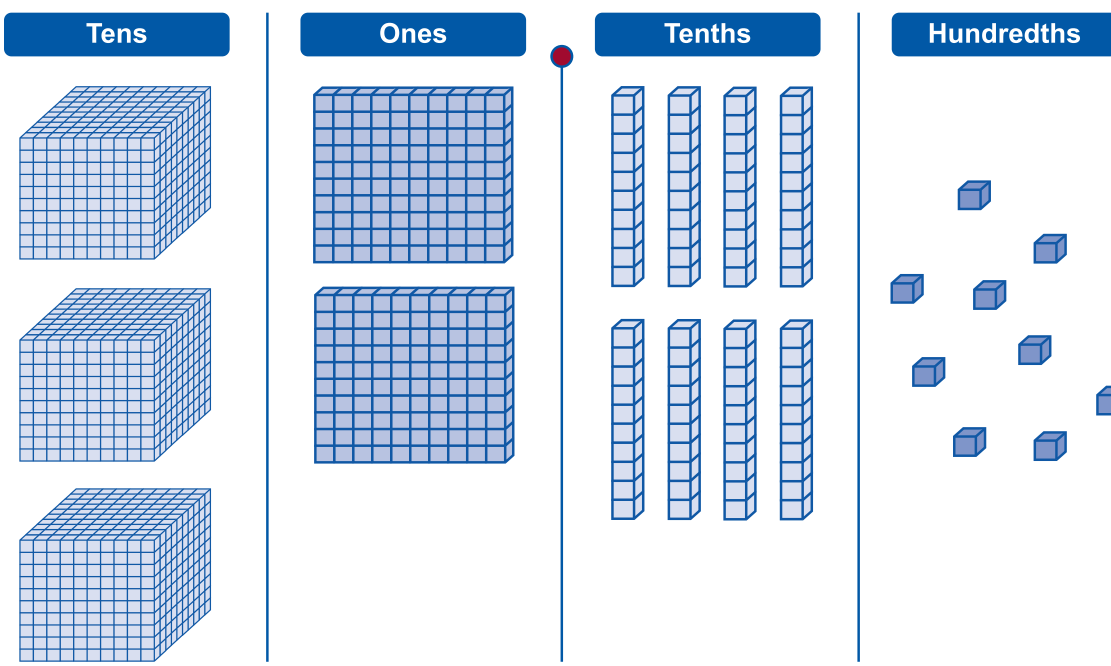
Example 3.3 depicts a teacher demonstrating how to use a familiar representation in a new way. In this example, upper-elementary students who are familiar with using base 10 blocks to represent whole-number operations can use these materials to represent decimal place value. The teacher and students discuss how, by reassigning the value of the blocks, they can also use the blocks to represent decimals. The teacher is helping students conceptualize how this same tool can be used to represent decimal place value. This representation is still proportional when used for decimals. After introducing students to this way of using base 10 blocks, the teacher might ask students to represent simple decimals, like 1.2 or 4.5.

Example 3.3. Teacher explains how to use base 10 blocks, with which the students are already familiar, to solve addition and subtraction problems with decimals.

Teacher: *When we are thinking about showing decimal amounts, the base 10 blocks can be used again to represent different units. The hundreds piece will represent ones, the tens unit will represent tenths, and the unit cubes will represent hundredths. Then the cube that previously represented thousands will now represent tens. You could imagine that the flat square piece is really a square baking pan full of brownies for a large family. If the family sliced the brownies into 10 long pieces, each piece would be* $\frac{1}{10}$ *or 0.1 of the pan of brownies. If they cut those 10 pieces into 10 equal parts from the other direction (shows horizontal cuts of a tenth into ten equal squares), each part is now* $\frac{1}{100}$ *or 0.01 of the whole.*

Here's how the number 32.89 would be represented using base 10 blocks. If the flat square pieces represent ones, then a group of 10 flat squares could be represented by a large cube. To represent 30, we would need 3 large cubes. To represent 2 ones, we'd need 2 flat square pieces. 8 rod shape units would represent 0.8, and 9 small unit cubes would represent 0.09.





3. Provide ample and meaningful opportunities for students to use representations to help solidify the use of representations as "thinking tools."

Students will need many opportunities to work with representations before they will successfully use them to model concepts and procedures and solve problems.[60] Allowing students to use representations only a few times will not be enough. Over multiple uses, students will begin to more deeply understand mathematics concepts and grasp how representations can be used as "thinking tools" in mathematics.[61] The goal is for representations to help students better understand mathematics concepts and procedures, and for students to grow comfortable using representations as tools to model problems and build their understanding.[62]

Representations can be used when students explain their thinking.[63] At first, students may need help articulating how they used the representations to depict the mathematical ideas. Pose prompting questions to help students explain how they represented the concepts and/or procedures.[64] As students become more comfortable using representations, routinely ask them to use the representations to explain their solution approach. This helps reinforce the mathematics not only for the student explaining her thinking, but also for the students who are listening to the explanation the student is giving.

4. Revisit concrete and semi-concrete representations periodically to reinforce and deepen understanding of mathematical ideas.

Students will use representations in high school (and beyond) such as algebra blocks, geometric models, and computer-based transformation tools for rigid motions.[65] As students grow more comfortable with the mathematics, they may use concrete or semi-concrete representations less often.[66]

Systematically revisit concrete and semi-concrete representations to reinforce and deepen students' understanding of mathematical ideas. Also, if students are not able to correctly solve problems or if they feel uncertain about how to approach a problem, encourage them to use a concrete or semi-concrete representation to represent or model the situation. Make concrete and semi-concrete representations available for students to use as necessary. Representations will help students become comfortable with the mathematics.[67]

Potential Obstacles and the Panel's Advice

OBSTACLE: *"I connected the abstract concepts and procedures to concrete and semi-concrete representations and then faded them, but I don't think my students fully understand the concepts."*

PANEL'S ADVICE: Only fade out concrete and semi-concrete representations as students become accurate with doing the work abstractly. If students do not fully understand the concepts, then fading out is not appropriate. Revisit concrete and semi-concrete representations periodically to clarify and reinforce the connection between abstract notations and concrete or semi-concrete representations. By periodically revisiting the connections, students will often experience deeper understanding and new insights about those connections.[68] Even after the abstract representations are used more consistently, it may be helpful to revisit concrete and semi-concrete representations to build and clarify the underlying mathematical concepts.[69]

OBSTACLE: *"My students just play with concrete representations and can't concentrate on the mathematics."*

PANEL'S ADVICE: Explain the expectations for appropriately using concrete representations as a learning tool. Students will need instruction in how to think about and use representations effectively.[70] Showing the connection between representations and the mathematics may not be enough for students to be able to use them independently. Students may also need instruction in how representations can be used to think through and solve a problem. Without instruction, students may not know how to use the representation in a way that will aid their understanding. Instruction can be done by modeling and talking through the steps of solving a problem, or it can be done by facilitating a discussion with students about how concepts and procedures can be represented. Students can provide their thoughts and model their use for the class.

OBSTACLE: *"My students are confused because different representations are used in different classes."*

PANEL'S ADVICE: Consistency in the types of representations shared in core classroom instruction and during intervention sessions, throughout the year and across grades, is critical.[71] Consistent use is particularly important for students who are struggling to grasp a concept or operation.[72] Using a core set of representations across settings and grades helps reinforce instruction on the same concepts. Keep the same set of core representations in use across grades: use the same representations as students move to the next grade. This level of consistency can be a part of a whole-school agreement where the goal is to align mathematics instruction through use of cohesive representations, language, notation, rules, and generalizations across grade levels.

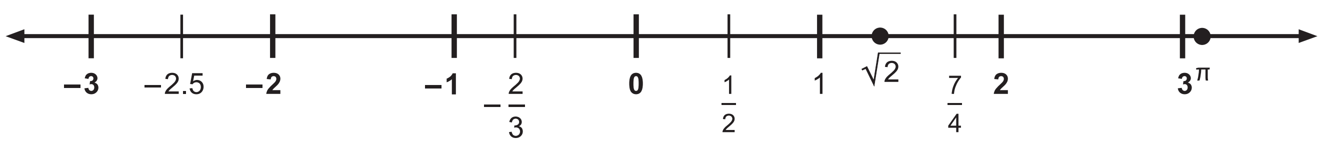
Recommendation 4: Number Lines

Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

Introduction

The number line is a unique mathematical representation that can concurrently represent all real numbers, including whole numbers and rational numbers, positive and negative numbers, and other sets of numbers. **Example 4.1** depicts examples of numbers that can be displayed on a number line. This ability to represent different sets of numbers makes the number line a powerful tool for helping students develop a unified understanding of numbers and for supporting their learning of advanced mathematics.[73]

Example 4.1. Number line representing magnitudes of whole, positive, negative, rational, and irrational numbers.



Number lines can be used to develop a variety of mathematical understandings and are included across several of the contemporary state standards.[74] Number lines are an important tool for teaching and understanding magnitude and operations for both whole numbers and fractions.[75] Number lines are also useful for demonstrating elapsed time problems,[76] graphing coordinates,[77] and displaying and analyzing data.[78] Vertical number lines are used to teach temperature and how to read thermometers, linear spring scales, or depth charts,[79] and can be paired with a horizontal line to form coordinate grids.[80] Consistent use of number lines can help students build understanding of the number system and improve their overall mathematics performance across a variety of mathematics content.[81]

Students who are proficient in mathematics often construct a mental number line as they solve problems.[82] When a teacher consistently uses number lines during intervention, students gradually develop the ability to visualize a number line when considering the magnitude of a number such as a fraction, determining strategies for solving mathematics problems, or evaluating the reasonableness of their answers after solving problems. It also sets the stage for more advanced work in middle and high school mathematics, when students acquire skill with negative numbers and solve linear inequalities through number lines.[83]

The WWC and the expert panel assigned a **strong level** of evidence to this recommendation based on 14 studies of the effectiveness of using number lines to facilitate the learning of mathematical concepts and procedures.[84] Eleven of the studies meet WWC group design standards without reservations,[85] and three studies meet WWC group design standards with reservations.[86] See **Appendix C** for a detailed rationale for the Level of Evidence for **Recommendation 4**.

This recommendation explains methods for using the number line to teach and support critical understanding of mathematics during intervention. Each step provides guidance for whole numbers in early elementary (grades K–2) and fractions and decimals in upper elementary (grades 3–6). This section describes strategies, examples, and tools that can support instructors in effectively using number lines in intervention settings.

How to carry out the recommendation

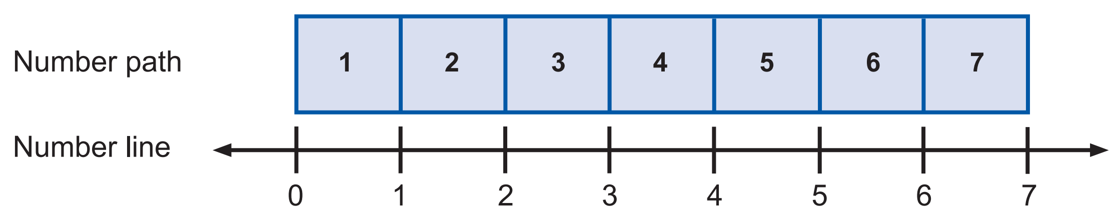
1. Represent whole numbers, fractions, and decimals on a number line to build students' understanding of numerical magnitude.

Early elementary (grades K–2)

Before using a number line, introduce students to a concrete version of a number line. Introduce students to number lines using a number path that students can walk, board games, or clotheslines; this may help students begin to form a visual image of what a number line looks like. When using these number lines, focus on the length of the units and the equivalency of the length unit. The distance between the positions of zero and one establishes the length of the unit and is the same distance between all whole numbers.

After exposing students to the concrete number line with a series of individual units lined up on a path, connect that idea to a number line on paper or projected on a screen. **Example 4.2** depicts how a concrete number path with a series of individual units corresponds to a number line.[87] Ask students to identify similarities and differences between the two representations. Draw their attention to the distance from zero to one and how that distance is the same length as one unit. This connection will help students understand that the 1 on a number line is not merely a tick mark, but also represents the full one-unit distance from zero.[88] Discuss with students how the concrete units represent the same numbers as a number line does.

Example 4.2. Connecting individual concrete units to a number line to represent positive whole numbers.



Explain the basic characteristics of number lines, such as those listed in the box below. Show students how each tick mark is equidistant from the previous tick mark and represents a unit of the length of 1. Notice how the whole numbers appear in the same predictable counting pattern. That is, when moving to the right, the magnitude of the number increases by one unit, just like counting by ones. Explain how numbers decrease by one unit as they move to the left and show students how zero is to the left of 1 on the number line. Introduce them to other counting patterns on the number line—such as skip-counting by twos, fives, or tens—by showing that the length of the new unit composed of 2, 5, or 10 units can be repeated across the distance of the number line.

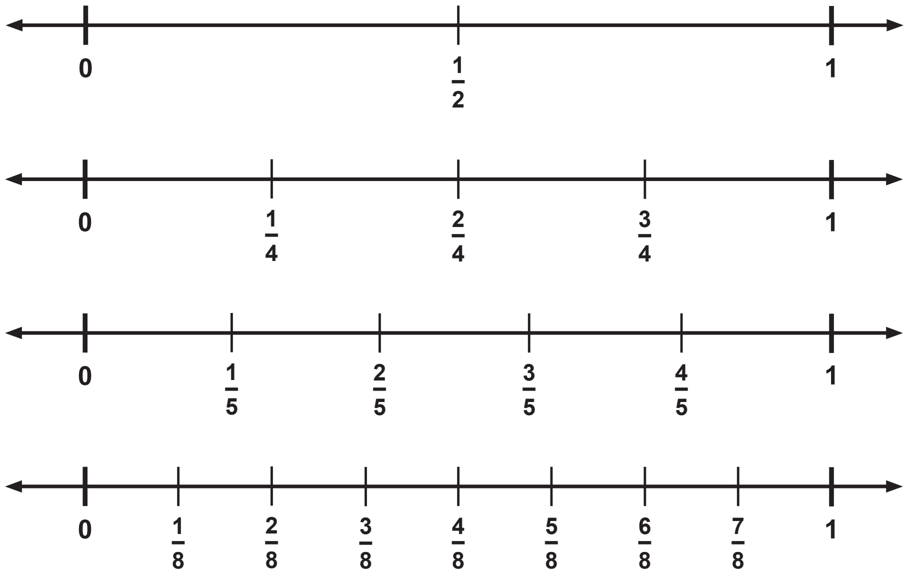
Upper elementary (grades 3–6)

Once students understand the concept of a fraction with concrete representations, show students how to represent fractions on a number line by connecting linear concrete representation to the semi-concrete number line. Demonstrate the location of fractions on the number line, starting with familiar fractions that are less than one. Ask students to fold a strip of paper in half to see two equal parts. Discuss with students how this represents partitioning a 0–1 segment of the number line into two equal parts. Ask students to mark the location of one-half. Then ask them to partition a strip of paper into four equal parts to represent the locations of one-fourth, two-fourths, and three-fourths on a 0–1 line segment. Ask students to demonstrate how the number line can be partitioned into additional parts, by showing a larger denominator, like eighths.[89] Reinforce the idea that the denominator represents the number of partitions in one whole. Include partitioning the number line with odd denominators which may be more difficult for students to partition equally. Number lines can be used to demonstrate the pattern of unit fractions and their corresponding magnitude.[90] The number lines in **Example 4.3** show the 0–1 portion of a number line partitioned into different size parts: halves, fourths, eighths, and fifths. Draw students' attention to the unit fraction on each number line to help students see the relative magnitude of each unit fraction.

Characteristics of number lines using whole numbers.

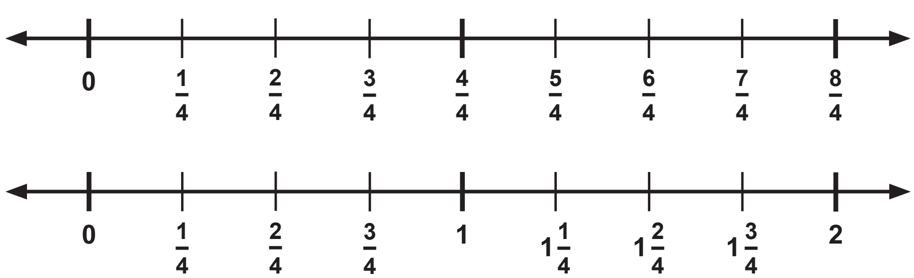
- Each whole number on a number line is equidistant from the next whole number.
- Whole numbers are represented in a predictable sequence.
- Number lines increase or decrease to infinity; you can always add or subtract one more unit.
- Display number lines with arrows on both ends to show that the units go infinitely in both directions.
- The numbers increase in value as you move to the right, and numbers decrease in value as you move to the left.
- Number lines can be presented with a pattern of numbers that represent numbers in a predictable way (e.g., 0, 10, 20, 30 or 0, 2, 4, 6, 8, 10).
- Number lines can be:
    - Three-dimensional/concrete representations (e.g., Cuisenaire rods®, a thermometer, or a ruler),
    - Two-dimensional/semi-concrete representations (e.g., a drawing of a number line, a picture of a ruler, a scale on a thermometer, or a coordinate grid), or
    - Virtual number lines on a screen or as a mental image in a student's head.

Example 4.3. Number line with halves, fourths, fifths, and eighths.



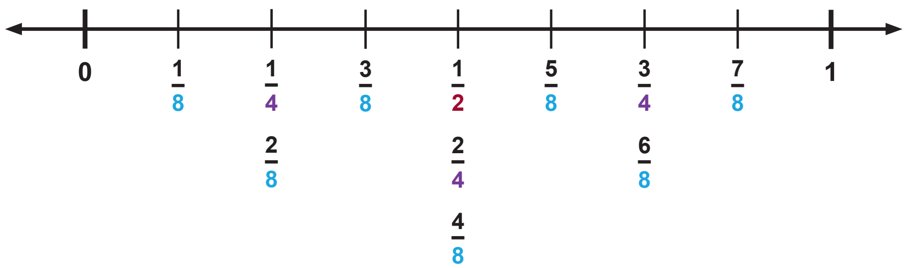
To ensure that students do not assume all fractions are less than one, expand the 0-1 segment to 0-2 to depict fractions equal to and greater than one. Show students that whole numbers can be represented as fractions and that similar fractions are located between other whole numbers. **Example 4.4** shows two number lines. The first includes fractions equal to and greater than one so that a student can see the pattern of counting by fourths as each number increases by one-fourth. The second number line shows fraction equivalences to the first number line with familiar fractions written between two whole numbers, which is how rulers are designed to measure length. Discuss how fractions greater than one can be expressed in two ways: with a numerator that is larger than the denominator as in the first number line and as a way to measure length, as in the second number line, which includes a whole number and a fraction less than one. This comparison of numbers expands students' ideas of fractions and measurement.[91]

Example 4.4. Fractions equal to, greater than, and less than 1.



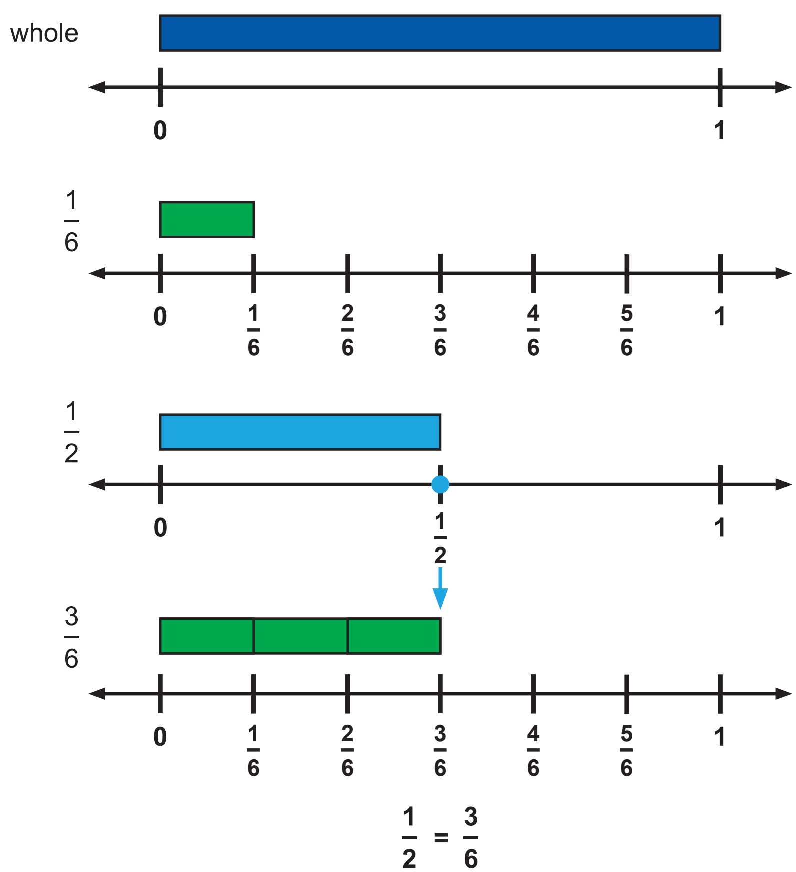
Once students conceptually understand and can articulate that the point where a fraction is located on the number line represents the length of units from zero to that position, then the same teaching steps can be used to concretely introduce the concept of equivalent fractions. Show students how different fractions are positioned at the same point on the number line by sequentially partitioning a number line into different units. Introduce new denominators over several lessons.[92] In **Example 4.5**, the number line is first partitioned in halves, then into fourths, and then into eighths. Explain that some fractions are positioned at the same location on the number line and are therefore equivalent (for example, $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are equivalent, and $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent).

Example 4.5. Equivalent fractions are positioned at the same point on the number line.



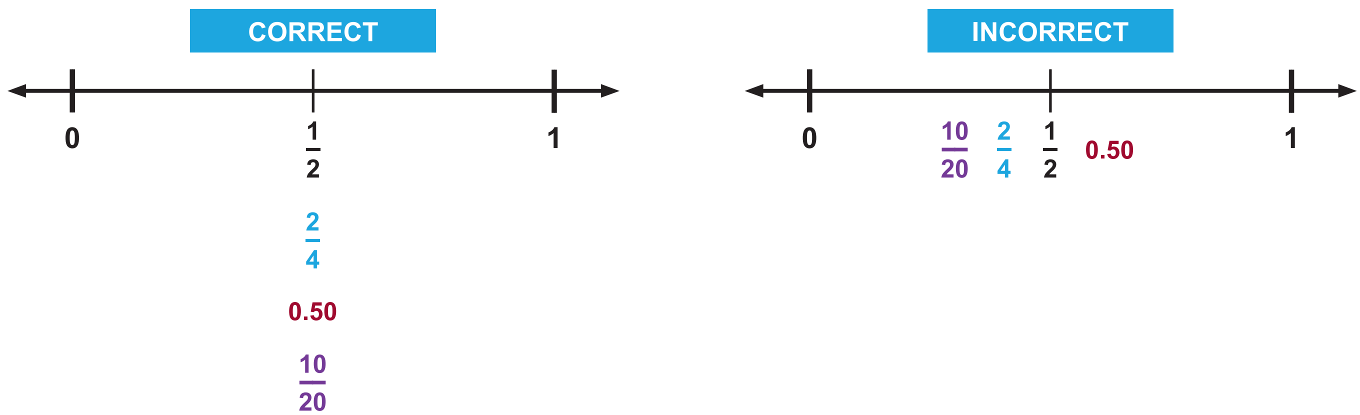
Incorporate other linear representations to show how two fractions with different denominators can be equivalent and occupy the same distance on the number line. **Example 4.6** shows how Cuisenaire rods® can be aligned with a number line to reinforce the equivalencies on a number line.

Example 4.6. Connecting a concrete representation of a length to a number line.



Expand the idea of equivalencies to include decimals and percentages so that students understand that these rational numbers are also equivalencies and there is an infinite number of equivalencies at any point on the number line. Reinforce this idea by writing the equivalent fractions *underneath each other to show the precise position*, rather than side by side when on the same number line. **Example 4.7** displays equivalent numbers underneath each other to ensure that the label aligns to the same tick mark.

Example 4.7. Label tick marks that represent the same equivalences vertically at the same position on the number line, rather than side by side.

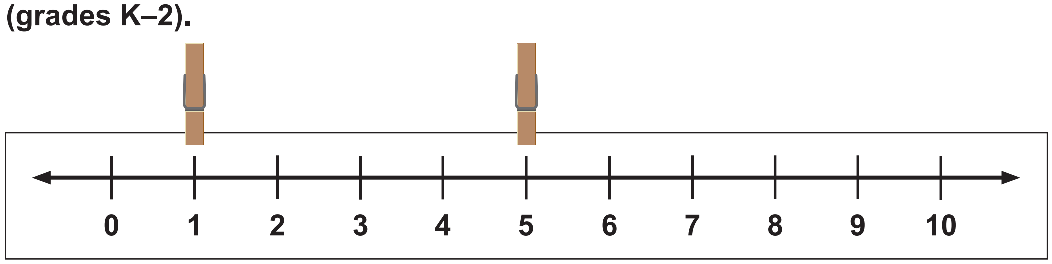


2. Compare numbers and determine their relative magnitude using a number line to help students understand quantity.

Early elementary (grades K–2)

Use number lines to teach the relative magnitude of whole numbers. Start by putting two numbers on a number line using equal units. Explain that each number's distance from zero represents the number's magnitude. Whole numbers with greater magnitude are further to the right. Whole numbers with lesser magnitude are further to the left and therefore closer to 0. Explain how to compare the two numbers and determine which is greater based on which is more equal units away from zero (farther to the right when working with positive whole numbers). In **Example 4.8**, students see that 5 is further to the right than 1.

Example 4.8. Use number lines to teach the relative magnitude of whole numbers in early elementary (grades K–2).



Upper elementary (grades 3–6)

Use number lines to compare the magnitude of fractions and decimals. Reinforce for students that fraction and decimal magnitude, like whole-number magnitude, is represented by how far to the right or left of zero a number is positioned. Before comparing fractions, students must understand that fractions have an infinite number of equivalences, as shown in **Example 4.6**.

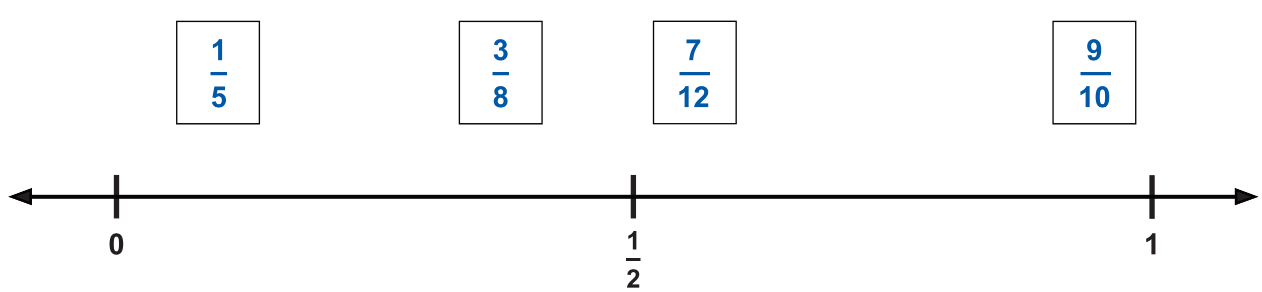
Comparing the relative magnitude of fractions with a number line supports students' understanding of fraction values and how they relate to one another. Help students compare fraction magnitude by thinking about "benchmark numbers," starting with 0, $\frac{1}{2}$, and 1 when thinking of fractions between 0 and 1. Benchmark numbers can be located on a number line, and then other fractions can be compared to the benchmark numbers to describe which fractions are greater than or less than other fractions. This knowledge can then be used as a strategy for students when they evaluate fraction magnitude during other activities, such as when comparing two fractions and showing the relationship using a greater-than or less-than sign or ordering a set of fractions from least to greatest or greatest to least.[93] Evaluating fraction magnitude can also be useful when predicting the approximate magnitude of a computational solution.

Early and upper elementary (grades K–6)

Provide students with practice determining the magnitude of whole numbers, fractions, and decimals using a number line. This type of activity will build familiarity with number lines.[94] Students receiving intervention will need ample practice using number lines to become more proficient in estimating magnitude. Present students with a number line with two points marked near the end (for example, 0-1, 0-2, 0-100, or 0-1,000) and ask them to estimate magnitude for whole numbers and/or fractions.

In **Example 4.9**, students work as a group to estimate fraction magnitude using benchmark numbers. Four fractions are given on index cards for students to place on the 0-1 number line. First, the students add the benchmark number one-half on the number line before estimating the magnitude of each fraction. For $\frac{7}{12}$, a student reasons that the fraction is a little to the right of $\frac{1}{2}$ because $\frac{7}{12}$ is just $\frac{1}{12}$ greater than $\frac{6}{12}$, which is equivalent to $\frac{1}{2}$. The next student also uses one-half as a benchmark for $\frac{3}{8}$, reasoning that the fraction is just $\frac{1}{8}$ less than $\frac{4}{8}$, which is also equivalent to $\frac{1}{2}$. For $\frac{1}{5}$, a student places it closer to 0 because it is a unit-fraction and less than $\frac{1}{2}$. For $\frac{9}{10}$, a student places it close to 1 because it is just $\frac{1}{10}$ away from $\frac{10}{10}$ or the whole. Ask students to discuss their reasoning for making the placement of each fraction with the group and pose additional questions to explore the depth of the conceptual understanding as necessary.

Example 4.9. Students estimate the location of four fractions using benchmark numbers and places the flashcards on the 0–1 number line.

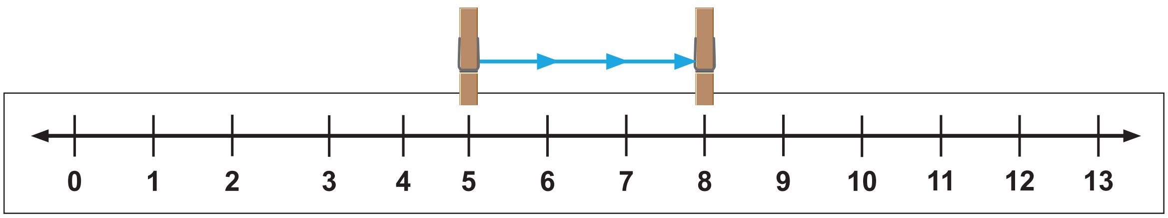


3. Use the number line to build students' understanding of the concepts underlying operations.

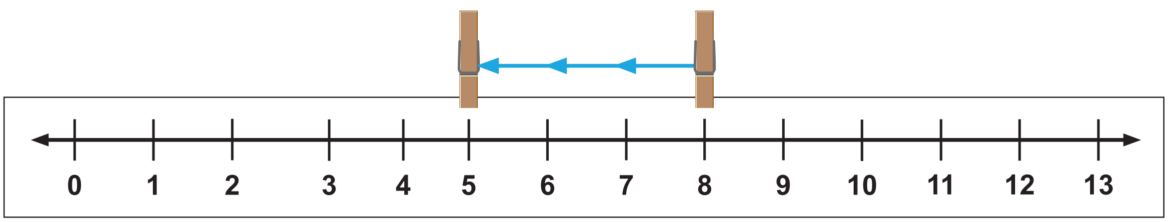
Early elementary (grades K–2)

Show students how to use number lines for addition and subtraction of whole numbers. After comparing whole numbers, students begin to learn addition and subtraction by looking at the distance between whole numbers. In **Example 4.10**, students first determine that 8 is greater than 5. Then, students see how much greater 8 is than 5 by counting up. They can visibly see that 8 is 3 units to the right of 5. The focus on the unit length, or distance, is key rather than counting the tick marks. When moving toward the right, students see that 5 units plus 3 more units is equal to 8 units. When moving to the left students see that starting with 8 and moving 3 units to the left is equal to 5, showing the subtraction equation 8 - 3 = 5.

Example 4.10. Show early elementary (grades K–2) students how to use number lines to add and subtract whole numbers.



5 + 3 = 8



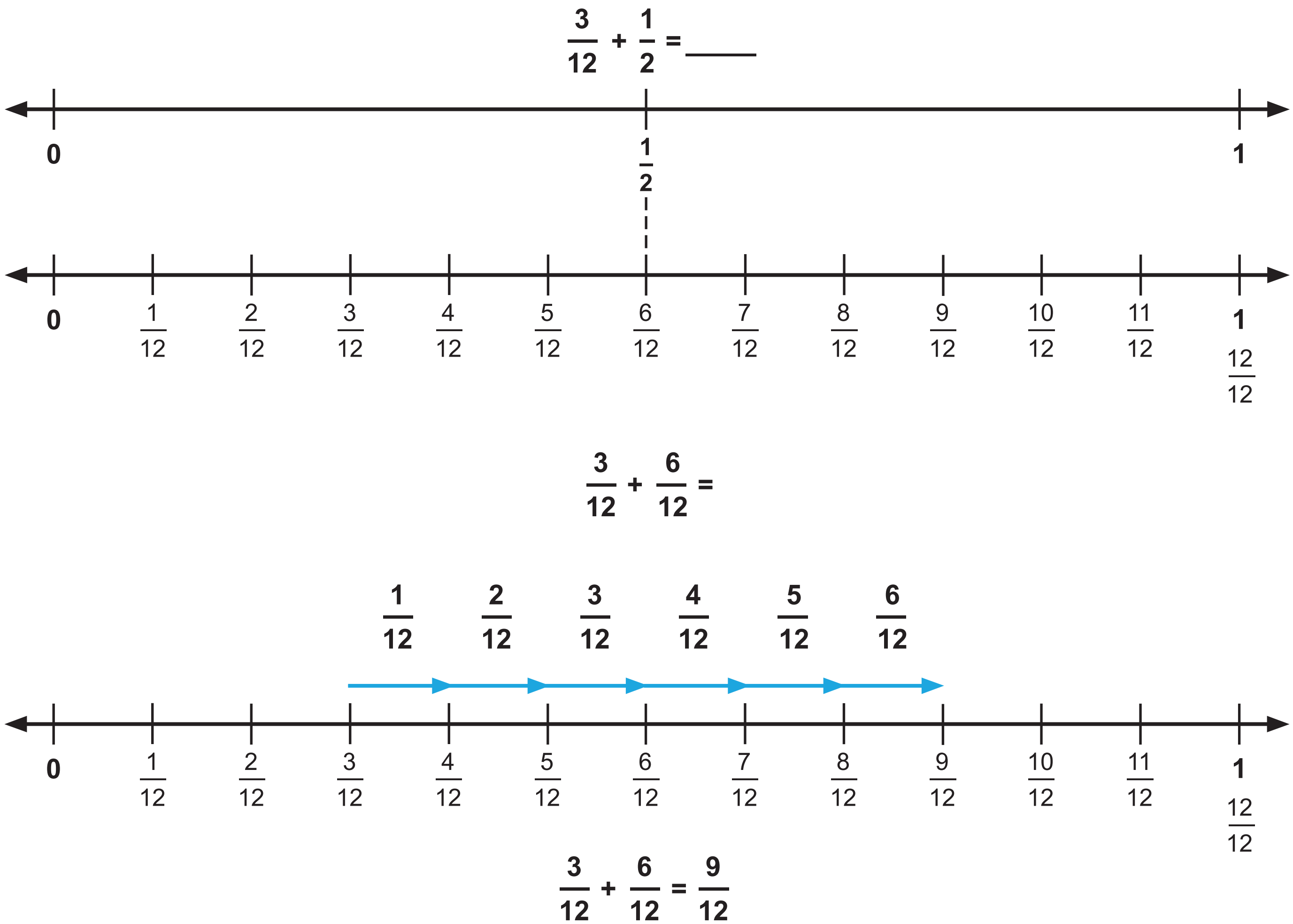
8 – 3 = 5

Always connect the equation to the number line when solving calculation problems with students. First, model a problem with the number line and have students write the corresponding equation. Then present an equation for students to model on the number line.

Upper elementary (grades 3-6)

A number line is also a powerful visual for demonstrating addition and subtraction of fractions.[95] Start by adding fractions with the same denominator. This can be done using one number line. When introducing addition and subtraction with unlike denominators, explain that number lines are particularly helpful. Number lines help make visible the concepts underlying addition and subtraction of fractions when the two addends have different denominators.[96] Using double number lines can make the equivalences more visible for students so that they can understand why finding an equivalent fraction is the necessary and correct approach for solving these types of problems.[97] **Example 4.11** depicts an addition problem where double number lines are used to show students that $\frac{1}{2}$ and $\frac{6}{12}$ are the same distance from 0, and therefore, the same magnitude. This illustration helps students see why they can use equivalent fractions when adding and subtracting fractions with unlike denominators.

Example 4.11. Use the number line to show students fraction addition.



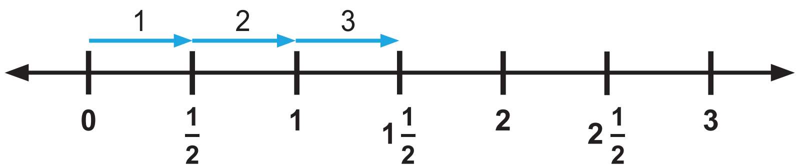
Concepts of multiplication and division can also be shown on a number line. When first introducing fraction multiplication and division, include whole numbers as one of the factors, divisors, or dividends. Start with a word problem to set the stage for understanding the concept underlying the operation. In these instances, a number line is useful for showing the problem because the number line effectively represents whole numbers and fractions.[98] **Examples 4.12** and **4.13** show how number lines are used to depict multiplication and division (respectively) with whole numbers as one of the operands.

Example 4.12. Multiplication with a fraction and a whole number.

Use a word problem to provide context for the operation:

Arya runs a $\frac{1}{2}$-mile loop in her neighborhood. She does this loop 3 times each morning for exercise. How many miles does Arya run for exercise each morning?

3 groups of $\frac{1}{2}$

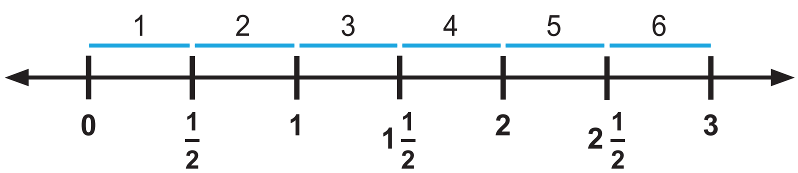


$$3 \times \frac{1}{2} = 1\frac{1}{2}$$

Example 4.13. Division with a fraction and a whole number.

Use a word problem to provide context for the operation:

Twyla baked 3 large brownies. She cuts each brownie in half to share with friends. How many pieces of brownie does Twyla have to share?



$$3 \div \frac{1}{2} = 6$$

Potential Obstacles and the Panel's Advice

OBSTACLE: *"I used the number line for fraction multiplication and my students were confused."*

PANEL'S ADVICE: Number lines are not always useful to help students understand all mathematical ideas.[99] Multiplication and division with two fractions less than 1 are not represented well on a number line, especially when fractions have large denominators. Instead, try using an area model for multiplication when the fractions are both less than one. See **Table 3.1** in **Recommendation 3** (Representations) for which representations fit the mathematics concepts best.

OBSTACLE: *"My students don't want to use the number line and benchmark fractions when comparing fractions because cross-multiplying is easier and faster."*

PANEL'S ADVICE: Many teachers use cross-multiplying to compare fractions, probably because it is easy and fast. The panel believes, however, that cross-multiplying does not help students understand fractions in a meaningful way. Help students see that using benchmarks and thinking about the relative magnitude of fractions will help them understand this operation with fractions more deeply. Do not allow students to revert to cross-multiplying during intervention and continue to focus on fractions concepts and the understanding of procedures, which will be more helpful to students later on.

OBSTACLE: *"My students don't seem to have a good grasp of the number line and what it represents."*

PANEL'S ADVICE: Concrete representations can be used at any grade level to support students' understanding of number lines.[100] Use concrete representations with length models to help transition students toward understanding the number line. Show students how to build a number line with manipulatives that are of consistent and equal length units. Fraction tiles and Cuisenaire rods® can be linked to number lines, for example. Be sure to show students which edge of the tile or rod represents the length of the unit or the fraction. Demonstrate this by lining up the rods to the number line equivalent. **Example 4.6** depicts connecting concrete representations to number lines as a way to reinforce equivalent fractions.

Recommendation 5: Word Problems

Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

Introduction

Learning to solve word problems is an important part of the elementary mathematics curriculum because word problems help students apply the mathematics they are learning, develop critical thinking skills, and begin to connect mathematics to a variety of scenarios or contexts.[101] Becoming successful at applying mathematics through solving word problems can deepen students' understanding of grade-level content and set students up for success in advanced mathematics courses and the workforce.[102]

Problem solving in the elementary grades is often done by presenting word problems that can be solved using computational procedures, such as addition, subtraction, multiplication, and division.[103] Unfortunately, learning calculations alone does not necessarily help students successfully solve word problems.[104] To set up and solve word problems successfully, students need to read and understand the problem's narrative, determine what the problem is asking them to find, and identify one or more mathematical operations that will solve the problem.[105] Students with or at risk for mathematics disabilities often have difficulty with one or more of these steps, which further impacts their ability to set up and solve problems correctly.[106] Thus, the panel recommends dedicating some instructional time during intervention to word problems. The WWC and the expert panel assigned a **strong level** of evidence to this recommendation based on 18 studies of the effectiveness of systematic word problem solving instruction.[107] Fifteen of the studies meet WWC group design standards without reservations,[108] and three studies meet WWC group design standards with reservations.[109] See **Appendix C** for a detailed rationale for the Level of Evidence for **Recommendation 5**.

This recommendation outlines approaches for supporting students in understanding and solving word problems. Not all of these approaches can be used in the first intervention lesson. Instead, introduce these approaches over the course of several lessons so that students build their capacity to understand word problems and execute all the steps needed to solve them. This section describes strategies, examples, and tools that can support instructors in effectively teaching students how to solve word problems.

How to carry out the recommendation

1. Teach students to identify word problem types that include the same type of action or event.

Introduce one problem type at a time. Begin by introducing a new problem type with a story which includes all quantities. This helps students think about what the quantities represent without needing to solve for an unknown.[110] Next, present the same story with a missing quantity (that is, a word problem). Connect the quantities between the story and the word problem so that the students see how they are the same.

What is a problem type?

A problem type includes all problems with the same set of quantities or salient features.[a] Identifying a problem type is different from determining the operation used to solve the problem. Even though the two can be related, the same operation may be used in different problem types, or the same problem type may require students to use a different operation. Consequently, it is not useful to associate a problem type with an operation.[b] Different programs may have different words for the same problem types. For example, what we refer to as a *Change* problem may be referred to as change over time, join, or separate, depending on the curricula. Other common types can include *Combine* problems, *Compare* or *Comparison* problems, and *Ratio* or *Proportion* problems.

a. Fuchs et al., 2005; Fuchs & Fuchs, 2001; Jitendra et al., 2016.
b. Fuchs et al., 2005; Fuchs & Fuchs, 2001; Jitendra et al., 2016.

Use role-playing,[111] gestures,[112] or concrete and/or semi-concrete manipulatives,[113] to help students visualize the problem and identify relevant information. This helps students see how the quantities relate to each other.

Show students additional examples of the problem type using different scenarios so that students can see how the quantities are still the same even though the situation differs. Discuss how and why each new problem belongs to the problem type you are teaching. Use these steps for each new problem type you introduce.

Example 5.1 shows how to introduce a *Change* problem, which depicts stories that include a change over time. The story and word problem are about children getting off the bus to show how the number of children changes. Use counters to help students visualize this problem. Then, present two other *Change* problems to students to depict additional scenarios where a change over time occurs. Emphasize that in one problem the quantity increases and in the other it decreases.

Example 5.1. Introducing a *Change* problem.

<u>Story with all quantities</u>

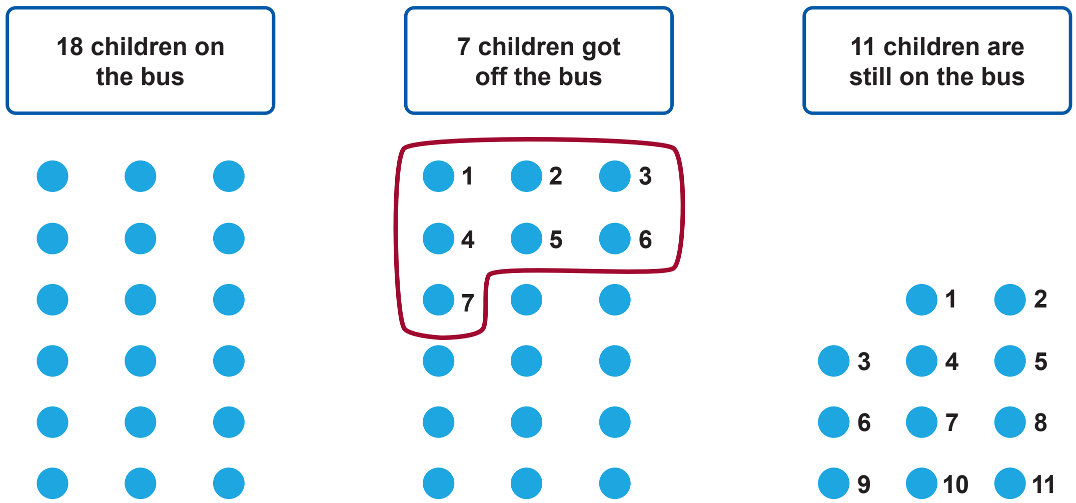
There were 18 children on the bus. 7 children got off the bus at the first stop.

11 children are still on the bus.

<u>Word problem with a missing quantity</u>

There were 18 children on the bus. 7 children got off the bus at the first stop.

How many children are still on the bus?



18 counters represent the children on the bus.

Teacher and students count and remove 7 counters.

Teacher and students count remaining children.

Additional examples of *Change* problems:

<u>Quantity increase</u>

The rose bush has 15 flowers blooming.

Then 12 more bloomed. How many flowers are blooming on the rose bush now?

<u>Quantity decrease</u>

Selina had 24 cupcakes. At her birthday the next day, she and her friends ate 16.

How many cupcakes does Selina have left to share with her family?

2. Teach students a solution method for solving each problem type.

Introduce a solution method using a worked-out example. Talk through the problem-solving process and connect the relevant problem information to the worked-out example. Say out loud the decisions that were made to solve the problem at each step. Then demonstrate how to apply the solution method by solving a problem with the students using that method. Discuss each decision you make and ask students guiding questions to engage them as you solve the problem. Solution methods may include graphic organizers, diagrams, tables, or equations that directly link to the problem type by connecting to and representing the underlying mathematics in the word problem.[114]

Students may need ongoing support to complete the multi-step process required for solving each problem type.[115] Provide students with a visual guide detailing steps to reference as they solve word problems. Some parts of the guide may apply to understanding the problem before solving it, such as “read the problem,” “name the problem type,” “identify the question,” and “find relevant information.” Other parts may be geared toward choosing a solution method that is specific to the problem type. As students become more comfortable solving problems, gradually fade the use of visual guides so students do not become overly reliant on them.

Example 5.2 demonstrates how a teacher solves an *Equal Groups* problem and uses a simple drawing to set up and solve the problem. The teacher refers to a visual prompt card that outlines steps for solving *Equal Groups* problems.

Example 5.2. Upper elementary (grade 3–6) teacher thinking aloud how she sets up and solves an *Equal Groups* problem using a prompt card.

Amber bought donuts for the teachers' lounge. Each box of donuts has 6 donuts inside. If she bought 24 donuts, how many boxes did she buy?

Teacher: *The prompt card says to read the problem.* (Teacher reads problem.) *This problem is about donuts. What's the problem asking?*

Students: *How many boxes did she buy?*

Teacher: *Right. The problem is asking how many boxes or groups of donuts she brought. The problem tells us there are 6 in each box and she brought 24. Is it an Equal Groups problem?*

Students: *Yes.*

Teacher: *How do you know when a word problem is an Equal Groups problem?*

Students: *In an Equal Groups problem, there is a number of groups, group size, and a total number of items. In this problem, we're trying to find the number of groups or boxes of donuts.*

Teacher: *Right! How many did she buy?*

Students: *24.*

Teacher: *Great. Right. To figure out what we should do next, let's look at the prompt card. The prompt card says to write down the information you need to solve the problem. What do we know?*

Students: *We know each box has 6 donuts inside. We need to figure out how many boxes we need to get 24 donuts.*

Teacher: *The next step on the prompt card says to draw the groups to find the missing amount. What would you do to draw the groups?*

Students: *I start by drawing a box with 6 donuts. I use circles to represent the donuts. Then, I draw another box with 6 more donuts.*

Teacher: *Let's see if we have 24 yet. What's 6 + 6 equal?*

Students: *12.*

Teacher: *The next step says to keep drawing the groups until you find the amount. So, we need to keep going. If we add another box of 6 donuts, how many donuts do we have?*

Students: *18. We are not done yet. Let's draw another box.*

Teacher: *Great. Let's try that. Now we have 4 boxes of 6 donuts. Do 4 boxes of 6 add up to 24 donuts?*

Students: *Yes! 6 + 6 + 6 + 6 = 24. We needed 4 boxes of donuts.*

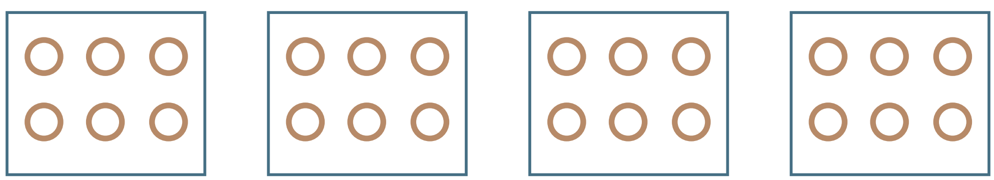
Teacher: *Right! That's right. To solve this problem, we used repeated addition. We added up the number of donuts in each box until we reached 24. The final step says to write the answer with a label.*

Students: *Let's write "4 boxes" for the answer.*

Teacher: *Great work!*

6 donuts in a box

24 donuts total



6 + 6 + 6 + 6 = 24

12 18 24

4 boxes

Note: Depending on where the students are with addition and multiplication, the teacher could connect the repeated addition to a multiplication number sentence. Teachers could also refer to a multiplication chart as an additional support for students making the connection between repeated addition and multiplication.

3. Expand students' ability to identify relevant information in word problems by presenting problem information differently.

Once students can recognize and solve the most accessible problems within a type, present word problems of that same type that are less familiar so that students broaden their understanding of that problem type.[116] In the opinion of the panel, it is essential to include problems that vary the unknown quantity to help students understand the mathematical structure in each problem type. Other problems that look different may require additional steps to solve or include irrelevant numerical information or information on a chart, graph, or diagram.[117] Teaching a variety of problems helps students more flexibly transfer their understanding of problem types to a wider range of problems.[118] **Example 5.3** provides examples of *Change* problems and *Ratio* problems that may be less familiar to students.

Example 5.3. Problem types with less familiar features.

Change problems

Result quantity unknown

Ana had 13 red apples. Then she gave 6 apples to her neighbor. How many apples does she have now?

Change quantity unknown

There were 24 people swimming at the pool. Some of them got out for lunch. Now there are 13 people in the pool. How many people got out for lunch?

Start quantity unknown

Alice gave her brother 32 baseball cards and now has only 15 left. How many baseball cards did she start with?

Change problem with multiple steps

There are 21 students at the lunch table. Eleven students got up to return their trays. Then, 3 students went to the bathroom. How many students are still eating at the lunch table?

Change problem with irrelevant information

There are 21 students at the lunch table and 4 parents. Eleven students got up to return their trays. How many students are still eating at the lunch table?

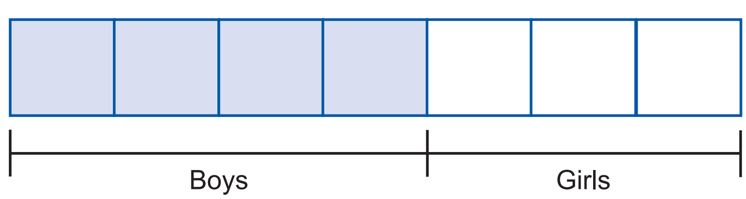
Ratio problems

Ratio problem with ratio given

Zahara bought some food at the farmer's market. For every 1 cucumber she bought, she bought 3 tomatoes. If she bought 12 tomatoes, how many cucumbers did she buy?

Ratio problem with a chart/graph/diagram

In Ms. Walker's class, there are more boys than girls. Below is a diagram representing the number of boys to girls. If there are 12 boys in the class, how many girls are there?



Ratio problem with irrelevant information

Sally loves to garden. She keeps 5 flower gardens and 1 vegetable garden. In the flower garden, for every 5 daisies she plants, she also plants 1 rose. If she planted 3 roses, how many daisies did she plant?

Ratio problem with multiple steps

Sally loves to plant flowers in her garden. For every 5 daisies she plants, she also plants 1 rose. If she planted 3 roses, how many flowers did she plant altogether?

Students may need ongoing support in identifying which quantities are relevant for solving problems once they have learned several variations of each problem type. This can be accomplished by continuing to help students visualize the problem by using concrete manipulatives such as counters or fraction tiles, or semi-concrete representations such as tables or simple diagrams of the quantitative relationships.

Another approach to identifying relevant information is to encourage students to reread the problem more than once and restate the problem in their own words. Refer back to the question to connect the unknown to the information given in the word problem to help determine what is important. By rereading the problem, students may have greater success discerning which information is relevant versus irrelevant.[119] Teachers may ask students to write down, circle, or underline information that will be used solve the problem and to cross out information that is not useful.

Example 5.4 presents a *Part-Part-Whole* problem. This problem includes three types of vegetables that are combined. It also includes irrelevant information that is not needed to solve the problem. Problems with irrelevant information challenge students to really think about which quantities are needed to set up and solve the problem correctly. The example includes counters and pictures of vegetables for students to visualize the problem information.

Example 5.4. Teacher guides students through identifying relevant information and using a concrete representation to visualize the story.

Word Problem

Roger has 6 apple trees and a vegetable garden. He picked 3 carrots, 9 onions, and 4 potatoes. How many vegetables did he pick?

Teacher: *In this problem, there is a lot of information. Let's use these counters to represent the relevant problem information as we solve the problem. What is this problem about?*

Student 1: *Roger's apple trees and vegetables.*

Teacher: *Great. Let's read the question, "How many vegetables did he pick?" Look back at the problem and see what it says about vegetables.*

Student 2: *I see it says vegetable garden.*

Teacher: *Right. Look at the problem, do you see what kind of vegetables he picked?*

Student 2: *Carrots, onions, and potatoes.*

Teacher: *Right, he picked carrots, onions, and potatoes. Do we use all the information in this problem or is there any irrelevant information we need to ignore?*

Student 2: *He also has 6 apple trees.*

Teacher: *Right. Are apple trees vegetables? Does knowing how many apple trees he has help us answer the question about vegetables?*

Student 2: *No.*

Teacher: *Good, what do we do with irrelevant information?*

Student 2: *We ignore this irrelevant information.*

Teacher: *Let's use some counters to visualize the problem. Each vegetable name is on an index card with a picture of the vegetable. Let's show the number of each vegetable with counters. How many carrots? Look back at the problem if you need to.*

Student 3: *3.*

Teacher: *Let's count out 3 counters for the carrots, 1, 2, 3. How many onions? Look back at the problem if you need to.*

Student 1: *9.*

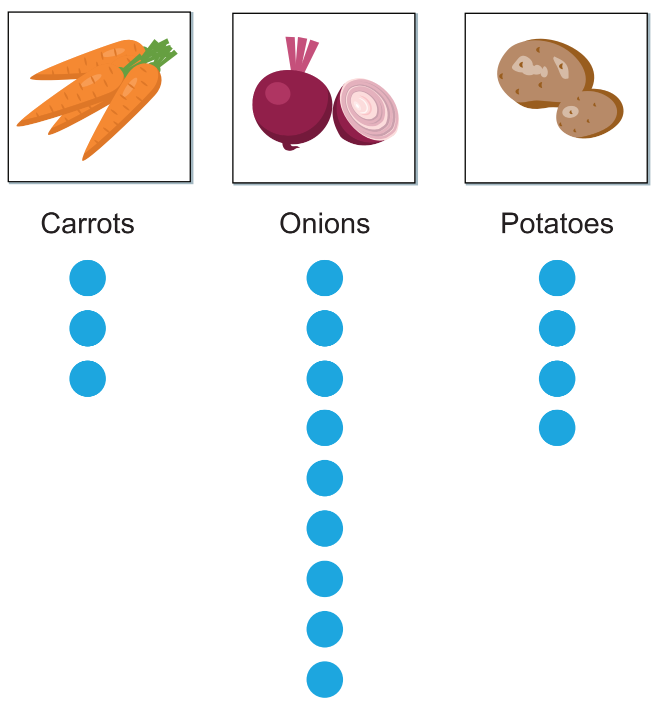
Teacher: *Let's count out 9 counters for the onions, 1, 2, 3, 4, 5, 6, 7, 8, 9. How many potatoes? Look back at the problem if you need to.*

Student 2: *4*

Teacher: *Let's look back at the problem. What was it asking again?*

Student 1: *How many vegetables he picked.*

Teacher: *To figure out how many vegetables he picked, what do we do?*



Student 1: *Put them altogether. Add the vegetables.*

Teacher: *Right. This problem is about combining vegetables. We can see here that we have 3 carrots, 9 onions, and the 4 potatoes. To solve the problem, we combine the number of vegetables. Let's answer the question, "How many vegetables did he pick?" Let's count.*

Student 1: *16 vegetables.*

4. Teach vocabulary or language often used in word problems to help students understand the problem.

When first introducing word problems, choose problems that are accessible, meaning that all the words in the story are familiar to students. However, as students learn the problem type, how to identify what it is asking for, and what strategies work for finding the solution, word problems can include more difficult language. Teach students the meaning of words and language constructions they may find difficult in the word problems.

Difficult words can be previewed before they are presented in a word problem to help students prepare for understanding them when used in word problems.[120] Before you start teaching, anticipate which words are critical for understanding the problem. Pay particular attention to words that relate to one another or share a categorical structure that may help students identify which information in the problem is relevant and which is irrelevant. Teach the meaning of words and continue to discuss them during problem solving to solidify their meaning.

Table 5.1 presents several types of words and language constructions presented in word problems that students may find difficult to understand when trying to make sense of the problem. This may include words that are familiar to students but have different meanings in a word problem context, categorical words and the subcategories that comprise them, words that involve comparisons, and words that signal events associated with the problem types you are teaching. Instruction in these words will increase students' success in identifying relevant and irrelevant information.

Table 5.1. Clarify words presented in word problems prior to students solving the problem.

What to teach	Sample word problem	Why focus on these words	What to do
Unknown words or familiar words that may be confusing in context	There is a 2-mile **relay** race at Brown Elementary School. Each **leg** of the race is 12 mile. How many children are needed to run the race?	In this problem, students may not understand the meaning of the word **relay** and how it includes many parts and a team of children racing. Students are likely to know the meaning of the word **leg** (as a part of a body) but may not understand what **leg of a race** means.	Teach students that a **relay** is a race that is made up of several parts. Explain these are often called **legs** and that one child runs only during one part or **leg of a race**. So, if there are four legs, four children are needed.
Categories and the things that comprise them	Sasha has 6 **pets**. Four of the **pets** are **turtles**, and the others are **puppies**. How many **puppies** does Sasha have?	Students may not know that **pets** is a category that can include many animals such as **turtles** and **puppies**. Categories are often found in ratio problems and part-part-whole problems.	Teach students about categories or words. For each new problem, discuss the category of words (e.g., **pets**, **turtles**, and **puppies**) ahead of time.
Words that compare two quantities: more, less, fewer, older, younger, taller, shorter, bigger, smaller, hotter, and colder.	Willa is 42 inches tall. Renaldo is 8 inches **shorter** than Willa. How tall is Renaldo?	Students need to understand that **shorter** means **fewer** inches or a **smaller** number of inches.	Teach students about these words by focusing on how the quantity is changing.
	Ava's sister is 8 years **older** than she is. Ava is 6 years old. How old is her sister? Ava is 6 years old, and her sister is 14 years old. How much **younger** is Ava than her sister?	Students also need to learn comparison words that relate to one another. In these examples, both problems are about comparing age, but one problem uses the word **older** and the other problem uses the word **younger**.	Pair similar word problems using reciprocal comparison words to help students understand the quantities when different comparison words are used.

5. Include a mix of previously and newly learned problem types throughout intervention.

After a problem type has been taught, distribute previously learned problem types throughout lessons so that students do not forget the problem types they have learned. By revisiting previously learned problems, students practice discriminating among problem types as they learn new ones. Include a mix of problems throughout intervention as cumulative review.

Include an activity where students identify and name problem types without solving them. Remind students to think about the differences among problem types and how to tell one problem type from another. This type of practice helps students become more fluent in identifying and distinguishing the various problem types and reinforces the importance of reading and thinking about each problem before solving it.[121] Students may need support to remember the salient features of different problem types, like a prompt card listing the features of a problem or a gesture that evokes the action in the problem.

Potential Obstacles and the Panel's Advice

OBSTACLE: *"This type of word problem instruction isn't in my curriculum. Should I develop my own materials?"*

PANEL'S ADVICE: The panel is not suggesting teachers create materials that include this type of instruction or these types of word problems. Instead, the panel suggests using this recommendation as a guideline for evaluating curricula to adopt. Work with a team, including a mathematics coach or special educator, to evaluate whether the curriculum aligns with the steps in this recommendation.

OBSTACLE: *"I don't want to teach my students how to solve a problem using a specific method. I want to encourage my students to come up with their own solution approach."*

PANEL'S ADVICE: While having students develop their own solution strategies can be meaningful and useful, students in intervention are sometimes less equipped to generate a solution strategy that is accurate and appropriate for the word problem.[122] By teaching specific solution strategies, you offer students a way to move through the problem-solving process successfully so that eventually they may be able to develop their own solution methods.

OBSTACLE: *"My students often don't know the operations to solve the word problems in our curriculum."*

PANEL'S ADVICE: Before introducing a strategy for solving a problem type, make sure students have the necessary prerequisite skills to be able to apply the method for solving the problem. Review and embed practice on these prerequisite skills throughout the instruction focused on word problems. Students can also solve problems in which they demonstrate conceptual understanding of what the problem is asking, the structure of the problem, and the steps needed to solve the problem. In these cases, the use of devices, like a calculator, can help students to carry out mathematical operations. Alternatively, the teacher could replace the numbers with those familiar to students to make it easier for students to solve.

OBSTACLE: *"I use the key word strategy, but I don't feel like my students understand the word problems."*

PANEL'S ADVICE: Avoid teaching key words that link specific words to operations. Key words remove the need to read and understand the entire problem and instead direct students to apply a mathematical operation (addition, subtraction, multiplication, division) that is often incorrect.[123] In **Table 5.2**, notice how the key word misleads the student for many problems. Determine the correct operation by understanding what the problem is asking. Additionally, because teaching key words encourages students to look for specific words or phrases without considering other information in the problem, word problems without taught key words often leave students unable to start the problem-solving process. Further, multi-step problems likely include more than one taught key word, causing even more confusion.

Table 5.2. Examples of key words matched to an operation and why they fail.

Key word	Supposed operation related to the key word	Sample problem in which the key word method fails	Example of failed operation
Altogether	addition	Alice bought 4 cartons of eggs with 12 eggs in each carton. How many eggs does Alice have **altogether**?	$4 + 12 = 16$
More	addition	Colin had some crayons. Then, he bought 12 **more** crayons. Now, he has 90 crayons. How many crayons did Colin have to start with?	$90 + 12 = 102$
Fewer	subtraction	Paulo picked apples. Zach picked 12 **fewer** apples. If Zach picked 20 apples, how many apples did Paulo pick?	$20 - 12 = 8$
Left	subtraction	Liz shared 55 candies equally with 3 friends. After sharing, how many candies were **left** over?	$55 - 3 = 52$
Each	multiplication	Miles had 3 trays of building blocks with the same number of blocks on **each** tray. If Miles had 75 blocks altogether, how many were on each tray?	$75 \times 3 = 225$
Double	multiplication	Margaret bought **double** the number of songs as her sister. If Margaret bought 12 songs, how many songs did her sister buy?	$12 \times 2 = 24$
Share	division	Sal collected 18 quarters to **share** equally among his friends. After sharing, he had 3 quarters remaining. How many quarters did Sal share?	$18 \div 3 = 6$
Divide	division	Cam **divided** 5 pieces of paper into fourths. How many pieces of paper does Cam have now?	$5 \div 4 = 1\frac{1}{4}$

OBSTACLE: *"My students often take so much time drawing pictures of each item in the problem that they don't have time to even begin the problem solving."*

PANEL'S ADVICE: Teach students how to draw simpler sketches, such as stick figures for people or circles to represent rocks or apples (see **Example 5.2**, where circles represented donuts). A simple sketch shows the quantitative relationships and helps the students determine what the problem is asking and what they need to do to solve it.[124] It may also be helpful to use concrete representations if students struggle with fine motor or spatial skills.[125] Make a direct connection between drawings or other representations and the equation.

Recommendation 6: Timed Activities

Regularly include timed activities as one way to build students' fluency in mathematics.

Introduction

Each recommendation in this guide supports students in becoming accurate, efficient, and flexible problem solvers. This recommendation offers one more, albeit short, way to support fluency building through timed activities. These timed activities last between 1 and 5 minutes and are not the entire focus of the intervention. Instead, they are one component embedded within a multi-component intervention. Add timed activities to intervention once students have been working on a concept over many lessons. Do not use timed activities to introduce and teach mathematics concepts and operations.

Quickly retrieving basic arithmetic facts (addition, subtraction, multiplication, and division) is not easy for students who experience difficulties in mathematics.[126] Without such retrieval, students will struggle to follow their teachers' explanations of new mathematical ideas.[127] Automatic retrieval gives students more mental energy to understand relatively complex mathematical tasks and execute multi-step mathematical procedures.[128] Thus, building automatic fact retrieval in students is one (of many) important goals of intervention.[129]

In addition to basic facts, timed activities may address other mathematical subtasks important for solving complex problems.[130] This could include, for example, *recalling equivalencies for fraction benchmarks* of $\frac{1}{2}$ and 1, which helps students compare the magnitude of fractions or solve rate problems more efficiently, or quickly *evaluating* and *estimating place value*, which helps students identify whether regrouping is necessary when solving double-digit addition and subtraction problems. The goal of these activities is to move students toward accurate and efficient performance of these smaller mathematical tasks so that this knowledge can be easily accessed when necessary for solving problems.

The panel does not recommend merely giving students timed worksheets or putting students on a computer-based program without supporting their learning. Timed activity can engage students by providing feedback in real time, including goals for improvement, and steadily increasing item difficulty. Timed activities can be structured similarly during intervention, regardless of whether the focus is on automaticity with basic arithmetic facts or building fluency in other mathematical subtasks.

The WWC and the expert panel assigned a **strong level** of evidence to this recommendation based on 27 studies of the effectiveness of activities to support automatic retrieval of basic facts and fluid performance of other tasks involved in solving complex problems.[131] Twenty-one of the studies meet WWC group design standards without reservations,[132] and six studies meet WWC group design standards with reservations.[133] See **Appendix C** for a detailed rationale for the Level of Evidence for **Recommendation 6**.

The steps in this recommendation address how to systematically set up and implement timed activities that support fluency and ensure student success with those activities. This section describes strategies, examples, and tools that can support instructors in effectively using activities to support fluency for students who struggle.

How to carry out the recommendation

1. Identify already-learned topics for activities to support fluency and create a timeline.

When planning activities to support fluency, think through what students need in order to understand and more easily apply the mathematics they are learning. Consider the mathematics topic that is the focus of intervention and whether basic facts and/or other subtasks might help students understand and perform that mathematics task more fluently. Think about which complex strategies or procedures the students will be learning. Break those into a series of smaller steps that are required to understand and accurately solve problems. Plan activities to support fluency in one of those areas. **Table 6.1** provides examples of intervention topics connected to options for activities to support fluency.

Table 6.1. Examples of activities that can support fluency for various intervention topics.

Intervention topic	Fluency focus	Relevancy to the intervention
Fractions intervention (grade 4 and up)	Multiplication basic facts	Relevant for finding equivalent fractions for fractions addition and subtraction
	Equivalencies for benchmark fractions of $\frac{1}{2}$ and 1	Relevant for using benchmark numbers as a strategy to compare or order fractions or to estimate fraction magnitude on a number line
Place value with multi-digit addition and subtraction (grade 2 and up)	Addition and subtraction basic facts	Relevant so that students can efficiently add or subtract each place value
	Evaluate the problem to determine if regrouping is necessary	Relevant so that students can determine if regrouping is needed as a standard practice when adding or subtracting numbers with multiple digits
Note: This list is not comprehensive.		

Pick one topic to build over time. For each topic, plan a schedule for introducing and conducting the activity to support fluency. At the start, choose easier items for the activity. To help students remain engaged in the topic, increase the difficulty of the items as students become more fluent with the easier items. For example, if working on addition facts, you might start with $n + 1$ or doubles at first. Then, increase the difficulty of the items to include other more difficult combinations. If working on multiplication facts, for example, focus on zeroes and ones first. Then integrate the tens and fives, and so on.

As you move on to harder facts, include easier facts so students are discriminating among problem types and fact sets and families. In this way, gradually move to include the full range of mathematics facts including addition and subtraction. The panel believes mixed practice develops students' ability to fluently discriminate between operations. Introduce the next topic after students have worked on the first topic over many weeks and demonstrated fluency for that topic.

2. Choose the activity and accompanying materials to use in the timed activity and set clear expectations.

Timed activities are brief (usually 1–5 minutes) and require students to generate many correct responses in that short amount of time.[134] Activities that support fluency can be done using flash cards, computer programs, or worksheets.[135] Using these materials, activities can be structured for students to work together as a group or individually. Periodically incorporate game-like features, such as keeping score or having students cooperate as a team to increase their score.

Activities to support fluency are well-suited for small-group intervention settings. Set up the activity with clear expectations of who responds and when. For example, students can respond one at a time going around the table, or the teacher can randomly call on students. Alternatively, all students can respond at once. Students may respond verbally, with response cards or white boards, or with gestures or hand signals (e.g., touching or pointing). If using worksheets for fluency, discuss students' answers after time has been called and ask students to correct and explain any missed items.

3. Ensure that students have an efficient strategy to use as they complete the timed activity.

Plan timed activities that focus on previously learned content.[136] Include the strategies you want students to use during timed activities during other portions of the intervention lessons. For example, when teaching addition facts, instruction may be organized around teaching number combinations, doubles, doubles plus one, or various combinations of 10 or other numbers. Include instruction on counting strategies for addition and subtraction. Be sure that students are competent in using these strategies before students begin the timed activity.[137]

Before starting the timed activity, remind students to use a strategy they know.[138] For example, reminding students of the "double plus one" strategy before starting the timed activity may help students use that strategy when they get to the problem 6 + 7 if they don't know it automatically. If counting on from the largest number has been taught as a strategy during intervention, remind students how to use it before starting the activity.

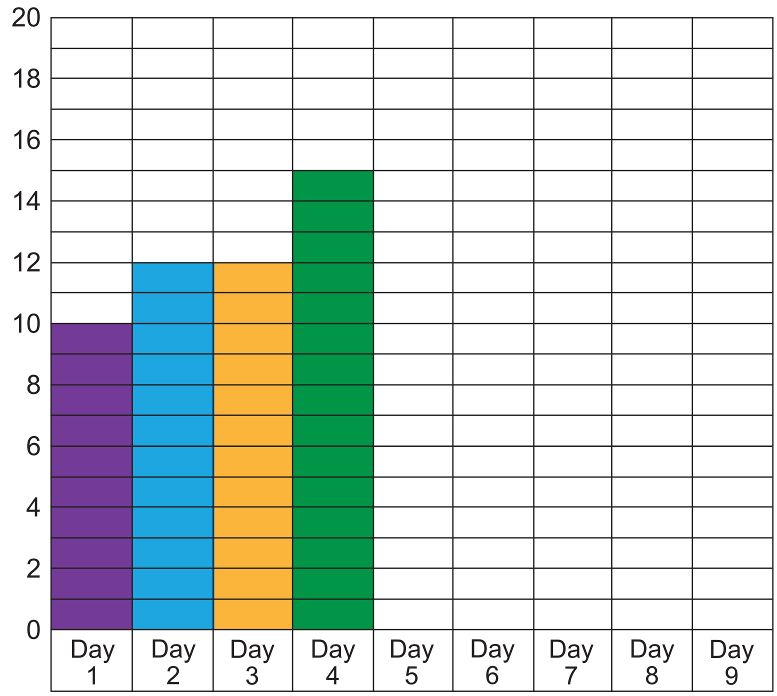
4. Encourage and motivate students to work hard by having them chart their progress.

The goal for activities that support fluency is for students to generate many correct responses in a short time.[139] Remind students that the goal is to produce answers that are accurate.

To keep students focused and motivated during these activities, have students record their scores over time on a chart or graph. As students see their scores improve over time, they may feel more excited and motivated to set goals and work hard. Goals to "meet or beat" a previously earned fluency score can be set for individuals or as a collective score for the intervention group. Working toward a goal as a group can reduce the pressure on individual students. If tracking progress individually, rather than as a group, make sure the graphs are kept private.

Example 6.1 shows a graph for 4 days of timed activities. Students beat their scores over days 1, 2, and 4. They met their score for day 3 (scores for days 2 and 3 were 12). The goal is to meet or beat the previous score, and students achieved this each day because scores did not decrease.

Example 6.1. Graph tracking scores for timed fluency activities.



5. Provide immediate feedback by asking students to correct errors using an efficient strategy.

When using flash cards or other activities that allow immediate feedback from a teacher, students may self-correct before feedback can be provided, which the panel believes is a positive indication that they are moving toward fluency. If students do not self-correct, immediately ask them to fix their incorrect answer and explain why the new answer is correct. If the student struggles, remind them of the efficient strategy they have learned. The student is responsible for using the taught strategy and correcting their answer before moving on.[140]

Often, computer-based programs reward students with fireworks or cute images when their answer is correct and a buzzer or refreshed screen with the same problem when their answer is incorrect. If students need additional help, instant tutorials are often built in. In this way, computer programs provide students with immediate feedback. Select computer games that require students to correct their own errors before moving on to the next problem.

Immediate feedback is sometimes not possible with worksheets, even in small groups. In the opinion of the panel, when working with worksheets, teachers should score and return them as soon as possible and then review with students the problems that need to be corrected and the effective strategies that could be used. When students correct their work, have them explain the new solution to the teacher. If the student appears not to have a known strategy for finding correct answers, reteach students an efficient strategy.

Potential Obstacles and the Panel's Advice

OBSTACLE: *"We do fluency worksheets every day, and my students are not improving."*

PANEL'S ADVICE: In the opinion of the panel, giving timed worksheets alone *does not* support fluency. Use the steps in the recommendation to think about how you could set your students up for success. Does the timed activity make sense for the intervention focus? Do your students have a way to find the answer if they don't know it automatically? Are you giving students feedback in an immediate and meaningful way? Are students observing their own progress and setting goals? Activities that support fluency need to address these elements to be effective.

OBSTACLE: *"Some students seem to race through and guess."*

PANEL'S ADVICE: Remind students that accuracy is the goal; not how many problems were attempted. Show students how their scores reflect their correct responses. Suggest that they slow down and aim for accuracy next time to see if they can improve their score. Stress that the goal of these activities is to support the student in growing their abilities to solve problems.

OBSTACLE: *"Some of my students have anxiety when doing timed activities, especially when completing an activity with a large number of problems."*

PANEL'S ADVICE: If using worksheets, students may feel anxiety when seeing a large number of problems all at once. Make sure students know they are not expected to finish all of the problems and that there are more items on the worksheet than they are expected to complete. Students may be less anxious when they do not perceive that there are a large number of items that they are supposed to finish. Instead of presenting a large list of problems to solve, use flashcards or other activities that do not present many problems at the onset. Having students work as a group to "meet or beat" their previous collective score can also decrease the pressure they may feel if asked to perform individually.

Glossary

A

Academic language - the language or words used in school, which could be discipline-specific like *isotope* or *vertices* or more general words like *infer* or *unknown*.

Automaticity - the ability to produce answers quickly without much mental energy.

B

Benchmark fractions - common fractions used as a comparison point to help students order or evaluate the magnitude of other fractions.

C

Curricular materials - educational tools and resources that teachers use to provide curriculum and instructional experiences.

F

Fluency - the ability to perform mathematics accurately and with ease.

G

Graphic organizer - a visual display of the relationships between facts or ideas.

Guiding questions - open-ended questions asked of students to direct their attention to key details without telling students the answer.

I

Instructional design - the architecture of learning experiences and curricular materials.

Intervention - focused, often more intense, instruction provided to students who are falling behind in core instruction, usually provided one-on-one or in small groups.

Irrelevant information - information provided in a word problem that is not related to the solution approach.

Isometric paper - triangular graph paper, often used for pseudo-three-dimensional views.

L

Learning outcomes - a clear statement of what the student is expected to learn or be able to do.

Linear representations - representations of mathematical concepts that are arranged along a straight or nearly straight line.

M

Mathematical concepts - abstract ideas of why the mathematics you are doing works.

Mathematical ideas - content that is central to the learning of mathematics such as whole numbers, even and odd numbers, addition, fractions, and decimals.

Mathematical procedures - the steps for performing mathematical tasks.

Multi-component intervention - an intervention that includes a bundle of instructional practices that are not disentangled when evaluating the impact of the intervention in a study.

Multi-Tiered Systems of Support (MTSS) (also referred to as Response to Intervention [RtI]) - a data-driven, systemic, problem-solving framework that helps educators provide academic and behavioral support for students with various needs.

Multiple-contrast studies - studies that evaluated multiple interventions using multiple intervention groups or compared the same intervention group to multiple comparison groups.

P

Prompts - opened-ended statements teachers tell students to direct their attention to key details without telling students the answer.

S

Sentence starters - prompts that begin a sentence that the student must complete.

Sequence of instruction - the efficient ordering of the content students will be learning to improve mathematical understanding and to meet learning outcomes.

Solution approach (also referred to as a solution strategy) - the general approach or strategy used to solve a mathematical problem, including the steps taken to solve the problem.

Student-friendly definitions - definitions suitable for students, designed with the needs and interests of students in mind.

U

Underlying mathematical structure (also referred to as the mathematical structure or underlying quantitative relationship) - the way quantities are set up and relate to one another in a problem that align with one of the four mathematical operations (addition, subtraction, multiplication, division).

Unknown - the quantity that is not shown in a mathematics equation or word problem.

V

Vernacular - language spoken by ordinary people.

W

Word wall - a collection of mathematical words, their definitions, and examples which are displayed with larger lettering on a wall.

Worked-out examples - examples that depict how a problem is solved with one solution.

Appendix A: Postscript from the Institute of Education Sciences

What is a Practice Guide?

The Institute of Education Sciences (IES) publishes practice guides to share coherent expert guidance addressing a particular educational challenge. Each recommendation in the practice guides is explicitly connected to supporting evidence from studies that meet What Works Clearinghouse™ (WWC) standards.

How are Practice Guides Developed?

To produce a practice guide, IES first selects a topic. Topic selection is informed by inquiries on the WWC website and requests sent to the WWC Help Desk, a limited literature search, and an assessment of the topic's evidence base. Next, working with a WWC contractor, IES selects a panel chair who has a national reputation and expertise in the topic, as well as additional panelists to co-author the guide. Panelists are selected based on their expertise in the topic area and the belief that they can work together to develop relevant, evidence-based recommendations. Panels include at least two current practitioners with expertise in the topic.

Relevant studies are identified through panel recommendations and a systematic literature search. These studies are then reviewed against the WWC design standards by certified reviewers who assess the internal validity of each study.[141] The panel synthesizes the evidence into recommendations. WWC staff summarize the research and draft the practice guide.

IES practice guides are then subjected to external peer review. This review is independent of the panel and the IES and WWC staff that supported the development of the guide. A critical task of the peer reviewers is to determine whether the evidence cited in support of particular recommendations is up-to-date and that studies of similar or better quality that point in a different direction have not been overlooked. Peer reviewers also evaluate whether the level of evidence category assigned to each recommendation is appropriate. WWC staff revise the guide to address concerns identified by the external peer reviewers and IES.

Levels of Evidence for What Works Clearinghouse Practice Guides

The level of evidence represents the quality and quantity of existing research supporting each recommendation. The WWC and the panel assign each recommendation one of the following three levels of evidence: strong evidence, moderate evidence, and minimal evidence.

A *strong* level of evidence rating refers to consistent evidence that the recommended strategies, programs, or practices improve relevant outcomes for a diverse population of students.[142] In other words, this level of evidence indicates that there is strong causal and generalizable evidence to support the panel's recommendation.

A *moderate* level of evidence rating refers either to evidence from studies that allow strong causal conclusions but cannot be generalized with assurance to the population on which a recommendation is focused (perhaps because the findings have not been widely replicated), or to evidence from studies that are generalizable but have some causal ambiguity.

A *minimal* level of evidence rating suggests that the panel and the WWC cannot point to a body of evidence that demonstrates the practice's positive effect on student outcomes. In some cases, this simply means that the recommended practices would be difficult to study in a rigorous, experimental or quasi-experimental fashion;[143] in other cases, it means that researchers have not yet studied this practice, or that there is weak or conflicting evidence of effectiveness. A minimal evidence rating does not indicate that the panel views the recommendation as any less important than other recommendations with a strong or moderate evidence rating.

To determine these levels of evidence, the WWC along with the panelists first conducts a careful review of the studies supporting each recommendation. For each recommendation, they examine the entire evidence base, taking into account the following considerations:

- Relevance of studies for representing the range of participants, settings, and comparisons on which the recommendation is focused.
- Whether findings from the studies can be attributed to the recommended practice.
- The weighted mean effect size from the fixed-effect meta-analysis for each relevant outcome domain.
- The extent of evidence meeting WWC standards.[144]
- How well the studies represent the range of participants and settings relevant to the recommendation.
- The panel's confidence in the effectiveness of the recommended practice.

In developing the levels of evidence, the panel and WWC consider each of the criteria in **Table A.1**. The level of evidence rating for a recommendation is determined based on findings for each of the criteria. For a recommendation to get a strong rating, the research must be rated as strong on each criterion. If at least one criterion receives a rating of moderate and none receives a rating of minimal, then the level of evidence for the recommendation is determined to be moderate. If one or more criteria receive a rating of minimal, then the level of evidence for the recommendation is determined to be minimal.

Table A.1. IES levels of evidence for What Works Clearinghouse practice guides

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
Extent of evidence	The research includes studies that meet WWC standards and provide a “medium to large” extent of evidence.[a]	The research includes at least one study that meets WWC standards and provides a “small” extent of evidence.[b]	The research may include evidence from studies that do not meet the criteria for moderate or strong evidence.
Effects on relevant outcomes	The research shows, for the relevant outcome domain(s), a preponderance of evidence of “positive effects” without contradictory evidence of “negative effects” or “potentially negative effects.”	The research shows, for the relevant outcome domain(s), a preponderance of evidence of “positive effects” or “potentially positive effects.” Contradictory evidence of “negative effects” or “potentially negative effects” must be discussed and considered with regard to relevance to the scope of the guide and the intensity of the recommendation as a component of the intervention evaluated.	There may be weak or contradictory evidence of effects.
Relevance to scope	The research has direct relevance to the scope—relevant context, sample, comparisons, and outcomes evaluated.	Relevance to the scope may vary. At least some research is directly relevant to the scope.	The research may be out of the scope of the practice guide.
Relationship between research and the recommendations	The research includes a direct test of the recommendation or the recommendation is a major component of the intervention tested in the studies.	Intensity of the recommendation as a component of the interventions evaluated in the studies may vary.	Studies for which the intensity of the recommendation as a component of the interventions evaluated in the studies is low, and/or the recommendation reflects expert opinion based on reasonable extrapolations from research.
Panel confidence	The panel has a high degree of confidence that a given practice is effective.	The panel determines that the research does not rise to the strong level of evidence but is more compelling than a minimal level of evidence. The panel may not be confident about whether the research has effectively controlled for other explanations or whether the practice would be effective in most or all contexts.	In the panel’s opinion, the recommendation must be addressed as part of the practice guide; however, the panel cannot point to a body of research that rises to the moderate or strong level of evidence.
Role of expert opinion	Not applicable.	Not applicable.	Expert opinion based on defensible interpretations of theory.

Criteria	STRONG Evidence Base	MODERATE Evidence Base	MINIMAL Evidence Base
When assessment is the focus of the recommendation	For assessments, research meets the standards of *The Standards for Educational and Psychological Testing*.[c]	For assessments, research provides evidence of reliability that meets *The Standards for Educational and Psychological Testing* but with evidence of validity from samples not adequately representative of the population on which the recommendation is focused.	Not applicable.

Note: A recommendation must satisfy all applicable requirements in the same column for the WWC to characterize the practice as supported by the evidence base at that level.

[a] This includes randomized controlled trials (RCTs) and quasi-experimental design studies (QEDs) for this practice guide.

[b] The research may include studies generally meeting WWC group design standards and supporting the effectiveness of a program, practice, or approach with small sample sizes and/or other conditions of implementation or analysis that limit generalizability.

[c] American Educational Research Association, American Psychological Association, & National Council on Measurement in Education (1999).

A Final Note About IES Practice Guides

Expert panels try to build a consensus, forging statements that all its members endorse. Practice guides do more than find common ground; they create a list of actionable recommendations. Where research clearly shows which practices are effective, the panelists use this evidence to guide their recommendations. However, in some cases, the research does not provide a clear indication of what works. In these cases, the panelists' interpretation of the existing, but incomplete, evidence plays an important role in developing the recommendations.

Appendix B: Methods and Processes for Developing This Practice Guide

Phase 1: Selecting the Panel; Establishing a Review Protocol

Expert Panel. The WWC established a 7-member expert panel to advise on the development of the practice guide. The panel consisted of researchers who were at the forefront of intervention research and practitioners with experience in implementing MTSS or working with students with or at-risk for disabilities, as well as mathematics educators.

Practice Guide Review Protocol. The WWC worked with the expert panel to develop the practice guide review protocol, available at **https://ies.ed.gov/ncee/wwc/Document/275**, which clarifies the practice guide's purpose and scope. Two questions were identified to guide the literature search and the evidence review effort:

- Which instructional practices or approaches recur in effective interventions for students in grades K–6 requiring intervention in mathematics?
- Are there effective intervention practices that impact student understanding and proficiency in any of the following topic areas: counting and cardinality, whole numbers, rational numbers, algebra and algebraic reasoning, geometry, and statistics?

The time frame for the literature search was 15 years, from January 2004 to December 2018. Older studies that were used as evidence in the original guide, *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* (2009), were also eligible for review if they met the screening criteria.

The eligible sample included students with learning disabilities in mathematics or those considered at risk: that is, experiencing difficulties in learning mathematics. Eligible study designs included randomized controlled trials (RCTs), quasi-experimental studies (QEDs), and regression discontinuity designs (RDDs). Only those mathematics interventions that were provided individually (1:1), in small groups (2 to 6 students), or in large groups (more than 6 students) for students with or at risk for disabilities were included. Interventions implemented in class-wide general mathematics classes were excluded. Only outcomes that fit into one of thirteen outcome domains addressing aspects of mathematics proficiency (e.g., knowledge, understanding, problem solving, computation) were eligible for inclusion. The thirteen domains were:

1. Algebra and Algebraic Reasoning
2. Counting and Cardinality
3. Geometry
4. General Mathematics Achievement
5. Rational Numbers Computation
6. Rational Numbers Knowledge
7. Rational Numbers Magnitude Understanding/Relative Magnitude Understanding
8. Rational Numbers Word Problems/Problem Solving
9. Statistics
10. Whole Numbers Computation
11. Whole Numbers Knowledge
12. Whole Numbers Magnitude Understanding/Relative Magnitude Understanding
13. Whole Numbers Word Problems/Problem Solving

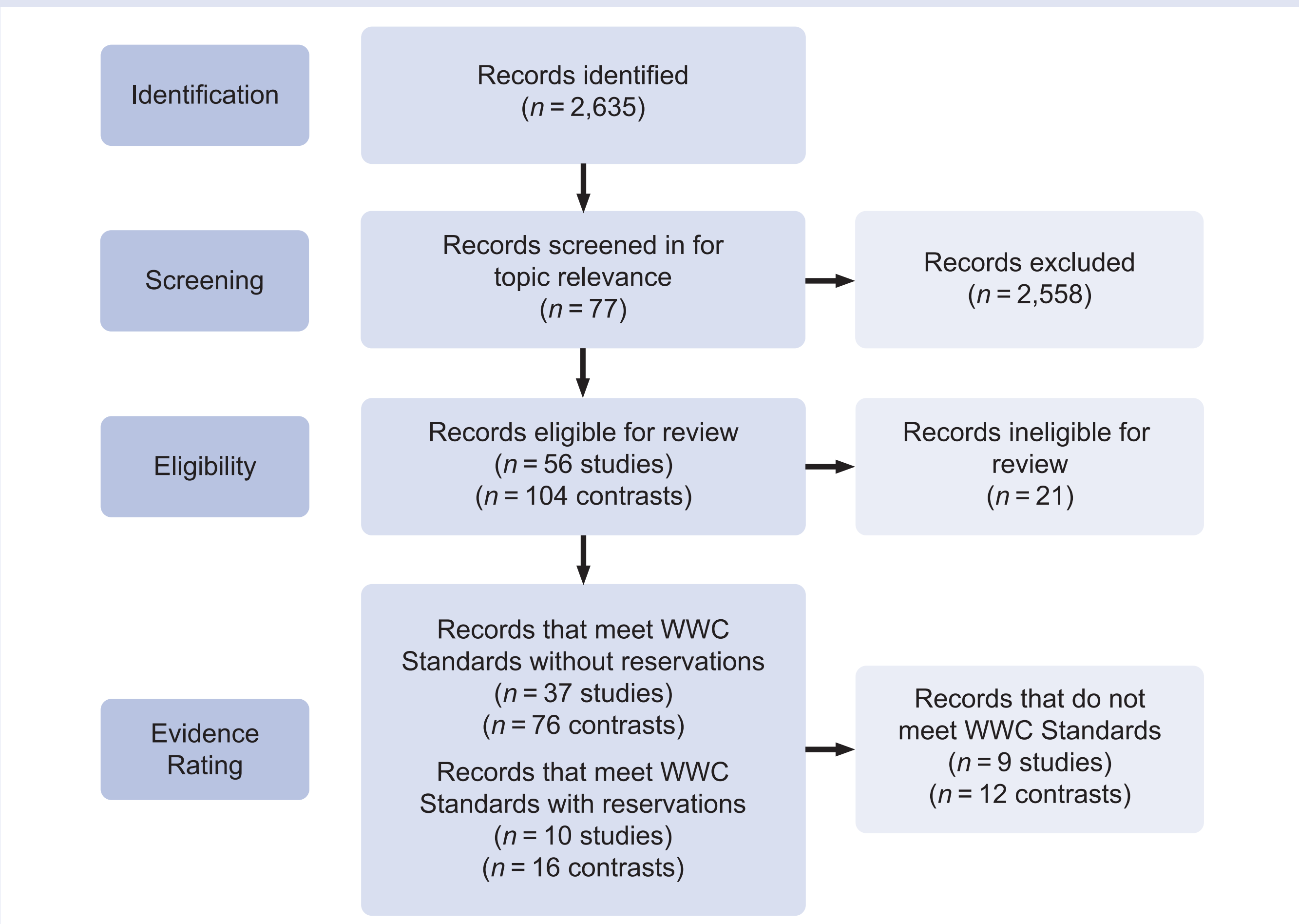
For additional details, the protocol can be accessed on the What Works Clearinghouse website.

Phase 2: Literature Search and Review

A targeted yet comprehensive search of electronic databases was conducted using keywords focused on mathematics content, intervention, population, and study design. Panel members also recommended studies that could potentially contribute to the guide.

A total of 2,635 records were identified and screened using a multi-stage screening process to determine if they focused on mathematics interventions and met the eligibility criteria (i.e., eligible mathematics interventions, sample, study designs, and outcomes). The sample eligibility definitions and corresponding search terms are delineated in the practice guide review protocol. This screening process resulted in 56 eligible studies. Of these, 31 studies included multiple contrasts–that is, they included more than two experimental conditions. Thus, it was possible to review more than one contrast per study from this set of 31 studies. WWC review teams selected one or more contrasts from these studies for review based on their relevance to the recommendations. From the 56 studies, a total of 104 experimental contrasts were reviewed using WWC 4.0 group design and RDD standards. For a study to meet WWC standards, at least one contrast must meet standards with or without reservations. See **Figure B.1**. for the number of records that went through the screening

Figure B.1. Studies identified, screened, and reviewed for this practice guide



and eligibility process and the number of studies and contrasts that were reviewed with the corresponding WWC evidence ratings.

Phase 3: Generating the Recommendations

WWC staff conducted a detailed examination of the studies that meet WWC standards to identify instructional practices that played a role in each intervention. In conjunction with the WWC, the panel identified six recommendations that were grounded in evidence provided by the 44 studies that meet WWC standards.[145] The panel then suggested ideas for carrying out the recommendations.

Phase 4: Drafting the Practice Guide

WWC staff worked with the panel to further expand and clarify each recommendation and delineate how to implement each recommendation. WWC staff then used an iterative process to draft the recommendations, soliciting feedback from the panel and revising as needed at each stage. WWC staff also compiled the level of evidence for each recommendation and drafted the technical appendices. The practice guide underwent several rounds of review, including an IES external peer review (as described in **Appendix A**).

Appendix C: Rationale for Evidence Ratings

Conducting Reviews of Eligible Studies

WWC-certified staff reviewed 56 studies to assess the quality of evidence supporting education programs and practices, using WWC group design standards version 4.0 and RDD standards version 4.0. The 44 studies that meet WWC standards provide the evidence for the recommendations. These studies are bolded in the endnotes and in the reference pages.

Determining Relevance to Recommendations

All 44 studies provide evidence for more than one recommendation, as the interventions in these studies include more than one practice (or component) for improving student outcomes. For example, one multi-component intervention might include systematic instruction (**Recommendation 1**), mathematical language (**Recommendation 2**), and number line (**Recommendation 4**), and thus be used as evidence for all three recommendations in this guide. It is not possible to identify whether one particular component or a combination of components within a multi-component intervention produced an effect. Thus, the calculated effect sizes reflect the effect of each full intervention package. The project staff determined which components were likely to cause an effect based on their prominence in the intervention program. Major intervention components in each study that meet standards were then assigned to the evidence base for the relevant recommendation. In **Table C.1**, the mapping between each study and the six recommendations is presented.

Table C.1. Mapping between studies and recommendations

	Recommendations					
Study	Systematic instruction	Mathematical language	Representations	Number lines	Word problems	Fluency
Barbieri, Rodrigues, Dyson, and Jordan (2019)*	X		X	X		X
Bryant, Bryant, Roberts, and Fall (2016)*	X	X	X			
Bryant et al. (2011)*	X		X			X
Clarke et al. (2017)[a]*	X	X	X			
Clarke et al. (2014)*	X		X			
Darch, Carnine, and Gersten (1984)[a]*	X				X	
Doabler et al. (2016)[a]*	X	X	X			
Dyson, Jordan, Beliakoff, and Hassinger-Das (2015)[a]***	X	X	X			X
Dyson, Jordan, Rodrigues, Barbieri, and Rinne (2018)*	X		X	X		X
Fien et al. (2016)*	X		X			X
Fuchs et al. (2005)*	X		X			X
Fuchs, Fuchs, Craddock, Hollenbeck, and Hamlett (2008)[a]*	X				X	
Fuchs et al. (2006)*	X					X

Study	Recommendations					
	Systematic instruction	Mathematical language	Representations	Number lines	Word problems	Fluency
Fuchs, Geary, et al. (2013)[a]***	X	X	X	X		X
Fuchs, Malone, et al. (2019)[a]**	X	X	X	X		X
Fuchs, Malone, et al. (2016)[a]**	X	X	X	X	X	X
Fuchs, Powell, et al. (2008)[a]*	X		X			X
Fuchs et al. (2010)[a]***	X				X	X
Fuchs et al. (2009)[a]**	X	X			X	X
Fuchs, Schumacher, et al. (2013)*	X	X	X	X		X
Fuchs, Schumacher, et al. (2016)[a]**	X	X	X	X	X	X
Fuchs et al. (2014)[a]**	X	X	X	X		X
Fuchs, Seethaler, et al. (2008)*	X				X	X
Fuchs, Seethaler, et al. (2019)[a]***	X	X	X	X	X	X
Gersten et al. (2015)*	X		X	X		X
Jayanthi et al. (2018)*	X	X	X	X		
Jitendra, Dupuis, et al. (2013)*	X				X	
Jitendra et al. (1998)*	X				X	
Jitendra, Rodriguez, et al. (2013)*	X				X	
Kanive, Nelson, Burns, and Ysseldyke (2014)[a]**	X		X			X
Malone, Fuchs, Sterba, Fuchs, and Foreman-Murray (2019)[a]**	X	X	X	X	X	X
Powell and Driver (2015)[a]***	X	X	X			X
Powell, Driver, and Julian (2015)[a]*	X		X			X
Powell, Fuchs, et al. (2015)[a]*	X		X			X
Powell, Fuchs, Fuchs, Cirino, and Fletcher (2009)[a]**	X			X		X
Smith, Cobb, Farran, Cordray, and Munter (2013)*		X	X			
Swanson (2014)[a]*	X				X	
Swanson, Lussier, and Orosco (2013)[a]*	X				X	
Swanson, Moran, Bocian, Lussier, and Zheng, (2013)[a]**	X				X	
Swanson, Moran, Lussier, and Fung (2014)[a]**	X				X	
Swanson, Orosco, and Lussier (2014)[a]*	X				X	
Tournaki (2003)[a]**	X					X
Wang et al. (2019)[a]*	X		X	X	X	X
Watt and Therrien (2016)*	X		X			

Note: The available WWC reviews for each study can be accessed via the hyperlinks on the * symbol.
[a] Indicates that the study is a multiple-contrast study.

Twenty-seven studies included more than one intervention condition. These multiple-contrast studies compared interventions to each other and to a comparison condition. For each recommendation, the project team identified the contrast most relevant to the recommendation and included that contrast in the evidence base for that recommendation. (The WWC classifies all contrasts that share an intervention or comparison group as part of the same study, and thus only one contrast can contribute to the level of evidence.)

In some cases, different contrasts from a single study were relevant to more than one recommendation. For example, a single study could include a contrast in which an intervention with a word problem practice is compared to a comparison condition and be included in the evidence for **Recommendation 5** on Word Problems, as well as a contrast in which an intervention with a fluency practice is compared with a comparison group and be included in the evidence for **Recommendation 6** on Timed Activities.

In some studies, there were multiple contrasts that were relevant to a single recommendation. For these studies, WWC staff worked with the panel to identify the contrast that was most relevant for each recommendation. In some studies, the most relevant contrast included an aggregated treatment group that combined multiple interventions compared with a single comparison condition.

Determining Relevant Outcomes

To simplify and focus the synthesis of evidence, the WWC worked with the panel to identify which outcome domains were relevant for each recommendation. The relevant domains for each recommendation are listed in **Table C.2**.

Table C.2. Relevant domains for each recommendation

Recommendation 1	Recommendations 2–5	Recommendation 6
1. Algebra and Algebraic Reasoning 2. Counting and Cardinality 3. Geometry 4. General Mathematics Achievement 5. Rational Numbers Computation 6. Rational Numbers Knowledge 7. Rational Numbers Magnitude Understanding/Relative Magnitude Understanding 8. Rational Numbers Word Problems/Problem Solving 9. Statistics 10. Whole Numbers Computation 11. Whole Numbers Knowledge 12. Whole Numbers Magnitude Understanding/Relative Magnitude Understanding 13. Whole Numbers Word Problems/Problem Solving	1. Algebra and Algebraic Reasoning 2. Counting and Cardinality 3. Geometry 4. General Mathematics Achievement 5. Rational Numbers Computation 6. Rational Numbers Magnitude Understanding/Relative Magnitude Understanding 7. Rational Numbers Knowledge 8. Statistics 9. Whole Numbers Computation 10. Whole Numbers Knowledge 11. Whole Numbers Magnitude Understanding/Relative Magnitude Understanding	1. Rational Numbers Word Problems/Problem Solving 2. Whole Numbers Word Problems/Problem Solving

Recommendation 1 focuses on overarching systematic instructional design regardless of the mathematical content; therefore, all 13 domains were relevant to **Recommendation 1**. Studies included in the evidence for Recommendations 2, 3, 4 and 6 focused on impacts on number knowledge and understanding, and not word problem performance. Impacts on word problem domains were most relevant to the Word Problem recommendation (**Recommendation 5**) and thus were included there. No studies that meet WWC standards included findings in the geometry or statistics domains.

The panel and staff considered only the findings in the predetermined relevant domains when determining the level of evidence for each recommendation. For brevity, only findings in relevant domains are presented in this appendix.

Estimating Fixed-Effects Meta-Analytic Effect Sizes

As discussed in **Appendix A**, the level of evidence determination for each recommendation relied on the extent of the evidence from the supporting studies. To synthesize the evidence across studies for each recommendation, the WWC calculated a weighted fixed-effects meta-analytic mean effect size for each relevant outcome domain in which at least two studies had findings.[146] This pooled estimate, which treats all of the studies contributing to that practice recommendation as a single study, means the WWC did not rely on a "vote counting" approach to assess evidence of positive effects on any relevant outcome. (For domains in which only one study had findings, the study's domain-level effect size was used in the level of evidence determination.) To calculate the meta-analytic weight, studies were weighted by the inverse of the variance of each study's effect size. Thus, large-scale studies received more weight than small-scale studies. The statistical significance of each effect size for each outcome domain was calculated using a z-test. For additional information on this process, see Appendix H of the WWC Version 4.1 WWC Procedures Handbook.

To ensure that the resulting effect sizes were statistically independent, only one contrast from each study was included in the analysis.[147] In the case of multiple-contrast studies, only the findings from the contrast most relevant to the recommendation were included in the meta-analytic effect size calculation. Relevant contrasts that compared the effectiveness of two treatments were excluded from the meta-analysis and are reported in this practice guide as supplemental evidence.

For consistency, the meta-analytic effect size calculation for each domain is based on outcomes measured closest to the end of the intervention. The effect sizes per domain for each study are listed in Tables C.4, C.6, C.8, C.10, C.12, and C.14. All other outcomes (follow-up measures, sub-scales) are presented as supplemental evidence.

Recommendation 1: Systematic Instruction

Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

Rationale for a Strong Level of Evidence

The WWC and the expert panel assigned **Recommendation 1** a strong level of evidence based on 43 studies.[148] Collectively, the studies have strong internal validity. Thirty-two studies meet WWC group design standards without reservations because they were RCTs with low sample attrition.[149] Eleven studies meet WWC group design standards with reservations because they were either compromised RCTs, RCTs with high sample attrition, or QEDs, but the analytic intervention and comparison groups in each satisfied the baseline equivalence requirement.[150] In addition, the 43 studies demonstrate strong external validity, with their samples collectively including 6,990 students and 490 schools across multiple states.[151]

Across the 43 studies, there were findings in 11 outcome domains (**Table C.3**) even though all 13 outcome domains were relevant for this recommendation. Ten domains had statistically significant, positive meta-analytic effect sizes: algebra and algebraic reasoning ($g = 0.60$, $p < 0.01$), counting and cardinality ($g = 0.34$, $p < 0.01$), general mathematics achievement ($g = 0.31$, $p < 0.01$), rational numbers computation ($g = 1.47$, $p < 0.01$), rational numbers knowledge ($g = 0.60$, $p < 0.01$), rational numbers magnitude understanding/relative magnitude understanding ($g = 0.98$, $p < 0.01$), rational numbers word problems/problem solving ($g = 0.55$, $p < 0.01$), whole numbers computation ($g = 0.52$, $p < 0.01$), whole numbers knowledge ($g = 0.28$, $p < 0.05$), and whole numbers word problems/problem solving ($g = 0.42$, $p < 0.01$). The final domain (whole numbers magnitude understanding/relative magnitude understanding) did not have a statistically significant meta-analytic effect size.

Table C.3. Domain-level effect sizes across the 43 studies supporting Recommendation 1

Domain	Number of studies (*k*)	Effect size[a]	95% Confidence interval	*p* Value
Algebra and Algebraic Reasoning	4	**0.60**	[0.38–0.82]	< 0.01
Counting and Cardinality	5	**0.34**	[0.21–0.47]	< 0.01
General Mathematics Achievement	14	**0.31**	[0.23–0.39]	< 0.01
Rational Numbers Computation	10	**1.47**	[1.35–1.58]	< 0.01
Rational Numbers Knowledge	10	**0.60**	[0.50–0.70]	< 0.01
Rational Numbers Magnitude Understanding/Relative Magnitude Understanding	9	**0.98**	[0.87–1.08]	< 0.01
Rational Numbers Word Problems/Problem Solving	4	**0.55**	[0.39–0.71]	< 0.01
Whole Numbers Computation	17	**0.52**	[0.44–0.61]	< 0.01
Whole Numbers Knowledge	2	**0.28**	[0.06–0.50]	< 0.05
Whole Numbers Magnitude Understanding/Relative Magnitude Understanding	3	**0.05**	[−0.09–0.18]	*ns*
Whole Numbers Word Problems/Problem Solving	19	**0.42**	[0.34–0.51]	< 0.01

Note: All effect sizes were calculated using a fixed-effects meta-analytic effect size across studies. *ns* = nonsignificant findings; *k* = number of studies with at least one outcome in the relevant domain and contributed to the meta-analytic effect size.

[a] Significant findings are bolded.

The 43 studies relevant to this recommendation have a preponderance of positive evidence, strong internal and external validity, and are closely aligned with the practices outlined in the recommendation. Consequently, the panel and the WWC determined that the recommendation receives a **strong evidence rating**. This rating is supported by the strength of the evidence according to the following criteria:

- **Consistency of effects on relevant outcomes.** Across the 11 domains with findings from studies that meet WWC standards, 10 had a statistically significant positive meta-analytic effect size. The other domain had uncertain effects (it did not have statistically significant meta-analytic effect size). No domains had statistically significant, negative meta-analytic effect sizes.
- **Extent of evidence.** The 43 studies related to this recommendation demonstrated positive effects with a medium to large extent of evidence. Seven of the 11 domains (algebra and algebraic reasoning, counting and cardinality, rational numbers computation, rational numbers knowledge, rational numbers magnitude understanding/relative magnitude understanding, whole numbers computation, and whole numbers word problems/problem solving) had statistically significant, positive meta-analytic effect sizes, with more than 50 percent of the meta-analytic weight from studies that meet WWC standards without reservations, and had samples of more than 350 students and multiple districts and states. These seven domains represent a preponderance of the outcome domains with findings for this recommendation.
- **Relationship between the evidence and recommendation.** The studies supporting this recommendation exhibit a strong relationship between the evidence and recommended practices because the 43 studies included at least one of the recommended practices as a major component (see below for more information).
- **Relevancy.** The 43 studies supporting this recommendation have relevant samples, contexts, comparisons, and outcomes. The studies included samples of students with, or at risk for mathematics difficulties in kindergarten through grade 6; examined interventions that were implemented as a supplement to Tier 1 instruction or in a resource room; and measured outcomes in relevant domains. The interventions ranged from roughly 8 days to 6–7 months in duration. Most studies' interventions were substantial in length; in 39 studies, the interventions lasted 8 weeks or longer.[152]

A Brief Summary of the Studies Providing Evidence for the Recommendation

The 43 studies were directly related to the recommendation, with all studies testing interventions that were described as using a systematic design or an explicit and systematic approach to instruction. Thirty-two studies examined interventions that addressed whole-numbers concepts,[153] nine addressed rational-number concepts,[154] and two studies addressed both whole-numbers and rational-numbers concepts.[155]

Forty-two studies examined teacher- or computer-led interventions that included all the recommendation's How-to Steps.[156] Specifically, these interventions reviewed previously learned material to help students maintain their understanding of concepts and procedures as they learned new material (How-to Step 1), used accessible numbers to teach concepts (How-to Step 2), and built mathematics concepts and procedures incrementally (How-to Step 3). Instructors provided students with ongoing visual and verbal supports (How-to Step 4) and immediate feedback (How-to Step 5). The one remaining study examined an intervention in which instructors provided students with ongoing visual and verbal supports (How-to Step 4).[157]

Table C.4. Studies providing evidence for Recommendation 1: Provide systematic instruction during intervention to develop student understanding of mathematical ideas.

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Barbieri et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions intervention vs. control Participants: 51 grade 6 students with mathematics difficulties Setting: 7 classrooms in 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Rational numbers computation: 0.17 Rational numbers knowledge: 1.09*
Bryant et al. (2016) *Meets WWC standards with reservations*	Design: Cluster RCT Contrast: Early numeracy intervention vs. control Participants: 71 kindergarten students with mathematics difficulties Setting: 32 classrooms in 16 schools in urban school districts in Texas	Duration: 25- to 28-minute sessions; 4 times per week; 23 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.86* General mathematics achievement: 0.99*
Bryant et al. (2011) *Meets WWC standards without reservations*	Design: RCT Contrast: Early numeracy tutoring vs. control Participants: 203 grade 1 students with mathematics difficulties Setting: 50 classrooms in 10 schools in 1 school district in Texas	Duration: 25-minute sessions; 4 times per week; 19 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	General mathematics achievement: 0.50*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Clarke et al. (2017) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 529 kindergarten students with mathematics difficulties Setting: 69 classrooms in 14 schools in 4 school districts in Oregon[†]	Duration: 20-minute sessions; 5 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Counting and cardinality: 0.38* General mathematics achievement: 0.19*
Clarke et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 88 grade 1 students with mathematics difficulties Setting: 9 schools in 2 suburban school districts in the Pacific Northwest region of the U.S.	Duration: 30-minute sessions; 3 times per week; 20 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.14 Whole numbers knowledge: 0.82* General mathematics achievement: 0.11
Darch et al. (1984) *Meets WWC standards without reservations*	Design: RCT Contrast: Explicit instruction (with or without extended practice) vs. basal instruction (with or without extended practice) Participants: 73 grade 4 students with mathematics difficulties Setting: 6 classrooms in 1 school district in Oregon	Duration: 30-minute sessions; 11-19 sessions total Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Instruction based on materials developed from four basal programs, and additional practice lessons for some students	Whole numbers word problems/problem solving: 1.43*

Recommendation 1				
Study and WWC rating	**Study description**[a]	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size**[b]
Doabler et al. (2016) *Meets WWC standards with reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 301 kindergarten students with mathematics difficulties Setting: 36 classrooms in 9 urban and suburban schools in 2 school districts in Boston, MA	Duration: 20-minute sessions; 5 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	General mathematics achievement: 0.28*
Dyson et al. (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Number sense intervention with number-fact practice or number-list practice vs. control Participants: 126 kindergarten students with mathematics difficulties Setting: 4 schools in 2 urban school districts†	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.63* Whole numbers computation: 0.71*
Dyson et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Fraction sense intervention vs. control Participants: 52 grade 6 students with mathematics difficulties Setting: 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 0.48* Rational numbers magnitude understanding/ relative magnitude understanding: 0.90* Rational numbers knowledge: 0.99*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fien et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole-number concepts intervention vs. control Participants: 238 grade 1 students with mathematics difficulties Setting: 26 classrooms in 9 schools in 2 suburban school districts in Eugene and Portland, OR	Duration: 15-minute sessions; 4 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.08 Whole numbers magnitude understanding/ relative magnitude understanding: 0.07 Whole numbers knowledge: 0.09*
Fuchs et al. (2005) *Meets WWC standards without reservations*	Design: RCT Contrast: Preventive mathematics tutoring vs. control Participants: 127 grade 1 students with mathematics difficulties Setting: 41 classrooms in 10 schools in 1 urban school district in the southeast region of the U.S.	Duration: 40-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.23 Whole numbers word problems/problem solving: 0.51* General mathematics achievement: 0.38*
Fuchs, Fuchs, et al. (2008) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem tutoring in Tier 2 (with or without word problem intervention in Tier 1) vs. no word problem tutoring in Tier 2 (with or without word problem intervention in Tier 1) Participants: 243 grade 3 students with mathematics difficulties Setting: 120 classrooms in 1 urban school district in the southeast region of the U.S.†	Duration: 20- to 30-minute sessions, 3 times per week; 13 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction, and word problem instruction as part of core instruction for some students	Whole numbers word problems/problem solving: 0.95*

Recommendation 1				
Study and WWC rating	Study description[a]	Intervention condition description	Comparison condition description	Outcome domain and WWC-calculated effect size[b]
Fuchs et al. (2006) *Meets WWC standards without reservations*	Design: RCT Contrast: Computer-assisted instruction for number combination skill vs. irrelevant control (computer-assisted spelling instruction) Participants: 33 grade 1 students with mathematics difficulties Setting: 9 classrooms in 3 schools in 1 urban school district	Duration: 10-minute sessions; 3 times per week; 18 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Computer-assisted instruction (CAI) similar to intervention condition but focused on presenting spelling words instead of number combinations	Whole numbers computation: 0.39 Whole numbers word problems/problem solving: 0.12
Fuchs, Geary, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge tutoring (with speeded or non-speeded practice) vs. control Participants: 591 grade 1 students with mathematics difficulties Setting: 233 classrooms in 40 schools in 1 urban school district in the southeast region of the U.S.[†]	Duration: 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.63* Whole numbers word problems/problem solving: 0.24* Whole numbers magnitude understanding/ relative magnitude understanding: −0.05
Fuchs, Malone, et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions magnitude intervention with error analysis vs. control Participants: 97 grade 4 and 5 students with mathematics difficulties Setting: 49 classrooms in 13 schools in 1 urban school district in the southeast region of the U.S.[†]	Duration: 40-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.98* Rational numbers magnitude understanding/ relative magnitude understanding: 1.33* Rational numbers knowledge: 0.11

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Malone, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in providing explanations or solving word problems vs. control Participants: 212 grade 4 students with mathematics difficulties Setting: 52 classrooms in 14 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.79* Rational numbers word problems/ problem solving: 0.51* Rational numbers magnitude understanding/ relative magnitude understanding: 0.93* Rational numbers knowledge: 0.86*
Fuchs, Powell, et al. (2008) *Meets WWC standards with reservations*	Design: RCT Contrast: Fact retrieval with procedural computation and computational estimation tutoring vs. irrelevant control (word-identification skill tutoring) Participants: 66 grade 3 students with mathematics difficulties Setting: 80 classrooms in 18 schools in Nashville, TN and Houston, TX†	Duration: 15- to 18-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Tutoring in word-identification skills	Whole numbers computation: 0.08 Whole numbers word problems/problem solving: 0.11 Whole numbers magnitude understanding/ relative magnitude understanding: 0.82* General mathematics achievement: 0.16
Fuchs et al. (2009) *Meets WWC standards without reservations*	Design: RCT Contrast: Tutoring in solving word problems or automatic retrieval vs. control Participants: 133 grade 3 students with mathematics difficulties Setting: 63 classrooms in 18 schools in 2 school districts in Nashville, TN and Houston, TX†	Duration: 20- to 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.59* Whole numbers word problems/problem solving: 0.41* Algebra and algebraic reasoning: 0.39*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs et al. (2010) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction (with or without strategic counting practice) vs. control Participants: 150 grade 3 students with mathematics difficulties Setting: 31 schools in 2 urban school districts in Nashville, TN and Houston, TX†	Duration: 20- to 30-minute sessions, 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.58* Whole numbers word problems/problem solving: 0.52* Algebra and algebraic reasoning: 0.87
Fuchs, Schumacher, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions understanding intervention vs. control Participants: 259 grade 4 students with mathematics difficulties Setting: 53 classrooms in 13 schools†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 2.50* Rational numbers magnitude understanding/ relative magnitude understanding: 1.46* Rational numbers knowledge: 0.92*
Fuchs, Schumacher, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in solving multiplicative or additive word problems vs. control Participants: 213 grade 4 students with mathematics difficulties Setting: 45 classrooms in 14 schools†	Duration: 35-minute sessions; 3 sessions per week; 12 weeks Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.34* Rational numbers word problems/ problem solving: 0.71* Rational numbers magnitude understanding/ relative magnitude understanding: 0.80* Rational numbers knowledge: 0.34*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions knowledge intervention with fluency activities or conceptual activities vs. control Participants: 243 grade 4 students with mathematics difficulties Setting: 49 classrooms in 14 schools in 1 urban school district[†]	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Rational numbers computation: 1.33* Rational numbers magnitude understanding/ relative magnitude understanding: 1.08* Rational numbers knowledge: 0.58*
Fuchs, Seethaler, et al. (2008) *Meets WWC standards with reservations*	Design: RCT Contrast: Tutoring in solving word problems vs. control Participants: 35 grade 3 students with mathematics difficulties Setting: 18 classrooms in 1 urban school district in the southeast region of the U.S.	Duration: 20- to 30-minute sessions; 3 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.49 Whole numbers word problems/problem solving: 0.97* General mathematics achievement: 0.19
Fuchs, Seethaler, et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem intervention (with or without language instruction) or number knowledge intervention vs. control Participants: 391 grade 1 students with mathematics difficulties Setting: 186 classrooms in 21 schools[†]	Duration: 30-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.65* Whole numbers word problems/problem solving: 0.49*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Gersten et al. (2015) *Meets WWC standards with reservations*	Design: Cluster RCT Contrast: Number operations intervention vs. control Participants: 881 grade 1 students with mathematics difficulties Setting: 76 schools in 4 urban school districts in 4 states in the south-central and southwest regions of the U.S.	Duration: 40-minute sessions; 3-4 times per week; 17 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	General mathematics achievement: 0.34*
Jayanthi et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Small-group fractions intervention vs. control Participants: 186 grade 5 students with mathematics difficulties Setting: 3 school districts in the west and southeast regions of the U.S.	Duration: 35-minute sessions; 3-4 times per week; 6-7 months Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.07* Rational numbers magnitude understanding/ relative magnitude understanding: 0.94* Rational numbers knowledge: 0.72*
Jitendra, Dupuis, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem intervention vs. control Participants: 109 grade 3 students with mathematics difficulties Setting: 28 classrooms in 9 schools in 1 large urban school district in the Midwest region of the U.S.	Duration: 30-minute sessions, 5 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Small-group tutoring in topics selected from the core mathematics curriculum (place value, whole numbers addition and subtraction computation strategies, and word problem solving)	Whole numbers word problems/problem solving: 0.46* General mathematics achievement: 0.34*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Jitendra et al. (1998) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction vs. control Participants: 34 students in grades 2–5 with mathematics disabilities or difficulties Setting: 4 classrooms in 4 schools in the northeast and southeast regions of the U.S.	Duration: 40- to 45-minute sessions; 17-20 sessions total Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Instruction based on a basal mathematics program	Whole numbers word problems/problem solving: 0.63
Jitendra, Rodriguez, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem intervention vs. control Participants: 136 grade 3 students with mathematics difficulties Setting: 35 classrooms in 12 schools in 1 urban school district in the Midwest region of the U.S.	Duration: 30-minute sessions; 5 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Small-group tutoring in topics selected from the core mathematics curriculum (place value, addition and subtraction, and word problem solving)	Whole numbers computation: −0.40 Whole numbers word problems/problem solving: 0.02* General mathematics achievement: 0.11
Kanive et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Conceptual understanding intervention vs. control Participants: 57 grade 4 and 5 students with mathematics difficulties Setting: 1 school in Minnesota[†]	Duration: 15-minute sessions; 1 time per week; 2 weeks Content: Whole numbers Relevance to Recommendation: Instruction included verbal and/or visual supports.	Business as usual core mathematics instruction	Whole numbers computation: 0.29 Whole numbers word problems/problem solving: 0.16

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Malone et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction or decimal magnitude instruction vs. control Participants: 225 grade 4 students with mathematics difficulties Setting: 58 classrooms in 12 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions and decimals Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Rational numbers computation: 1.53* Rational numbers word problems/ problem solving: 0.39* Rational numbers magnitude understanding/ relative magnitude understanding: 0.48* Rational numbers knowledge: 0.12
Powell and Driver (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Addition tutoring (with or without embedded vocabulary component) vs. control Participants: 98 grade 1 students with mathematics difficulties Setting: 58 classrooms in 18 schools in 2 school districts in the mid-Atlantic region of the U.S.†	Duration: 10- to 15-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.34
Powell, Driver, et al. (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Standard equations tutoring or combined (standard and nonstandard) equations tutoring vs. control Participants: 51 grade 2 students with mathematics difficulties Setting: 31 classrooms in 10 schools in 2 school districts in the mid-Atlantic region of the U.S.†	Duration: 10- to 15-minute sessions; 3 times per week; 4 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.23 Algebra and algebraic reasoning: 0.80*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Powell, Fuchs, et al. (2015) *Meets WWC standards with reservations*	Design: Cluster QED Contrast: Word-problem intervention or calculation intervention vs. control Participants: 265 grade 2 students with mathematics difficulties Setting: 110 classrooms in 25 schools in 1 urban school district[†]	Duration: Tier 1 portion: 40- to 45-minute sessions; 2 times per week; 17 weeks; Tier 2 portion (beginning week 4 of the Tier 1 portion): 25- to 30-minute sessions; 3 times per week Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.80*
Powell et al. (2009) *Meets WWC standards without reservations*	Design: RCT Contrast: Fact retrieval practice or conceptual instruction with fact retrieval practice vs. control Participants: 101 grade 3 students with mathematics difficulties Setting: 75 classrooms in 17 schools in Nashville, TN and Houston, TX[†]	Duration: 22- to 25-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 0.71*
Swanson (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction—verbal plus visual strategies condition vs. control Participants: 33 grade 3 students with mathematics difficulties Setting: 22 classrooms in 2 schools in 1 school district in the southwest region of the U.S.[†]	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers word problems/problem solving: 0.04 General mathematics achievement: 0.19

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Swanson, Lussier, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction with heuristic strategy plus visual schematic diagrams vs. control Participants: 38 grade 3 students with mathematics difficulties Setting: 21 classrooms[†]	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers word problems/problem solving: 0.57 General mathematics achievement: 0.62*
Swanson, Moran, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction—complete condition vs. control (students with mathematics difficulties only) Participants: 33 grade 3 students with mathematics difficulties Setting: 12 classrooms in 4 schools in 2 school districts in the southwest region of the U.S.[†]	Duration: 30-minute sessions; 2 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Small-group tutoring in topics selected from the core mathematics curriculum	Whole numbers word problems/problem solving: 0.39
Swanson, Moran, et al. (2014) *Meets WWC standards with reservations*	Design: RCT Contrast: Word problem instruction—complete condition vs. control (students with mathematics difficulties only) Participants: 45 grade 3 students with mathematics difficulties Setting: 12 classrooms in 4 schools in 2 school districts in the southwest region of the U.S.[†]	Duration: 30-minute sessions; 2 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Small-group instruction in topics selected from the core mathematics curriculum	Whole numbers word problems/problem solving: 0.14

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Swanson, Orosco, et al. (2014) *Meets WWC standards with reservations*	Design: RCT Contrast: Word problem instruction—material, verbal, and visual strategies condition vs. materials-only condition Participants: 29 grade 3 students with mathematics difficulties Setting: 18 classrooms in 1 school district in California†	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	The same word problem instruction as the intervention condition without the specific strategy instruction	Whole numbers word problems/problem solving: −0.01 General mathematics achievement: 0.69
Tournaki (2003) *Meets WWC standards without reservations*	Design: RCT Contrast: Minimum-addend strategy instruction vs. control Participants: 28 grade 2 students with mathematics disabilities or difficulties Setting: 1 urban school district in New York†	Duration: 15-minute sessions; 5 times per week; 8 days Content: Whole numbers Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction	Whole numbers computation: 2.30*
Wang et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction (with or without self-regulation) vs. control Participants: 84 grade 3 students with mathematics difficulties Setting: 29 classrooms in 8 schools in 1 urban school district†	Duration: 35-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Business as usual core mathematics instruction and any school-provided intervention	Whole numbers computation: 0.59* Rational numbers computation: 1.27* Rational numbers word problems/problem solving: 0.69* Rational numbers magnitude understanding/relative magnitude understanding: 0.95* Rational numbers knowledge: 0.85*

Recommendation 1				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Watt and Therrien (2016) *Meets WWC standards with reservations*	Design: RCT Contrast: Pre-teaching plus concrete-representational-abstract instructional sequence vs. supplemental reading instruction Participants: 32 grade 6 students with mathematics difficulties Setting: 4 classrooms in 2 schools	Duration: 30-minute sessions; 5 times per week; 2 weeks Content: Fractions Relevance to Recommendation: Instruction included review, feedback, and strategic supports. Mathematics content was sequenced to build incrementally across lessons. Students learned strategies to solve problems.	Small-group supplemental reading instruction	Algebra and algebraic reasoning: 0.09

Note: Each row in this table represents a study, defined by the WWC as an examination of the effect of an intervention on a distinct sample.

[a] Sample size represents the maximum number of participants in the study. In some studies, the number of participants varied across the outcome measures.

[b] Effect sizes presented are from the posttest closest to the end of the intervention. For brevity, only the domain average effect size and statistical significance are reported in this table. For studies that included multiple outcomes in a domain, reported effect sizes and statistical significance are for the domain and calculated as described in the WWC Procedures Handbook (version 4.0).

* Significant at $p \leq 0.05$.

† Indicates that the information is for the entire study (across all conditions).

Supplemental Findings for Recommendation 1

Supplemental findings (including impacts for follow-up, sub-scale, and distal measures) for 13 studies are available at the corresponding study pages on the WWC website.[158]

Recommendation 2: Mathematical Language

Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.

Rationale for a Strong Level of Evidence

The WWC and the expert panel assigned **Recommendation 2** a strong level of evidence based on 16 studies.[159] The 16 studies collectively have strong internal validity. Twelve studies meet WWC group design standards without reservations because they were RCTs with low sample attrition.[160] Four studies meet WWC group design standards with reservations because they were either compromised RCTs or RCTs with high sample attrition, but the analytic intervention and comparison groups in each satisfied the baseline equivalence requirement.[161] In addition, the 16 studies demonstrate strong external validity as their samples collectively included 3,060 students and 182 schools across multiple states.[162]

Across the 16 studies, there were findings in eight of the key outcome domains for this recommendation (**Table C.5**). The meta-analytic effect sizes for six of these domains were statistically significant and positive: counting and cardinality ($g = 0.47$, $p < 0.01$), general mathematics achievement ($g = 0.93$, $p < 0.01$), rational numbers computation ($g = 1.60$, $p < 0.01$), rational numbers knowledge ($g = 0.52$, $p < 0.01$), rational numbers magnitude understanding/relative magnitude understanding ($g = 0.99$, $p < 0.01$), and whole numbers computation ($g = 0.39$, $p < 0.01$). In the whole numbers magnitude understanding/relative magnitude understanding and algebra and algebraic reasoning domains, there were not statistically significant effect sizes.

Table C.5. Domain-level effect sizes across the 16 studies supporting Recommendation 2

Domain	Number of studies (*k*)	Effect size[a]	95% Confidence interval	*p* Value
Algebra and Algebraic Reasoning	1	0.23	NA	*ns*
Counting and Cardinality	3	**0.47**	[0.31–0.63]	< 0.01
General Mathematics Achievement	4	**0.93**	[0.81–1.04]	< 0.01
Rational Numbers Computation	7	**1.60**	[1.48–1.73]	< 0.01
Rational Numbers Knowledge	7	**0.52**	[0.40–0.63]	< 0.01
Rational Numbers Magnitude Understanding/Relative Magnitude Understanding	7	**0.99**	[0.88–1.11]	< 0.01
Whole Numbers Computation	6	**0.39**	[0.30–0.49]	< 0.01
Whole Numbers Magnitude Understanding/Relative Magnitude Understanding	1	−0.05	NA	*ns*

Note: All effect sizes were calculated using a fixed-effects meta-analytic effect size across studies except for the algebra and algebraic reasoning and whole numbers magnitude understanding/relative magnitude understanding domains. These two domains had findings from just one study each; the effect sizes presented here are the WWC-calculated domain-level average effect sizes for the individual relevant study for each domain. *ns* = nonsignificant findings; NA = not applicable; *k* = number of studies with at least one outcome in the relevant domain and contributed to the meta-analytic effect size.

[a] Significant findings are bolded.

The 16 studies relevant to this recommendation have a preponderance of positive evidence, strong internal and external validity, and are closely aligned with the practices outlined in the recommendation. Therefore, the panel and the WWC determined that the recommendation receives a **strong evidence rating**. This rating is supported by the strength of the evidence according to the following criteria:

- **Consistency of effects on relevant outcomes.** Across the eight relevant domains with findings from studies that meet WWC standards, six had a statistically significant positive meta-analytic effect size. The other two domains had uncertain effects (they did not have statistically significant effect sizes). No domains had statistically significant, negative effect sizes.
- **Extent of evidence.** The 16 studies related to this recommendation demonstrated positive effects with a medium to large extent of evidence. Five of the eight relevant domains (counting and cardinality, rational numbers computation, rational numbers knowledge, rational numbers magnitude understanding/relative magnitude understanding, and whole numbers computation) had statistically significant, positive meta-analytic effect sizes, with more than 50 percent of the meta-analytic weight from studies that meet WWC standards without reservations, and had samples of more than 350 students and multiple districts and states. These five domains represent a preponderance of the key outcome domains with findings for this recommendation.
- **Relationship between the evidence and recommendation.** The 16 studies supporting this recommendation exhibit a strong relationship between the evidence and recommended practices because all 16 studies include at least one of the recommended practices as a major component (see below for more information). Multiple studies supported each How-to Step in the recommendation.
- **Relevancy.** The 16 studies supporting this recommendation have relevant samples, contexts, comparisons, and outcomes. The studies included samples of students with, or at risk for mathematics difficulties in kindergarten through grade 5; examined interventions that were implemented as a supplement to Tier 1 instruction or in a resource room; and measured outcomes in relevant domains. The interventions ranged from roughly 8 weeks to 6–7 months in duration.

A Brief Summary of the Studies Providing Evidence for the Recommendation

The 16 studies were directly related to the recommendation, with all studies testing interventions that included the recommended practices as a major component. All 16 studies included mathematical language to help boost student understanding of mathematics concepts and procedures. Nine studies examined interventions that addressed whole-numbers concepts,[163] six addressed rational-numbers concepts,[164] and one study addressed both whole-numbers and rational-numbers concepts.[165]

Multiple studies related to each of the recommendation's How-to Steps. In 12 studies,[166] students were taught a focused set of mathematical vocabulary words and their definitions (How-to Step 1). Integrating and using mathematical language throughout instruction occurred in 13 studies (How-to Step 2).[167] In 14 studies,[168] students explained their mathematical thinking and verbalized solution methods; however, in three of these studies,[169] students were held accountable for high-quality explanations using precise mathematical language which is the preference of the panel (How-to Step 3).

Table C.6. Studies providing evidence for Recommendation 2: Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts

Recommendation 2				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Bryant et al. (2016) *Meets WWC standards with reservations*	Design: Cluster RCT Contrast: Early numeracy intervention vs. control Participants: 71 kindergarten students with mathematics difficulties Setting: 32 classrooms in 16 schools in urban school districts in Texas	Duration: 25- to 28-minute sessions; 4 times per week; 23 weeks Content: Whole numbers Relevance to Recommendation: Students learned mathematical vocabulary and instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.86* General mathematics achievement: 0.99*
Clarke et al. (2017) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 529 kindergarten students with mathematics difficulties Setting: 69 classrooms in 14 schools in 4 school districts in Oregon†	Duration: 20-minute sessions; 5 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Students learned how to provide verbal explanations of their problem-solving processes.	Business as usual core mathematics instruction	Counting and cardinality: 0.38* General mathematics achievement: 0.19*
Doabler et al. (2016) *Meets WWC standards with reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 301 kindergarten students with mathematics difficulties Setting: 36 classrooms in 9 urban or suburban schools in 2 school districts in Boston, MA	Duration: 20-minute sessions; 5 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Students learned how to provide verbal explanations of their problem-solving processes.	Business as usual core mathematics instruction	General mathematics achievement: 0.28*

Recommendation 2				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Dyson et al. (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Number-sense intervention with number-fact practice or number-list practice vs. control Participants: 126 kindergarten students with mathematics difficulties Setting: 4 schools in 2 urban school districts†	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students learned mathematical vocabulary and instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.63* Whole numbers computation: 0.71*
Fuchs, Geary, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge tutoring (with speeded or non-speeded practice) vs. control Participants: 591 grade 1 students with mathematics difficulties Setting: 233 classrooms in 40 schools in 1 urban school district in the southeast region of the U.S.†	Duration: 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal explanations of their problem-solving processes. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction	Whole numbers computation: 0.63* Whole numbers magnitude understanding/ relative magnitude understanding: −0.05
Fuchs, Malone, et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions magnitude intervention with error analysis vs. control Participants: 97 grade 4 and 5 students with mathematics difficulties Setting: 49 classrooms in 13 schools in 1 urban school district in the southeast region of the U.S.†	Duration: 40-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal explanations of their problem-solving processes while incorporating the vocabulary they learned. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.98* Rational numbers magnitude understanding/ relative magnitude understanding: 1.33* Rational numbers knowledge: 0.11

Recommendation 2				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Malone, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in providing explanations vs. control Participants: 143 grade 4 students with mathematics difficulties Setting: 52 classrooms in 14 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal and written explanations of their problem-solving processes. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction	Rational numbers computation: 1.94* Rational numbers magnitude understanding/ relative magnitude understanding: 1.04* Rational numbers knowledge: 0.57*
Fuchs et al. (2009) *Meets WWC standards without reservations*	Design: RCT Contrast: Tutoring in automatic retrieval vs. control Participants: 91 grade 3 students with mathematics difficulties Setting: 63 classrooms in 18 schools in 2 school districts in Nashville, TN and Houston, TX†	Duration: 20- to 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students learned how to provide verbal explanations of their problem-solving processes and instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction	Whole numbers computation: 0.56* Algebra and algebraic reasoning: 0.23
Fuchs, Schumacher, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions understanding intervention vs. control Participants: 259 grade 4 students with mathematics difficulties Setting: 53 classrooms in 13 schools†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal explanations of their problem-solving processes. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 2.50* Rational numbers magnitude understanding/ relative magnitude understanding: 1.46* Rational numbers knowledge: 0.92*
Fuchs, Schumacher, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in solving multiplicative or additive word problems vs. control Participants: 213 grade 4 students with mathematics difficulties Setting: 45 classrooms in 14 schools†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal explanations of their problem-solving processes. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.34* Rational numbers magnitude understanding/ relative magnitude understanding: 0.80* Rational numbers knowledge: 0.34*

Recommendation 2				
Study and WWC rating	**Study description**[a]	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size**[b]
Fuchs et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions knowledge intervention with fluency activities or conceptual activities vs. control Participants: 243 grade 4 students with mathematics difficulties Setting: 49 classrooms in 14 schools in 1 urban school district†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal explanations of their problem-solving processes. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction	Rational numbers computation: 1.33* Rational numbers magnitude understanding/ relative magnitude understanding: 1.08* Rational numbers knowledge: 0.58*
Fuchs, Seethaler, et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge intervention vs. control Participants: 196 grade 1 students with mathematics difficulties Setting: 186 classrooms in 21 schools†	Duration: 30-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students learned the meanings of mathematical symbols and how to provide verbal explanations of their problem-solving processes.	Business as usual core mathematics instruction	Whole numbers computation: 0.59*
Jayanthi et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Small-group fractions intervention vs. control Participants: 186 grade 5 students with mathematics difficulties Setting: 3 school districts in the west and southeast regions of the U.S.	Duration: 35-minute sessions; 3-4 times per week; 6-7 months Content: Fractions Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal and written explanations of their problem-solving processes. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.07* Rational numbers magnitude understanding/ relative magnitude understanding: 0.94* Rational numbers knowledge: 0.72*
Malone et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction or decimal magnitude instruction vs. control Participants: 225 grade 4 students with mathematics difficulties Setting: 58 classrooms in 12 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions and decimals Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal explanations of their problem-solving processes. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction	Rational numbers computation: 1.53* Rational numbers magnitude understanding/ relative magnitude understanding: 0.48* Rational numbers knowledge: 0.12

Recommendation 2				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Powell and Driver (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Addition tutoring with embedded vocabulary component vs. control Participants: 63 grade 1 students with mathematics difficulties Setting: 58 classrooms in 18 schools in 2 school districts in the mid-Atlantic region of the U.S.†	Duration: 10- to 15-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students learned mathematical vocabulary and how to provide verbal definitions. Instruction was provided using mathematically precise terminology.	Business as usual core mathematics instruction	Whole numbers computation: 0.18
Smith et al. (2013) *Meets WWC standards with reservations*	Design: RCT Contrast: Intensive one-to-one tutoring in arithmetical knowledge vs. control Participants: 775 grade 1 students with mathematics difficulties Setting: 20 schools in 5 school districts in 2 states†	Duration: 30-minute sessions; 4-5 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Students learned to provide verbal explanations of their problem-solving processes.	Business as usual core mathematics instruction	Whole numbers computation: 0.14* General mathematics achievement: 1.82*

Note: Each row in this table represents a study, defined by the WWC as an examination of the effect of an intervention on a distinct sample.

[a] Sample size represents the maximum number of participants in the study. In some studies, the number of participants varied across the outcome measures.

[b] Effect sizes from presented are from the posttest closest to the end of the intervention. For brevity, only the domain average effect size and statistical significance are reported in this table. For studies that included multiple outcomes in a domain, reported effect sizes and statistical significance are for the domain and calculated as described in the WWC Procedures Handbook (version 4.0).

* Significant at $p \leq 0.05$.

† Indicates that the information is for the entire study (across all conditions).

Supplemental Findings for Recommendation 2

Supplemental findings (including impacts for follow-up, sub-scale, and distal measures) for four studies are available at the corresponding study pages on the WWC website.[170]

From the multiple contrast studies, one study included a contrast between two treatment conditions that assessed whether teaching vocabulary definitions within an intervention focused on addition computation impacted computation performance.[171] Because both treatments integrated the use of these mathematics vocabulary words into each lesson (How-to Step 2), this contrast is only looking at the value added by providing separate instruction on these word meanings (How-to Step 1). Impacts for whole numbers computation were not statistically significant and therefore inconclusive ($g = -0.32$).

Recommendation 3: Representations

Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.

Rationale for a Strong Level of Evidence

The WWC and the expert panel assigned **Recommendation 3** a strong level of evidence based on 28 studies.[172] Together the studies have strong internal validity. Nineteen studies meet WWC group design standards without reservations because they were RCTs with low sample attrition.[173] Nine studies meet WWC group design standards with reservations because they were either compromised RCTs, RCTs with high sample attrition, or QEDs, but the analytic intervention and comparison groups in each satisfied the baseline equivalence requirement.[174] In addition, the 28 studies demonstrate strong external validity as their samples collectively included 6,272 students and 404 schools across multiple states.[175]

Across the 28 studies, there were findings in nine of the relevant domains for this recommendation (**Table C.7**). The meta-analytic effect sizes for eight domains were statistically significant and positive: algebra and algebraic reasoning ($g = 0.48$, $p < 0.05$), counting and cardinality ($g = 0.34$, $p < 0.01$), general mathematics achievement ($g = 0.64$, $p < 0.01$), rational numbers computation ($g = 1.46$, $p < 0.01$), rational numbers knowledge ($g = 0.58$, $p < 0.01$), rational numbers magnitude understanding/relative magnitude understanding ($g = 0.99$, $p < 0.01$), whole numbers computation ($g = 0.43$, $p < 0.01$), and whole numbers knowledge ($g = 0.28$, $p < 0.05$). In the whole numbers magnitude understanding/relative magnitude understanding domain, there was not a statistically significant effect size.

Table C.7. Domain-level effect sizes across the 28 studies supporting Recommendation 3

Domain	Number of studies (*k*)	Effect size[a]	95% Confidence interval	*p* Value
Algebra and Algebraic Reasoning	2	**0.48**	[0.01–0.95]	< 0.05
Counting and Cardinality	5	**0.34**	[0.21–0.47]	< 0.01
General Mathematics Achievement	9	**0.64**	[0.56–0.71]	< 0.01
Rational Numbers Computation	10	**1.46**	[1.35–1.57]	< 0.01
Rational Numbers Knowledge	10	**0.58**	[0.48–0.69]	< 0.01
Rational Numbers Magnitude Understanding/Relative Magnitude Understanding	9	**0.99**	[0.88–1.09]	< 0.01
Whole Numbers Computation	11	**0.43**	[0.34–0.51]	< 0.01
Whole Numbers Knowledge	2	**0.28**	[0.06–0.50]	< 0.05
Whole Numbers Magnitude Understanding/Relative Magnitude Understanding	3	0.05	[−0.09–0.18]	*ns*

Note: All effect sizes were calculated using a fixed-effects meta-analytic effect size across studies. *ns* = nonsignificant findings; *k* = number of studies with at least one outcome in the relevant domain and contributed to the meta-analytic effect size.
[a] Significant findings are bolded.

The 28 studies relevant to this recommendation have a preponderance of positive evidence, strong internal and external validity, and are closely aligned with the practices outlined in the recommendation. Therefore, the WWC and the panel determined that the recommendation receives a **strong evidence rating**. This rating is supported by the strength of the evidence according to the following criteria:

- **Consistency of effects on relevant outcomes.** Of the nine relevant domains with findings from studies that meet WWC standards, eight domains had statistically significant, positive meta-analytic effect sizes. The other domain had uncertain effects (it did not have a statistically significant meta-analytic effect size). No domains had statistically significant, negative effect sizes.
- **Extent of evidence.** The 28 studies related to this recommendation demonstrated positive effects with a medium to large extent of evidence. Five of the nine relevant domains (counting and cardinality, rational numbers computation, rational numbers knowledge, rational numbers magnitude understanding/relative magnitude understanding, and whole numbers computation) had statistically significant, positive meta-analytic effect sizes, with more than 50 percent of the meta-analytic weight from studies that meet WWC standards without reservations, and had samples of more than 350 students and multiple districts and states. These five domains represent a preponderance of the relevant domains with findings for this recommendation.
- **Relationship between the evidence and recommendation.** The studies supporting this recommendation exhibit a strong relationship between the evidence and recommended practices because all 28 studies include at least one of the recommended practices as a major component (see below for more information).
- **Relevancy.** The 28 studies supporting this recommendation have relevant samples, contexts, comparisons, and outcomes. The studies included samples of students with, or at risk for, mathematics difficulties in kindergarten through grade 6; examined interventions that were implemented as a supplement to Tier 1 instruction or in a resource room; and measured outcomes in relevant domains. The interventions ranged from roughly 2 weeks to 6–7 months in duration.

A Brief Summary of the Studies Providing Evidence for the Recommendation

The 28 studies were directly related to the recommendation, with all studies testing interventions that included the recommended practices as a major component. All 28 studies used three-dimensional, concrete representations (e.g., connecting cubes, fraction tiles, manipulatives) and two-dimensional, semi-concrete representations (e.g., visual diagrams, figures, pictures). In eight studies, number lines were the only semi-concrete representation that was used.[176] We discuss number lines in greater detail in **Recommendation 4**. Seventeen studies examined interventions that addressed whole-number concepts,[177] nine studies addressed rational number concepts,[178] and two studies addressed both.[179]

Multiple studies related to most of the recommendation's How-to Steps, while one How-to Step is based on the panel's expertise. The panel provides advice on how to select representations based on the concept being taught (How-to Step 1). In all 28 studies, representations were used to help students visualize the mathematics they were learning. In 25 studies, teachers connected concrete and/or semi-concrete representations to the abstract representation or mathematical notation (How-to Step 2).[180] Students used concrete and/or semi-concrete representations as a "thinking tool" to model the mathematics they were learning in 17 studies (How-to Step 3).[181] In 11 studies, the intervention revisited the use of representations periodically to reinforce and deepen student learning (How-to Step 4).[182]

Table C.8. Studies providing evidence for Recommendation 3: Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Barbieri et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions intervention vs. control Participants: 51 grade 6 students with mathematics difficulties Setting: 7 classrooms in 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Students learned to connect concrete and/or semi-concrete representations to abstract representations.	Business as usual core mathematics instruction	Rational numbers computation: 0.17 Rational numbers knowledge: 1.09*
Bryant et al. (2016) *Meets WWC standards with reservations*	Design: Cluster RCT Contrast: Early numeracy intervention vs. control Participants: 71 kindergarten students with mathematics difficulties Setting: 32 classrooms in 16 schools in urban school districts in Texas	Duration: 25- to 28-minute sessions; 4 times per week; 23 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.86* General mathematics achievement: 0.99*
Bryant et al. (2011) *Meets WWC standards without reservations*	Design: RCT Contrast: Early numeracy tutoring vs. control Participants: 203 grade 1 students with mathematics difficulties Setting: 50 classrooms in 10 schools in 1 school district in Texas	Duration: 25-minute sessions; 4 times per week; 19 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations.	Business as usual core mathematics instruction	General mathematics achievement: 0.50*
Clarke et al. (2017) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 529 kindergarten students with mathematics difficulties Setting: 69 classrooms in 14 schools in 4 school districts in Oregon†	Duration: 20-minute sessions; 5 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Students used concrete and/or semi-concrete representations to model mathematical thinking.	Business as usual core mathematics instruction	Counting and cardinality: 0.38* General mathematics achievement: 0.19*

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Clarke et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 88 grade 1 students with mathematics difficulties Setting: 9 schools in 2 suburban school districts in the Pacific Northwest region of the U.S.	Duration: 30-minute sessions; 3 times per week; 20 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.14 Whole numbers knowledge: 0.82* General mathematics achievement: 0.11
Doabler et al. (2016) *Meets WWC standards with reservations*	Design: RCT Contrast: Whole numbers understanding intervention vs. control Participants: 301 kindergarten students with mathematics difficulties Setting: 36 classrooms in 9 urban and suburban schools in 2 school districts in Boston, MA	Duration: 20-minute sessions; 5 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Students used concrete and/or semi-concrete representations to model mathematical thinking.	Business as usual core mathematics instruction	General mathematics achievement: 0.28*
Dyson et al. (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Number sense intervention with number-fact practice or number-list practice vs. control Participants: 126 kindergarten students with mathematics difficulties Setting: 4 schools in 2 urban school districts[†]	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.63* Whole numbers computation: 0.71*

Recommendation 3				
Study and WWC rating	**Study description**[a]	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size**[b]
Dyson et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Fraction sense intervention vs. control Participants: 52 grade 6 students with mathematics difficulties Setting: 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Students learned to connect concrete and/or semi-concrete representations to abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 0.48* Rational numbers magnitude understanding/ relative magnitude understanding: 0.90* Rational numbers knowledge: 0.99*
Fien et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole-number concepts intervention vs. control Participants: 238 grade 1 students with mathematics difficulties Setting: 26 classrooms in 9 schools in 2 suburban school districts in Eugene and Portland, OR	Duration: 15-minute sessions; 4 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Using a computer-based program, students connected semi-concrete representations to abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.08 Whole numbers magnitude understanding/ relative magnitude understanding: 0.07 Whole numbers knowledge: 0.09*
Fuchs et al. (2005) *Meets WWC standards without reservations*	Design: RCT Contrast: Preventive mathematics tutoring vs. control Participants: 127 grade 1 students with mathematics difficulties Setting: 41 classrooms in 10 schools in 1 urban school district in the southeast region of the U.S.	Duration: 40-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking.	Business as usual core mathematics instruction	Whole numbers computation: 0.23 General mathematics achievement: 0.38*
Fuchs, Geary, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge tutoring (with speeded or non-speeded practice) vs. control Participants: 591 grade 1 students with mathematics difficulties Setting: 233 classrooms in 40 schools in 1 urban school district in the southeast region of the U.S.†	Duration: 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking.	Business as usual core mathematics instruction	Whole numbers computation: 0.63* Whole numbers magnitude understanding/ relative magnitude understanding: −0.05

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Malone, et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions magnitude intervention (with or without error analysis) vs. control Participants: 143 grade 4 and 5 students with mathematics difficulties Setting: 49 classrooms in 13 schools in 1 urban school district in the southeast region of the U.S.†	Duration: 40-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.72* Rational numbers magnitude understanding/ relative magnitude understanding: 1.35* Rational numbers knowledge: 0.05
Fuchs, Malone, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in providing explanations or solving word problems vs. control Participants: 212 grade 4 students with mathematics difficulties Setting: 52 classrooms in 14 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.79* Rational numbers magnitude understanding/ relative magnitude understanding: 0.93* Rational numbers knowledge: 0.86*
Fuchs, Powell, et al. (2008) *Meets WWC standards with reservations*	Design: RCT Contrast: Fact retrieval with procedural computation and computational estimation tutoring vs. irrelevant control (word-identification skill tutoring) Participants: 66 grade 3 students with mathematics difficulties Setting: 80 classrooms in 18 schools in Nashville, TN and Houston, TX†	Duration: 15- to 18-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Tutoring in word-identification skills	Whole numbers computation: 0.08 Whole numbers magnitude understanding/ relative magnitude understanding: 0.82* General mathematics achievement: 0.16

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Schumacher, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions understanding intervention vs. control Participants: 259 grade 4 students with mathematics difficulties Setting: 53 classrooms in 13 schools†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 2.50* Rational numbers magnitude understanding/ relative magnitude understanding: 1.46* Rational numbers knowledge: 0.92*
Fuchs, Schumacher, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in solving multiplicative or additive word problems vs. control Participants: 213 grade 4 students with mathematics difficulties Setting: 45 classrooms in 14 schools†	Duration: 35-minute sessions; 3 sessions per week; 12 weeks Content: Fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.34* Rational numbers magnitude understanding/ relative magnitude understanding: 0.80* Rational numbers knowledge: 0.34*
Fuchs et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions knowledge intervention with fluency activities or conceptual activities vs. control Participants: 243 grade 4 students with mathematics difficulties Setting: 49 classrooms in 14 schools in 1 urban school district†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction	Rational numbers computation: 1.33* Rational numbers magnitude understanding/ relative magnitude understanding: 1.08* Rational numbers knowledge: 0.58*

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Seethaler, et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge intervention vs. control Participants: 196 grade 1 students with mathematics difficulties Setting: 186 classrooms in 21 schools†	Duration: 30-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations.	Business as usual core mathematics instruction	Whole numbers computation: 0.59*
Gersten et al. (2015) *Meets WWC standards with reservations*	Design: Cluster RCT Contrast: Number operations intervention vs. control Participants: 881 grade 1 students with mathematics difficulties Setting: 76 schools in 4 urban school districts in 4 states in the south-central and southwest regions of the U.S.	Duration: 40-minute sessions; 3-4 times per week; 17 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations.	Business as usual core mathematics instruction	General mathematics achievement: 0.34*
Jayanthi et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Small-group fractions intervention vs. control Participants: 186 grade 5 students with mathematics difficulties Setting: 3 school districts in the west and southeast regions of the U.S.	Duration: 35-minute sessions; 3-4 times per week; 6-7 months Content: Fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.07* Rational numbers magnitude understanding/ relative magnitude understanding: 0.94* Rational numbers knowledge: 0.72*

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Kanive et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Conceptual understanding intervention vs. control Participants: 57 grade 4 and 5 students with mathematics difficulties Setting: 1 school in Minnesota†	Duration: 15-minute sessions; 1 time per week; 2 weeks Content: Whole numbers Relevance to Recommendation: Students learned how to use representations to model mathematical thinking.	Business as usual core mathematics instruction	Whole numbers computation: 0.29
Malone et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction or decimal magnitude instruction vs. control Participants: 225 grade 4 students with mathematics difficulties Setting: 58 classrooms in 12 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions and decimals Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Business as usual core mathematics instruction	Rational numbers computation: 1.53* Rational numbers magnitude understanding/ relative magnitude understanding: 0.48* Rational numbers knowledge: 0.12
Powell and Driver (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Addition tutoring (with or without embedded vocabulary component) vs. control Participants: 98 grade 1 students with mathematics difficulties Setting: 58 classrooms in 18 schools in 2 school districts in the mid-Atlantic region of the U.S.†	Duration: 10- to 15-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking.	Business as usual core mathematics instruction	Whole numbers computation: 0.34

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Powell, Driver, et al. (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Standard equations tutoring or combined (standard and nonstandard) equations tutoring vs. control Participants: 51 grade 2 students with mathematics difficulties Setting: 31 classrooms in 10 schools in 2 school districts in the mid-Atlantic region of the U.S.†	Duration: 10- to 15-minute sessions; 3 times per week; 4 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking.	Business as usual core mathematics instruction	Whole numbers computation: 0.23 Algebra and algebraic reasoning: 0.80*
Powell, Fuchs, et al. (2015) *Meets WWC standards with reservations*	Design: Cluster QED Contrast: Calculation intervention vs. control Participants: 174 grade 2 students with mathematics difficulties Setting: 110 classrooms in 25 schools in 1 urban school district†	Duration: Tier 1 portion: 40- to 45-minute sessions; 2 times per week; 17 weeks; Tier 2 portion (beginning week 4 of the Tier 1 portion): 25- to 30-minute sessions; 3 times per week Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking.	Business as usual core mathematics instruction	Whole numbers computation: 1.19*
Smith et al. (2013) *Meets WWC standards with reservations*	Design: RCT Contrast: Intensive one-to-one tutoring in arithmetical knowledge vs. control Participants: 775 grade 1 students with mathematics difficulties Setting: 20 elementary schools in 5 school districts in 2 states†	Duration: 30-minute sessions; 4-5 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations	Business as usual core mathematics instruction	Whole numbers computation: 0.14* General mathematics achievement: 1.82*

Recommendation 3				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Wang et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction (with or without self-regulation) vs. control Participants: 84 grade 3 students with mathematics difficulties Setting: 29 classrooms in 8 schools in 1 urban school district[†]	Duration: 35-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking.	Business as usual core mathematics instruction and any school-provided intervention	Whole numbers computation: 0.59* Rational numbers computation: 1.27* Rational numbers magnitude understanding/ relative magnitude understanding: 0.95* Rational numbers knowledge: 0.85*
Watt and Therrien (2016) *Meets WWC standards with reservations*	Design: RCT Contrast: Pre-teaching plus concrete-representational-abstract instructional sequence vs. supplemental reading instruction Participants: 32 grade 6 students with mathematics difficulties Setting: 4 classrooms in 2 schools	Duration: 30-minute sessions; 5 times per week; 2 weeks Content: Fractions Relevance to Recommendation: Students connected concrete and/or semi-concrete representations to abstract representations and learned to use representations to model mathematical thinking. Instruction reduced emphasis on concrete and semi-concrete representations as students began to understand abstract representations.	Small-group supplemental reading instruction	Algebra and algebraic reasoning: 0.09

Note: Each row in this table represents a study, defined by the WWC as an examination of the effect of an intervention on a distinct sample.

[a] Sample size represents the maximum number of participants in the study. In some studies, the number of participants varied across the outcome measures.

[b] Effect sizes presented are from the posttest closest to the end of the intervention. For brevity, only the domain average effect size and statistical significance are reported in this table. For studies that included multiple outcomes in a domain, reported effect sizes and statistical significance are for the domain and calculated as described in the WWC Procedures Handbook (version 4.0).

* Significant at $p \leq 0.05$.

[†] Indicates that the information is for the entire study (across all conditions).

Supplemental Findings for Recommendation 3

Supplemental findings (including impacts for follow-up, sub-scale, and distal measures) for eight studies are available at the corresponding study pages on the WWC website.[183]

Recommendation 4: Number Lines

Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.

Rationale for a Strong Level of Evidence

The WWC and the expert panel assigned **Recommendation 4** a strong level of evidence based on 14 studies.[184] Together, these studies have strong internal validity. Eleven studies meet WWC group design standards without reservations because they were RCTs with low sample attrition.[185] Three studies meet WWC group design standards with reservations because they were either compromised RCTs or RCTs with high sample attrition, but the analytic intervention and comparison groups in each satisfied the baseline equivalence requirement.[186] In addition, the 14 studies demonstrate strong external validity as their samples collectively included 3,331 students and 246 schools across multiple states.[187]

Across the 14 studies, there were findings in 6 key outcome domains for this recommendation (**Table C.9**). Five of these domains had statistically significant, positive effect sizes: general mathematics achievement ($g = 0.34$, $p < 0.01$), rational numbers computation ($g = 1.46$, $p < 0.01$), rational numbers knowledge ($g = 0.62$, $p < 0.01$), rational numbers magnitude understanding/relative magnitude understanding ($g = 1.00$, $p < 0.01$), and whole numbers computation ($g = 0.62$, $p < 0.01$). The other domain (whole numbers magnitude understanding/relative magnitude understanding) did not have a statistically significant effect size.

Table C.9. Domain-level effect sizes across the 14 studies supporting Recommendation 4

Domain	Number of studies (*k*)	Effect size[a]	95% Confidence interval	*p* Value
General Mathematics Achievement	1	**0.34**	NA	< 0.01
Rational Numbers Computation	10	**1.46**	[1.35–1.58]	< 0.01
Rational Numbers Knowledge	10	**0.62**	[0.51–0.72]	< 0.01
Rational Numbers Magnitude Understanding/Relative Magnitude Understanding	9	**1.00**	[0.89–1.11]	< 0.01
Whole Numbers Computation	4	**0.62**	[0.48–0.75]	< 0.01
Whole Numbers Magnitude Understanding/ Relative Magnitude Understanding	1	−0.05	NA	*ns*

Note: All effect sizes were calculated using a fixed-effects meta-analytic effect size across studies except for the general mathematics achievement and whole numbers magnitude understanding/relative magnitude understanding domains. These two domains had findings from just one study each; the effect sizes presented here are the WWC-calculated domain-level average effect sizes for the individual relevant study for each domain. *ns* = nonsignificant findings; NA = not applicable; *k* = number of studies with at least one outcome in the relevant domain and contributed to the meta-analytic effect size.

[a] Significant findings are bolded.

The 14 studies relevant to this recommendation have a preponderance of positive evidence on relevant outcomes, strong internal and external validity, and are closely aligned with the practices outlined in the recommendation. Therefore, the WWC and the panel determined that the recommendation receives a **strong evidence rating**. This rating is supported by the strength of the evidence according to the following criteria:

- **Consistency of effects on relevant outcomes.** Five of the six relevant outcome domains with findings had statistically significant, positive effect sizes. The sixth domain had uncertain effects (it did not have a statistically significant effect size). No domains had statistically significant, negative effect sizes.
- **Extent of evidence.** The 14 studies related to this recommendation demonstrated positive effects with a medium to large extent of evidence. Four of the six relevant domains (rational numbers computation, rational numbers knowledge, rational numbers magnitude understanding/relative magnitude understanding, and whole numbers computation) had statistically significant, positive meta-analytic effect sizes, with more than 50 percent of the meta-analytic weight from studies that meet WWC standards without reservations, and had samples of more than 350 students and multiple districts and states. These four domains represent a preponderance of the key outcome domains with findings for this recommendation.
- **Relationship between the evidence and recommendation.** The 14 studies supporting this recommendation exhibit a strong relationship between the evidence and recommended practices because all 14 studies include at least one of the recommended practices as a major component (see below for more information). Multiple studies supported each How-to Step in the recommendation.
- **Relevancy.** The 14 studies supporting this recommendation have relevant samples, contexts, comparisons, and outcomes. All studies include samples of students with, or at-risk for, mathematics difficulties in grades 1 through 6; examined interventions that were implemented as a supplement to Tier 1 instruction or in a resource room; and measured outcomes in relevant domains. The interventions ranged from roughly 6 weeks to 6–7 months in duration.

A Brief Summary of the Studies Providing Evidence for the Recommendation

The 14 studies were directly related to the recommendation, with all studies testing interventions that included the recommended practices as a major component. All 14 studies used three-dimensional, concrete representations (e.g., connecting cubes, fraction tiles) or semi-concrete representations (e.g., visual diagrams, figures, pictures) to help students understand how number lines work and represent numerical magnitude. Four studies examined interventions that addressed whole-numbers concepts,[188] eight addressed rational-numbers concepts,[189] and two studies addressed both.[190]

Multiple studies related to each of the recommendation's How-to Steps. In 13 studies, teachers used number lines to represent numbers and their magnitude (How-to Step 1).[191] Students were taught to estimate and compare the relative magnitude of numbers using number lines in 10 studies (How-to Step 2).[192] In 2 studies, number lines were used to model mathematical operations (How-to Step 3).[193]

Table C.10. Studies providing evidence for Recommendation 4: Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics

Recommendation 4				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Barbieri et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions intervention vs. control Participants: 51 grade 6 students with mathematics difficulties Setting: 7 classrooms in 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction	Rational numbers computation: 0.17 Rational numbers knowledge: 1.09*
Dyson et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Fraction sense intervention vs. control Participants: 52 grade 6 students with mathematics difficulties Setting: 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 0.48* Rational numbers magnitude understanding/ relative magnitude understanding: 0.90* Rational numbers knowledge: 0.99*
Fuchs, Geary, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge tutoring (with speeded or non-speeded practice) vs. control Participants: 591 grade 1 students with mathematics difficulties Setting: 233 classrooms in 40 schools in 1 urban school district in the southeast region of the U.S.[†]	Duration: 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students represented numbers on a number line.	Business as usual core mathematics instruction	Whole numbers computation: 0.63* Whole numbers magnitude understanding/ relative magnitude understanding: −0.05

Recommendation 4				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Malone, et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions magnitude intervention (with or without error analysis) vs. control Participants: 143 grade 4 and 5 students with mathematics difficulties Setting: 49 classrooms in 13 schools in 1 urban school district in the southeast region of the U.S.†	Duration: 40-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.72* Rational numbers magnitude understanding/ relative magnitude understanding: 1.35* Rational numbers knowledge: 0.05
Fuchs, Malone, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in providing explanations or solving word problems vs. control Participants: 212 grade 4 students with mathematics difficulties Setting: 52 classrooms in 14 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.79* Rational numbers magnitude understanding/ relative magnitude understanding: 0.93* Rational numbers knowledge: 0.86*
Fuchs, Schumacher, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions understanding intervention vs. control Participants: 259 grade 4 students with mathematics difficulties Setting: 53 classrooms in 13 schools†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 2.50* Rational numbers magnitude understanding/ relative magnitude understanding: 1.46* Rational numbers knowledge: 0.92*

Recommendation 4				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Schumacher, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in solving multiplicative or additive word problems vs. control Participants: 213 grade 4 students with mathematics difficulties Setting: 45 classrooms in 14 schools†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.34* Rational numbers magnitude understanding/ relative magnitude understanding: 0.80* Rational numbers knowledge: 0.34*
Fuchs et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions knowledge intervention with fluency activities or conceptual activities vs. control Participants: 243 grade 4 students with mathematics difficulties Setting: 49 classrooms in 14 schools in 1 urban school district†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction	Rational numbers computation: 1.33* Rational numbers magnitude understanding/ relative magnitude understanding: 1.08* Rational numbers knowledge: 0.58*
Fuchs, Seethaler, et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge intervention vs. control Participants: 196 grade 1 students with mathematics difficulties Setting: 186 classrooms in 21 schools†	Duration: 30-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students represented numbers on a number line.	Business as usual core mathematics instruction	Whole numbers computation: 0.59*

Recommendation 4				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Gersten et al. (2015) *Meets WWC standards with reservations*	Design: Cluster RCT Contrast: Number operations intervention vs. control Participants: 881 grade 1 students with mathematics difficulties Setting: 76 schools in 4 urban school districts in 4 states in the south-central and southwest regions of the U.S.	Duration: 40-minute sessions; 3-4 times per week; 17 weeks Content: Whole numbers Relevance to Recommendation: Students represented numbers on a number line.	Business as usual core mathematics instruction	General Mathematics Achievement: 0.34*
Jayanthi et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Small-group fractions intervention vs. control Participants: 186 grade 5 students with mathematics difficulties Setting: 3 school districts in the west and southeast regions of the U.S.	Duration: 35-minute sessions; 3-4 times per week; 6-7 months Content: Fractions Relevance to Recommendation: Students used a number line to represent and compare numbers. Students also learned to represent operations using number lines.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.07* Rational numbers magnitude understanding/ relative magnitude understanding: 0.94* Rational numbers knowledge: 0.72*
Malone et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with decimal magnitude instruction vs. control Participants: 152 grade 4 students with mathematics difficulties Setting: 58 classrooms in 12 schools in 1 school district[†]	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions and decimals Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction	Rational numbers computation: 1.55* Rational numbers magnitude understanding/ relative magnitude understanding: 0.41* Rational numbers knowledge: 0.28
Powell et al. (2009) *Meets WWC standards without reservations*	Design: RCT Contrast: Conceptual fact retrieval instruction with practice vs. control Participants: 68 grade 3 students with mathematics difficulties Setting: 75 classrooms in 17 schools in Nashville, TN and Houston, TX[†]	Duration: 22- to 25-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students learned to represent operations using number lines.	Business as usual core mathematics instruction	Whole numbers computation: 0.62*

Recommendation 4				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Wang et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction (with or without self-regulation) vs. control Participants: 84 grade 3 students with mathematics difficulties Setting: 29 classrooms in 8 schools in 1 urban school district[†]	Duration: 35-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students used a number line to represent and compare numbers.	Business as usual core mathematics instruction and any school-provided intervention	Whole numbers computation: 0.59* Rational numbers computation: 1.27* Rational numbers magnitude understanding/ relative magnitude understanding: 0.95* Rational numbers knowledge: 0.85*

Note: Each row in this table represents a study, defined by the WWC as an examination of the effect of an intervention on a distinct sample.

[a] Sample size represents the maximum number of participants in the study. In some studies, the number of participants varied across the outcome measures.

[b] Effect sizes presented are from the posttest closest to the end of the intervention. For brevity, only the domain average effect size and statistical significance are reported in this table. For studies that included multiple outcomes in a domain, reported effect sizes and statistical significance are for the domain and calculated as described in the WWC Procedures Handbook (version 4.0).

* Significant at $p \leq 0.05$.

[†] Indicates that the information is for the entire study (across all conditions).

Supplemental Findings for Recommendation 4

Supplemental findings (including impacts for follow-up, sub-scale, and distal measures) for three studies are available at the corresponding study pages on the WWC website.[194]

Recommendation 5: Word Problems

Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.

Rationale for a Strong Level of Evidence

The WWC and the expert panel assigned **Recommendation 5** a strong level of evidence based on 18 studies that meet WWC standards and had outcomes in the relevant domains for this recommendation.[195] The studies collectively have strong internal validity. Fifteen studies meet WWC group design standards without reservations because they were RCTs with low sample attrition.[196] Three studies meet WWC group design standards with reservations because they were either compromised RCTs or RCTs with high sample attrition, but the analytic intervention and comparison groups in each satisfied the baseline equivalence requirement.[197] In addition, the 18 studies have strong external validity as their samples collectively include 1,751 students and 153 schools.[198]

There were two key outcome domains for this recommendation (**Table C.11**). Across the 18 studies, the fixed-effects meta-analytic effect size was statistically significant and positive for both domains: rational numbers word problems/problem solving ($g = 0.93$, $p < 0.01$) and whole numbers word problems/problem solving ($g = 0.54$, $p < 0.01$).

Table C.11. Domain-level effect sizes across the 18 studies supporting Recommendation 5

Domain	Number of studies (*k*)	Effect size[a]	95% Confidence interval	*p* Value
Rational Numbers Word Problems/Problem Solving	4	**0.93**	[0.74–1.11]	< 0.01
Whole Numbers Word Problems/Problem Solving	14	**0.54**	[0.43–0.65]	< 0.01

Note: All effect sizes were calculated using a fixed-effects meta-analytic effect size across studies. *k* = number of studies with at least one outcome in the relevant domain and contributed to the meta-analytic effect size.
[a] Significant findings are bolded.

The 18 studies relevant to this recommendation have a preponderance of positive evidence, strong internal and external validity, and are closely aligned with the practices outlined in the recommendation. Therefore, the WWC and the panel determined that the recommendation receives a **strong evidence rating**. This rating is supported by the strength of the evidence according to the following criteria:

- **Consistency of effects on relevant outcomes.** Of the two relevant domains with findings for this recommendation, both have a significant, positive meta-analytic effect size. Neither domain has a negative, statistically significant meta-analytic effect size.
- **Extent of evidence.** The 18 studies supporting this recommendation demonstrated positive effects with a medium to large extent of evidence. One of the two relevant domains (whole numbers word problems/problem solving) had a statistically significant, positive meta-analytic effect size, with more than 50 percent of the meta-analytic weight from studies that meet WWC standards without reservations, and had a sample of more than 350 students and multiple districts and states. This domain represents half of the relevant domains for the recommendation.

- **Relationship between the evidence and recommendation.** The 18 studies supporting this recommendation exhibit a strong relationship between the evidence and recommended practices because all 18 studies include at least one of the recommended practices as a major component (see below for more information).
- **Relevancy.** The 18 studies supporting this recommendation have relevant samples, contexts, comparisons, and outcomes. All studies have samples of students with, or at-risk for mathematics difficulties in grades 1 through 5; examined interventions that were implemented as a supplement to Tier 1 instruction or in a resource room; and measured outcomes in relevant domains. The interventions ranged from roughly 3 weeks to 16 weeks in duration.

A Brief Summary of the Studies Providing Evidence for the Recommendation

The 18 studies were directly related to the recommendation, with all studies testing interventions that included the recommended practices as a major component. In most of the studies solving word problems was the focus of the entire intervention, while in four studies,[199] word problems were considered a major component because they were included in every lesson as part of a multi-component intervention that also focused on other mathematics concepts. Fourteen studies examined interventions that addressed whole-numbers concepts,[200] three studies that addressed rational-numbers concepts,[201] and one study that addressed both.[202]

Multiple studies related to each of the recommendation's How-to Steps. In twelve studies,[203] students were taught to identify word problem types based on the underlying mathematical structure (How-to Step 1) and to apply a solution strategy based on that word problem type (How-to Step 2). This approach is referred to in the research as schema-based instruction. In these twelve studies, students' understanding of each problem type was expanded by presenting problem information differently, changing which quantity was unknown, or including problems with more than one step (How-to Step 3). Additionally, in seven of these studies,[204] instruction on difficult language used in word problems was incorporated into the intervention (How-to Step 4).

The remaining six studies used other approaches for teaching students to comprehend and solve word problems that are included in How-to Step 3.[205] One study taught students to identify the operation and to restate the problem in their own words.[206] Two studies looked at restating the problem and identifying relevant and irrelevant information.[207] Three studies looked at use of cognitive strategies to understand and set up solutions.[208] Thirteen studies included a mix of previously learned and newly learned problems throughout the intervention (How-to Step 5).[209]

Table C.12. Studies providing evidence for Recommendation 5: Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas

Recommendation 5				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Darch et al. (1984) *Meets WWC standards without reservations*	Design: RCT Contrast: Explicit instruction (with or without extended practice) vs. basal instruction (with or without extended practice) Participants: 73 grade 4 students with mathematics difficulties Setting: 6 classrooms in 1 school district in Oregon	Duration: 30-minute sessions; 11-19 sessions total Content: Whole numbers Relevance to Recommendation: Students learned to discriminate between word problem operations and solve them.	Instruction based on materials developed from four basal programs, and additional practice lessons for some students	Whole numbers word problems/ problem solving: 1.43*
Fuchs, Fuchs, et al. (2008) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem tutoring in Tier 2 (with or without word problem intervention in Tier 1) vs. no word problem tutoring in Tier 2 (with or without word problem intervention in Tier 1) Participants: 243 grade 3 students with mathematics difficulties Setting: 120 classrooms in 1 urban school district in the southeast region of the U.S.†	Duration: 20- to 30-minute sessions, 3 times per week; 13 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction, and word problem instruction as part of core instruction for some students	Whole numbers word problems/ problem solving: 0.95*

Recommendation 5				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Malone, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in solving multiplicative word problems vs. control Participants: 139 grade 4 students with mathematics difficulties Setting: 52 classrooms in 14 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction	Rational numbers word problems/ problem solving: 1.19*
Fuchs et al. (2009) *Meets WWC standards without reservations*	Design: RCT Contrast: Tutoring in solving word problems vs. control Participants: 89 grade 3 students with mathematics difficulties Setting: 63 classrooms in 18 schools in 2 school districts in Nashville, TN and Houston, TX†	Duration: 20- to 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction	Whole numbers word problems/ problem solving: 0.57*
Fuchs et al. (2010) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction (with or without strategic counting practice) vs. control Participants: 150 grade 3 students with mathematics difficulties Setting: 31 schools in 2 urban school districts in Nashville, TN and Houston, TX†	Duration: 20- to 30-minute sessions, 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction	Whole numbers word problems/ problem solving: 0.52*

Recommendation 5				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Schumacher, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in solving multiplicative word problems vs. control Participants: 142 grade 4 students with mathematics difficulties Setting: 45 classrooms in 14 schools†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers word problems/ problem solving: 1.08*
Fuchs, Seethaler, et al. (2008) *Meets WWC standards with reservations*	Design: RCT Contrast: Tutoring in solving word problems vs. control Participants: 35 grade 3 students with mathematics difficulties Setting: 18 classrooms in 1 urban school district in the southeast region of the U.S.	Duration: 20- to 30-minute sessions; 3 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction	Whole numbers word problems/ problem solving: 0.97*
Fuchs, Seethaler, et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem intervention (with or without language instruction) vs. control Participants: 299 grade 1 students with mathematics difficulties Setting: 186 classrooms in 21 schools†	Duration: 30-minute sessions; 3 sessions per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction and any school-provided intervention	Whole numbers word problems/ problem solving: 0.49*

Recommendation 5				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Jitendra, Dupuis, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem intervention vs. control Participants: 109 grade 3 students with mathematics difficulties Setting: 28 classrooms in 9 schools in 1 large urban school district in the Midwest region of the U.S.	Duration: 30-minute sessions, 5 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Small-group tutoring in topics selected from the core mathematics curriculum (place value, whole numbers addition and subtraction computation strategies, and word problem solving)	Whole numbers word problems/ problem solving: 0.46*
Jitendra et al. (1998) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction vs. control Participants: 34 students in grades 2–5 with mathematics disabilities or difficulties Setting: 4 classrooms in 4 schools in the northeast and southeast regions of the U.S.	Duration: 40- to 45-minute sessions; 17-20 sessions total Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Instruction based on a basal mathematics program	Whole numbers word problems/ problem solving: 0.63
Jitendra, Rodriguez, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem intervention vs. control Participants: 136 grade 3 students with mathematics difficulties Setting: 35 classrooms in 12 schools in 1 urban school district in the Midwest region of the U.S.	Duration: 30-minute sessions; 5 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Small-group tutoring in topics selected from the core mathematics curriculum (place value, addition and subtraction, and word problem solving)	Whole numbers word problems/ problem solving: 0.02*

Recommendation 5				
Study and WWC rating	**Study description**[a]	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size**[b]
Malone et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in additive word problems vs. control Participants: 149 grade 4 students with mathematics difficulties Setting: 58 classrooms in 12 schools in 1 school district in a large U.S. city†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction	Rational numbers word problems/ problem solving: 0.69*
Swanson (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction—verbal plus visual strategies condition vs. control Participants: 33 grade 3 students with mathematics difficulties Setting: 22 classrooms in 2 schools in 1 school district in the southwest region of the U.S.†	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify relevant and irrelevant information and determine what operation was needed.	Business as usual core mathematics instruction	Whole numbers word problems/ problem solving: 0.04
Swanson, Lussier, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction with heuristic strategy plus visual schematic diagrams vs. control Participants: 38 grade 3 students with mathematics difficulties Setting: 21 classrooms†	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify relevant and irrelevant information and determine what operation was needed.	Business as usual core mathematics instruction	Whole numbers word problems/ problem solving: 0.57

Recommendation 5				
Study and WWC rating	**Study description**[a]	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size**[b]
Swanson, Moran, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction - complete condition vs. control (students with mathematics difficulties only) Participants: 33 grade 3 students with mathematics difficulties Setting: 12 classrooms in 4 schools in 2 school districts in the southwest region of the U.S.†	Duration: 30-minute sessions; 2 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Students learned to restate (paraphrase) the word problems and identify and restate relevant and irrelevant information from the problem.	Small-group tutoring in topics selected from the core mathematics curriculum	Whole numbers word problems/ problem solving: 0.39
Swanson, Moran, et al. (2014) *Meets WWC standards with reservations*	Design: RCT Contrast: Word problem instruction - complete condition vs. control (students with mathematics difficulties only) Participants: 45 grade 3 students with mathematics difficulties Setting: 12 classrooms in 4 schools in 2 school districts in the southwest region of the U.S.†	Duration: 30-minute sessions; 2 times per week; 10 weeks Content: Whole numbers Relevance to Recommendation: Students learned to restate (paraphrase) the word problems and identify and restate relevant and irrelevant information from the problem.	Small-group instruction in topics selected from the core mathematics curriculum	Whole numbers word problems/ problem solving: 0.14
Swanson, Orosco, et al. (2014) *Meets WWC standards with reservations*	Design: RCT Contrast: Word problem instruction - material, verbal, and visual strategies condition vs. materials-only condition Participants: 29 grade 3 students with mathematics difficulties Setting: 18 classrooms in 1 school district in California†	Duration: 30-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students learned to identify relevant and irrelevant information and determine what operation was needed.	The same word problem instruction as the intervention condition without the specific strategy instruction	Whole numbers word problems/ problem solving: −0.01

Recommendation 5				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Wang et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction (with or without self-regulation) vs. control Participants: 84 grade 3 students with mathematics difficulties Setting: 29 classrooms in 8 schools in 1 urban school district[†]	Duration: 35-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students learned to identify word problems based on their underlying mathematical structure and apply a solution method based on the type of word problem.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers word problems/ problem solving: 0.69*

Note: Each row in this table represents a study, defined by the WWC as an examination of the effect of an intervention on a distinct sample.

[a] Sample size represents the maximum number of participants in the study. In some studies, the number of participants varied across the outcome measures.

[b] Effect sizes presented here are from the posttest closest to the end of the intervention. For brevity, only the domain average effect size and statistical significance are reported in this table. For studies that included multiple outcomes in a domain, reported effect sizes and statistical significance are for the domain and calculated as described in the WWC Procedures Handbook (version 4.0).

* Significant at $p \leq 0.05$.

[†] Indicates that the information is for the entire study (across all conditions).

Supplemental Findings for Recommendation 5

Supplemental findings (including impacts for follow-up, sub-scale, and distal measures) for four studies are available at the corresponding study pages on the WWC website.[210]

Three multiple contrasts studies included treatment-versus-treatment contrasts that were reviewed to provide supplemental support for some of the How-to Steps in this recommendation.[211] In one study,[212] the contrast looked at whether embedding word problem language instruction improved word-problem performance. Findings demonstrated a significant and positive effect ($g = 0.47$). In two studies,[213] the contrast looked at whether teaching students a strategy for identifying relevant information in a word problem improved performance, compared to students who were only taught to restate the word problem before solving it. Findings were not significant in either study and are therefore inconclusive ($g = 0.38$ and $g = 0.07$).

Recommendation 6: Timed Activities

Regularly include timed activities to build students' retrieval of basic facts and fluent use of critical steps for more complex mathematics.

Rationale for a Strong Level of Evidence

The WWC and the expert panel assigned **Recommendation 6** a strong level of evidence based on 27 studies.[214] Collectively, the studies have strong internal validity. Twenty-one studies meet WWC group design standards without reservations because they were RCTs with low sample attrition.[215] Six studies meet WWC group design standards with reservations because they were either not as well-implemented RCTs, RCTs with high sample attrition, or QEDs, but the analytic intervention and comparison groups in each satisfied the baseline equivalence requirement.[216] In addition, the 27 studies demonstrate strong external validity as their samples collectively included 4,336 students and 403 schools.[217]

Across the 27 studies, there were findings in nine of the key outcome domains for this recommendation (**Table C.13**). The effect sizes for all nine of these domains were statistically significant and positive: algebra and algebraic reasoning ($g = 0.55$, $p < 0.01$), counting and cardinality ($g = 0.27$, $p < 0.05$), general mathematics achievement ($g = 0.35$, $p < 0.01$), rational numbers computation ($g =1.55$, $p < 0.01$), rational numbers knowledge ($g = 0.57$, $p < 0.01$), rational numbers magnitude understanding/relative magnitude understanding ($g = 0.97$, $p < 0.01$), whole numbers computation ($g = 0.64$, $p < 0.01$), whole numbers knowledge ($g = 0.09$, $p < 0.05$) and whole numbers magnitude understanding/relative magnitude understanding ($g = 0.26$, $p < 0.01$).

Table C.13. Domain-level effect sizes across the 27 studies supporting Recommendation 6

Domain	Number of studies (*k*)	Effect size[a]	95% Confidence interval	*p* Value
Algebra and Algebraic Reasoning	3	**0.55**	[0.29–0.81]	< 0.01
Counting and Cardinality	2	**0.27**	[0.04–0.49]	< 0.05
General Mathematics Achievement	5	**0.35**	[0.24–0.47]	< 0.01
Rational Numbers Computation	9	**1.55**	[1.42–1.67]	< 0.01
Rational Numbers Knowledge	9	**0.57**	[0.46–0.68]	< 0.01
Rational Numbers Magnitude Understanding/Relative Magnitude Understanding	8	**0.97**	[0.86–1.09]	< 0.01
Whole Numbers Computation	16	**0.64**	[0.54–0.74]	< 0.01
Whole Numbers Knowledge	1	**0.09**	NA	< 0.05
Whole Numbers Magnitude Understanding/ Relative Magnitude Understanding	3	**0.26**	[0.11–0.41]	< 0.01

Note: All effect sizes were calculated using a fixed-effects meta-analytic effect size across studies except for the whole numbers knowledge domain. This domain had findings from just one study; the effect size presented here is the WWC-calculated domain-level average effect size for the individual relevant study. NA = not applicable; *k* = number of studies with at least one outcome in the relevant domain and contributed to the meta-analytic effect size.

[a] Significant findings are bolded.

The 27 studies relevant to this recommendation have a preponderance of positive evidence, strong internal and external validity, and are closely aligned with the practices outlined in the recommendation. Therefore, the WWC and the panel determined that the recommendation receives a **strong evidence rating**. This rating is supported by the strength of the evidence according to the following criteria:

- **Consistency of effects on relevant outcomes.** Across the nine relevant domains with findings from studies that meet WWC standards, all have a statistically significant positive meta-analytic effect size. No domains have statistically significant, negative effect sizes.
- **Extent of evidence.** The 27 studies related to this recommendation demonstrated positive effects with a medium to large extent of evidence. Five of the nine relevant domains (rational numbers computation, rational numbers knowledge, rational numbers magnitude understanding/relative magnitude understanding, whole numbers computation, whole numbers magnitude understanding/relative magnitude understanding) had statistically significant, positive meta-analytic effect sizes, with more than 50 percent of the meta-analytic weight from studies that meet WWC standards without reservations, and had samples of more than 350 students and multiple districts and states. These five domains represent a preponderance of the relevant domains with findings for this recommendation.
- **Relationship between the evidence and recommendation.** The 27 studies supporting this recommendation exhibit a strong relationship between the evidence and recommended practices because all 27 studies include at least one of the recommended practices as a major component (see below for more information).
- **Relevancy.** The 27 studies supporting this recommendation have relevant samples, contexts, comparisons, and outcomes. The studies included samples of students with, or at-risk for, mathematics difficulties in kindergarten through grade 6; examined interventions that were implemented as a supplement to Tier 1 instruction or in a resource room; and measured outcomes in relevant domains. The interventions ranged from roughly 8 days to 19 weeks in duration. Most studies had interventions of substantial length, as 22 studies' interventions lasted at least 8 weeks.[218]

A Brief Summary of the Studies Providing Evidence for the Recommendation

The 27 studies were directly related to the recommendation, with all studies testing interventions that included the recommended practices as a major component. Eighteen studies examined interventions that addressed whole-numbers concepts,[219] seven addressed rational-numbers concepts,[220] and two focused on both whole-number and rational-numbers concepts.[221]

All the studies included timed fluency activities within intervention. Eighteen studies focused on fluency of mathematics facts (sometimes called number combinations),[222] five studies focused on fluent retrieval of other important mathematical information material (e.g., fractions equivalent to one-half),[223] and four studies focused on both.[224] Two studies where the intervention focused only on fractions concepts,[225] fluency activities focused on whole numbers basic mathematics facts. This was the only part of the lesson not focused on fractions.

In 19 studies,[226] students were taught an efficient strategy for solving a problem that helped them solve fluency-building activities (How-to Step 3). Students tracked their progress in 17 studies (How-to Step 4).[227] In 26 studies,[228] students received immediate feedback from the teacher or computer (How-to Step 5). The first and second How-to Steps advise teachers on how to select fluency-building topics and how to choose the activity and materials for implementation. These steps are based on panel advice.

Table C.14. Studies providing evidence for Recommendation 6: Regularly include timed activities to build students' retrieval of basic facts and fluent use of critical steps for more complex mathematics

Recommendation 6				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Barbieri et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions intervention vs. control Participants: 51 grade 6 students with mathematics difficulties Setting: 7 classrooms in 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Students participated in activities to support their fluency development and received immediate corrective feedback.	Business as usual core mathematics instruction	Rational numbers computation: 0.17 Rational numbers knowledge: 1.09*
Bryant et al. (2011) *Meets WWC standards without reservations*	Design: RCT Contrast: Early numeracy tutoring vs. control Participants: 203 grade 1 students with mathematics difficulties Setting: 50 classrooms in 10 schools in 1 school district in Texas	Duration: 25-minute sessions; 4 times per week; 19 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development.	Business as usual core mathematics instruction	General mathematics achievement: 0.50*
Dyson et al. (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Number sense intervention with number-fact practice vs. control Participants: 86 kindergarten students with mathematics difficulties Setting: 4 schools in 2 school districts†	Duration: 30-minute sessions. 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received corrective feedback.	Business as usual core mathematics instruction	Counting and cardinality: 0.82* Whole numbers computation: 0.69*
Dyson et al. (2018) *Meets WWC standards without reservations*	Design: RCT Contrast: Fraction sense intervention vs. control Participants: 52 grade 6 students with mathematics difficulties Setting: 2 schools in the northeast region of the U.S.	Duration: 45-minute sessions; 5 times per week; 6 weeks Content: Fractions Relevance to Recommendation: Students participated in activities to support their fluency development and received immediate corrective feedback.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 0.48* Rational numbers magnitude understanding/ relative magnitude understanding: 0.90* Rational numbers knowledge: 0.99*

Recommendation 6				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fien et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Whole-number concepts intervention vs. control Participants: 238 grade 1 students with mathematics difficulties Setting: 26 classrooms in 9 schools in 2 suburban school districts in Eugene and Portland, OR	Duration: 15-minute sessions; 4 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Using a computer-based program, students participated in activities to support their fluency development and received corrective feedback.	Business as usual core mathematics instruction and any school-provided intervention	Counting and cardinality: 0.08 Whole numbers magnitude understanding/ relative magnitude understanding: 0.07 Whole numbers knowledge: 0.09*
Fuchs et al. (2005) *Meets WWC standards without reservations*	Design: RCT Contrast: Preventive mathematics tutoring vs. control Participants: 127 grade 1 students with mathematics difficulties Setting: 41 classrooms in 10 schools in 1 urban school district in the southeast region of the U.S.	Duration: 40-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Using a computer-based program, students participated in activities to support their fluency development and received immediate corrective feedback.	Business as usual core mathematics instruction	Whole numbers computation: 0.23 General mathematics achievement: 0.38*
Fuchs et al. (2006) *Meets WWC standards without reservations*	Design: RCT Contrast: Computer-assisted instruction for number combination skill vs. irrelevant control (computer-assisted spelling instruction) Participants: 33 grade 1 students with mathematics difficulties Setting: 9 classrooms in 3 schools in 1 urban school district	Duration: 10-minute sessions; 3 times per week; 18 weeks Content: Whole numbers Relevance to Recommendation: Using a computer-based program, students participated in activities to support their fluency development and received immediate corrective feedback.	Computer-assisted instruction (CAI) similar to intervention condition but focused on presenting spelling words instead of number combinations	Whole numbers computation: 0.39

Recommendation 6				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Geary, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge tutoring with speeded practice vs. control Participants: 401 grade 1 students with mathematics difficulties Setting: 233 classrooms in 40 schools in 1 urban school district in the southeast region of the U.S.†	Duration: 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction	Whole numbers computation: 0.78* Whole numbers magnitude understanding/ relative magnitude understanding: 0.29*
Fuchs, Malone, et al. (2019) *Meets WWC standards with reservations*	Design: RCT Contrast: Fractions magnitude intervention (with or without error analysis) vs. control Participants: 143 grade 4 and 5 students with mathematics difficulties Setting: 49 classrooms in 13 schools in 1 urban school district in the southeast region of the U.S.†	Duration: 40-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.72* Rational numbers magnitude understanding/ relative magnitude understanding: 1.35* Rational numbers knowledge: 0.05
Fuchs, Malone, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in providing explanations or solving word problems vs. control Participants: 212 grade 4 students with mathematics difficulties Setting: 52 classrooms in 14 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned strategies to support their fluency skills and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.79* Rational numbers magnitude understanding/ relative magnitude understanding: 0.93* Rational numbers knowledge: 0.86*

Recommendation 6				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Powell, et al. (2008) *Meets WWC standards with reservations*	Design: RCT Contrast: Fact retrieval with procedural computation and computational estimation tutoring vs. irrelevant control (word-identification skill tutoring) Participants: 66 grade 3 students with mathematics difficulties Setting: 80 classrooms in 18 schools in Nashville, TN and Houston, TX†	Duration: 15- to 18-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson. Fluency activities were both computer-based and with an instructor.	Tutoring in word-identification skills	Whole numbers computation: 0.08 Whole numbers magnitude understanding/ relative magnitude understanding: 0.82* General mathematics achievement: 0.16
Fuchs et al. (2009) *Meets WWC standards without reservations*	Design: RCT Contrast: Tutoring in automatic retrieval vs. control Participants: 91 grade 3 students with mathematics difficulties Setting: 63 classrooms in 18 schools in 2 school districts in Nashville, TN and Houston, TX†	Duration: 20- to 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked and graphed their progress. Fluency activities were both computer-based and with an instructor.	Business as usual core mathematics instruction	Whole numbers computation: 0.56* Algebra and algebraic reasoning: 0.23
Fuchs et al. (2010) *Meets WWC standards without reservations*	Design: RCT Contrast: Word problem instruction with strategic counting practice vs. control Participants: 101 grade 3 students with mathematics difficulties Setting: 84 classrooms in 31 schools in 2 urban school districts in Nashville, TN and Houston, TX†	Duration: 20- to 30-minute sessions; 3 times per week; 16 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction	Whole numbers computation: 0.76* Algebra and algebraic reasoning: 0.76*

Study and WWC rating	Study description[a]	Intervention condition description	Comparison condition description	Outcome domain and WWC-calculated effect size[b]
Recommendation 6				
Fuchs, Schumacher, et al. (2013) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions understanding intervention vs. control Participants: 259 grade 4 students with mathematics difficulties Setting: 53 classrooms in 13 schools†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 2.50* Rational numbers magnitude understanding/ relative magnitude understanding: 1.46* Rational numbers knowledge: 0.92*
Fuchs, Schumacher, et al. (2016) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with instruction in solving multiplicative or additive word problems vs. control Participants: 213 grade 4 students with mathematics difficulties Setting: 45 classrooms in 14 schools†	Duration: 35-minute sessions; 3 sessions per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction and any school-provided intervention	Rational numbers computation: 1.34* Rational numbers magnitude understanding/ relative magnitude understanding: 0.80* Rational numbers knowledge: 0.34*
Fuchs et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions knowledge intervention with fluency building activities vs. control Participants: 164 grade 4 students with mathematics difficulties Setting: 49 classrooms in 14 schools in 1 urban school district†	Duration: 30-minute sessions; 3 times per week; 12 weeks Content: Fractions Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction	Rational numbers computation: 1.44* Rational numbers magnitude understanding/ relative magnitude understanding: 0.96* Rational numbers knowledge: 0.64*
Fuchs, Seethaler, et al. (2008) *Meets WWC standards with reservations*	Design: RCT Contrast: Tutoring in solving word problems vs. control Participants: 35 grade 3 students with mathematics difficulties Setting: 18 classrooms in 1 urban school district in the southeast region of the U.S.	Duration: 20- to 30-minute sessions; 3 times per week; 12 weeks Content: Whole numbers Relevance to Recommendation: Students participated in activities to support their fluency development. Students tracked and graphed their progress and received corrective feedback.	Business as usual core mathematics instruction	Whole numbers computation: 0.49 General mathematics achievement: 0.19

Recommendation 6				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Fuchs, Seethaler, et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Number knowledge intervention vs. control Participants: 196 grade 1 students with mathematics difficulties Setting: 186 classrooms in 21 schools†	Duration: 30-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked and graphed their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction	Whole numbers computation: 0.59*
Gersten et al. (2015) *Meets WWC standards with reservations*	Design: Cluster RCT Contrast: Number operations intervention vs. control Participants: 881 grade 1 students with mathematics difficulties Setting: 76 schools in 4 urban school districts in 4 states in the south-central and southwest regions of the U.S.	Duration: 40-minute sessions; 3-4 times per week; 17 weeks Content: Whole numbers Relevance to Recommendation: Students participated in activities to support their fluency development and received immediate corrective feedback.	Business as usual core mathematics instruction	General mathematics achievement: 0.34*
Kanive et al. (2014) *Meets WWC standards without reservations*	Design: RCT Contrast: Computer-based practice vs. control Participants: 56 grade 4 and 5 students with mathematics difficulties Setting: 1 school in Minnesota†	Duration: 15-minute sessions; 1 time per week; 2 weeks Content: Whole numbers Relevance to Recommendation: Using a computer-based program, students participated in activities to support their fluency development and received corrective feedback.	Business as usual core mathematics instruction	Whole numbers computation: 0.50
Malone et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction or decimal magnitude instruction vs. control Participants: 225 grade 4 students with mathematics difficulties Setting: 58 classrooms in 12 schools in 1 school district†	Duration: 35-minute sessions; 3 times per week; 12 weeks Content: Fractions, decimals Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction	Rational numbers computation: 1.53* Rational numbers magnitude understanding/ relative magnitude understanding: 0.48* Rational numbers knowledge: 0.12

Recommendation 6				
Study and WWC rating	**Study description**[a]	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size**[b]
Powell and Driver (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Addition tutoring (with or without embedded vocabulary component) vs. control Participants: 98 grade 1 students with mathematics difficulties Setting: 58 classrooms in 18 schools in 2 school districts in the mid-Atlantic region of the U.S.†	Duration: 10- to 15-minute sessions; 3 times per week; 8 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students also tracked and graphed their progress.	Business as usual core mathematics instruction	Whole numbers computation: 0.34
Powell, Driver, et al. (2015) *Meets WWC standards without reservations*	Design: RCT Contrast: Standard equations tutoring or combined (standard and nonstandard) equations tutoring vs. control Participants: 51 grade 2 students with mathematics difficulties Setting: 31 classrooms in 10 schools in 2 school districts in the mid-Atlantic region of the U.S.†	Duration: 10- to 15-minute sessions; 3 times per week; 4 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students also tracked and graphed their progress.	Business as usual core mathematics instruction	Whole numbers computation: 0.23 Algebra and algebraic reasoning: 0.80*
Powell, Fuchs, et al. (2015) *Meets WWC standards with reservations*	Design: Cluster QED Contrast: Calculation intervention vs. control Participants: 174 grade 2 students with mathematics difficulties Setting: 110 classrooms in 25 schools in 1 urban school district†	Duration: Tier 1 portion: 40- to 45-minute sessions; 2 times per week; 17 weeks; Tier 2 portion (beginning week 4 of the Tier 1 portion): 25- to 30-minute sessions; 3 times per week Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students also tracked and graphed their progress.	Business as usual core mathematics instruction	Whole numbers computation: 1.19*

Recommendation 6				
Study and WWC rating	**Study description[a]**	**Intervention condition description**	**Comparison condition description**	**Outcome domain and WWC-calculated effect size[b]**
Powell et al. (2009) *Meets WWC standards without reservations*	Design: RCT Contrast: Conceptual fact retrieval instruction with practice vs. control Participants: 63 grade 3 students with mathematics difficulties Setting: 75 classrooms in 17 schools in Nashville, TN and Houston, TX[†]	Duration: 22- to 25-minute sessions; 3 times per week; 15 weeks Content: Whole numbers Relevance to Recommendation: Students learned strategies to support their fluency development and received corrective feedback. Students also tracked and graphed their progress. Fluency activities were both computer-based and with an instructor.	Business as usual core mathematics instruction	Whole numbers computation: 0.62*
Tournaki, N. (2003) *Meets WWC standards without reservations*	Design: RCT Contrast: Minimum-addend strategy instruction vs. control Participants: 28 grade 2 students with mathematics disabilities or difficulties Setting: 1 urban school district in New York[†]	Duration: 15-minute sessions; 5 times per week; 8 days Content: Whole numbers Relevance to Recommendation: Students learned a strategy to support their fluency development and received immediate corrective feedback.	Business as usual core mathematics instruction	Whole numbers computation: 2.30*
Wang et al. (2019) *Meets WWC standards without reservations*	Design: RCT Contrast: Fractions intervention with word problem instruction (with or without self-regulation) vs. control Participants: 84 grade 3 students with mathematics difficulties Setting: 29 classrooms in 8 schools in 1 urban school district[†]	Duration: 35-minute sessions; 3 times per week; 13 weeks Content: Whole numbers and fractions Relevance to Recommendation: Students learned strategies to support their fluency development and received immediate corrective feedback. Students tracked their progress and worked to improve their scores each lesson.	Business as usual core mathematics instruction and any school-provided intervention	Whole numbers computation: 0.59* Rational numbers computation: 1.27* Rational numbers magnitude understanding/ relative magnitude understanding: 0.95* Rational numbers knowledge: 0.85*

Note: Each row in this table represents a study, defined by the WWC as an examination of the effect of an intervention on a distinct sample.

[a] Sample size represents the maximum number of participants in the study. In some studies, the number of participants varied across the outcome measures.

[b] Effect sizes presented here are from the posttest closest to the end of the intervention. For brevity, only the domain average effect size and statistical significance are reported in this table. For studies that included multiple outcomes in a domain, reported effect sizes and statistical significance are for the domain and calculated as described in the WWC Procedures Handbook (version 4.0).

* Significant at $p \leq 0.05$.

[†] Indicates that the information is for the entire study (across all conditions).

Supplemental Findings for Recommendation 6

Supplemental findings (including impacts for follow-up, sub-scale, and distal measures) for seven studies are available at the corresponding study pages on the WWC website.[229]

Six studies included a contrast between two treatment conditions. In two studies, one treatment condition focused on strategy development for fluency activities related to mathematics facts, while the other treatment did not support students in developing or using a strategy (How-to Step 3). In one study,[230] a large significant effect was found for whole numbers computation, (g = 1.48*). In the other study,[231] findings were positive but not significant for the same domain (g = 0.37).

In three studies, one treatment included a timed fluency activity and the other treatment was identical except for being an untimed activity, which is highly relevant to the recommendation because it contrasts timed activities with those that are untimed. All impacts were positive, however not all were statistically significant across domains. In one study,[232] impacts for counting and cardinality were significant, (g = 0.42*) but nonsignificant for whole numbers computation (g = 0.09). In the second study,[233] findings were significant for whole numbers computation (g = 0.36*). In the third study,[234] findings were positive but not significant (g = 0.20).

One study focused on number line concepts related to building fluency in number combinations during other parts of the intervention and the other treatment condition did not.[235] This contrast addressed the value added of linking a specific concept to the fluency activity. These findings were inconclusive for whole numbers computation (g = 0.06).

Appendix D: About the Panel and Key WWC Staff

Panel

Lynn S. Fuchs, Ph.D. (Panel Chair), is Professor of Special Education and the Family Endowed Chair of Psychoeducational Assessment, Department of Special Education, Peabody College of Vanderbilt University; Alexander Heard Distinguished Service Professor at Vanderbilt University; and Professor of Pediatrics at Vanderbilt University Medical Center. She has conducted programmatic research on instructional methods for improving the mathematics outcomes of students at risk for and with learning disabilities, on assessment methods for enhancing instructional planning, and on the cognitive and linguistic characteristics associated with mathematics development. She has published more than 500 empirical studies in peer-reviewed journals, sits on the editorial boards of a variety of journals, and has been identified as one of the most frequently cited researchers in the social sciences. She has received a variety of awards to acknowledge her research accomplishments that have enhanced mathematics outcomes for children with and without disabilities. This includes the American Educational Research Association's Distinguished Contributions to Research in Education Award and the Council for Exceptional Children's Career Research Award.

Nicole Bucka, M.A., is a Multi-Tiered Systems of Support (MTSS) Specialist for the State of Rhode Island's BRIDGE-RI (Bridging Research, Implementation, & Data to Guide Education in Rhode Island). Targeting mathematics as a statewide need in 2012, she noted the lack of implementation guidance and recruited five middle schools to pilot implementation of the recommendations in the original IES practice guide, *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*. Based on the lessons learned from this collaboration, she developed training, tools, and implementation guidance to support scale-up. She brings a diverse skill set to the work, in part because of the many roles she has held in her education career: English teacher, English language development teacher, special educator and department chair, Response to Intervention district coordinator, and social-emotional facilitator. She has also served as a coach for the National Center on Intensive Intervention and as a senior advisor for the Office of Special Education Programs (OSEP)-funded PROGRESS Center, and has presented at national conferences on issues related to intensive intervention, MTSS in mathematics, and academic vocabulary.

Ben Clarke, Ph.D., is an Associate Professor in the School Psychology Program and the Associate Director of the Center on Teaching and Learning at the University of Oregon. His research focuses broadly on the development of students' mathematical thinking and how school systems can support all students' mathematical learning needs. He has led multiple federal grants from the Institute of Education Sciences, Office of Special Education Programs, and the National Science Foundation to develop and test the efficacy of mathematics intervention programs spanning the spectrum of grades K–6 in both traditional and technology-based formats. Also, he has designed and validated screening and progress monitoring assessments in the area of early mathematics and number sense. He has published more than 40 articles and 10 book chapters in the area of mathematics instruction and assessment. He has contributed to the development of multi-tiered instructional models, including the IES practice guide *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*. He was a practicing school psychologist for three years, where he led district efforts to implement response to intervention and multi-tier instructional models.

Barbara Dougherty, Ph.D., is the director of the Curriculum Research and Development Group for the College of Education at the University of Hawai'i. Her work focuses on mathematics education for students who struggle, specifically on improving algebraic concepts and skills. She is the co-Principle Investigator (co-PI) on an IES development and innovation grant to develop grade 3 intervention materials related to multiplicative relationships. Her previous IES funding was to develop MTSS modules for middle grades and to construct screening and progress monitoring assessments. She was a K–12 special education teacher for more than 10 years and taught mathematics in K–12 for over 18 years. Much of her educational research has been conducted in classrooms while collaborating with practitioners to develop curriculum and professional development materials.

Nancy Jordan, Ph.D., is Dean Family Endowed Chair and Professor of Education at the University of Delaware. Her research focuses on how children learn mathematics and why so many struggle. She has received numerous grants to fund her research, including ones from the National Institute of Child Health and Human Development, IES, and the Spencer Foundation. Notably, she was PI of the Center for Improving Learning of Fractions (funded by IES), where she studied the development of grade 3–6 students at-risk for failure in mathematics. She is the author of many highly cited journal articles on children's mathematics. She has also developed number sense screeners and interventions for at-risk learners. She is a fellow of the American Educational Research Association and the Association for Psychological Science. She was the recipient of the 2020 Kauffman-Hallahan-Pullin Distinguished Researcher Award of the Council for Exceptional Children. Recently, she served as Chair of the Governing Board of the Mathematical Cognition and Learning Society. She also served on the Committee on Early Childhood Mathematics of the National Research Council of the National Academies and as an expert panel member for the IES practice guide on teaching math to young children.

Karen Karp, Ed.D., is a Professor in the School of Education at Johns Hopkins University. Previously, she was a Professor of Mathematics Education at the University of Louisville, where she received the Distinguished Teaching Award and the Distinguished Service Award for a Career of Service. She is the co-author of the most widely used methods textbook on teaching mathematics in K–8 education—*Elementary and Middle School Mathematics: Teaching Developmentally*. She has also authored or co-authored 45 journal articles and more than 30 books in mathematics education, including subjects such as teaching and learning in mathematics, creating a school-wide mathematics pact, student-centered mathematics, and ways to inspire girls to think mathematically. She served on the board of directors of the National Council of Teachers of Mathematics (NCTM) and was president of the Association of Mathematics Teacher Educators (AMTE). She served as a co-PI with Russell Gersten on a grant funded by NSF, the goal of which was to bring teams of mathematics educators and special educators together to combine efforts in producing research on MTSS in mathematics.

John Woodward, Ph.D., is currently Distinguished Professor Emeritus and past Dean of the School of Education at the University of Puget Sound in Tacoma, Washington. The majority of his research since 1989 has focused on mathematics education for low-achieving and special education students. He has published over 80 peer-reviewed articles in professional education journals, as well as 15 book chapters. His research is cited in considerable detail in the recent Instructional Practices Report from the National Mathematics Panel (2008) and the IES practice guide *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* (2009). He also chaired the IES practice guide Improving Mathematical Problem Solving in Grades 4 through 8 (2012). In addition to conducting research, he is the senior author of a widely used mathematics intervention curriculum for middle-grade students. He is also the senior author of professional development materials based on the Common Core Standards. He has presented on issues in mathematics education in the U.S., Canada, Germany, Australia, Japan, Korea, and the People's Republic of China.

Key WWC Staff

Madhavi Jayanthi, Ed.D., research director at Instructional Research Group (IRG), served as project director for this practice guide. She also participated in various capacities in developing four other practice guides on a range of topics (English learners, response to intervention in mathematics, mathematical problem solving, and dropout prevention). Jayanthi served as a co-PI of several multi-site RCTs funded by National Center for Education Research, National Center on Education and the Economy, and National Science Foundation. She currently serves as a co-PI for an OSEP-funded study focused on building algebraic reasoning for at-risk middle school students. Jayanthi is certified in WWC 4.1 group design standards. Her research interests include examining effective instructional practices in mathematics and reading for struggling learners.

Russell Gersten, Ph.D., executive director at IRG and professor emeritus of educational research at the University of Oregon, served as the principal investigator. Gersten developed the very first practice guide and created the concept of "roadblocks" in practice guides. He led, as either the panel chair or PI, the teams that developed five practice guides. Gersten has played a considerable role in the WWC virtually since its inception. He was senior author of a meta-analysis of mathematics interventions for students with learning disabilities published in the *Review of Educational Research*, and he has authored over 160 articles in scholarly journals, including major pieces on screening and intervention in MTSS in mathematics, and the role of the number line and number sense in interventions.

Rebecca Newman-Gonchar, Ph.D., senior research associate at IRG, served as a recommendation writer. She has contributed to six practice guides on a range of topics, including the first practice guide focused on English learners, response to intervention in mathematics, response to intervention in reading, and mathematical problem-solving; and two practice guide updates focused on English learners and dropout prevention. She served as a co-PI for two research syntheses—one focused on studies of mathematics professional development and one on reading interventions for struggling students—and she is currently serving as a co-PI for an NSF-funded meta-analysis of rational number interventions for struggling students. Newman-Gonchar is certified in 4.1 group design standards.

Robin F. Schumacher, Ph.D., senior research associate at IRG, served as a recommendation writer. She is a certified WWC reviewer in 4.1 group design standards. She has reviewed studies using WWC group design standards for several topic areas and projects. She recently co-authored a professional learning community (PLC) guide based on the recommendations from the practice guide focused on mathematical problem-solving. She serves as co-PI for an NSF-funded meta-analysis of rational number interventions and for an OSEP-funded project on improving algebraic reasoning of middle school students with mathematics learning disabilities. Her research focuses on developing and implementing mathematics intervention as part of MTSS across grades K–8.

Julia Lyskawa, M.P.P., researcher at Mathematica, served as the project lead for the Mathematica subcontract with IRG. She has worked in various capacities on three practice guides on a range of topics (secondary writing, algebra knowledge, and teaching math to young children). Lyskawa oversaw implementation of the WWC's dissemination strategy for practice guides and other products, created supplemental resources to support each practice guide, and promoted practice guides at conferences and through webinars and social media. She created the concept of practice guide summaries and has written several for numerous practice guides. She is a certified WWC reviewer in the version 4.1 group design standards and has conducted reviews and reconciliations across a range of topic areas for practice guides and intervention reports.

Betsy Keating, M.P.P., researcher at Mathematica, served as the evidence coordinator for this practice guide. She served as the deputy practice lead and deputy evidence lead on two practice guides that focused on algebra knowledge and foundational reading. Keating co-authored *Tips for Supporting Reading Skills at Home*, a supplemental product providing tips for parents or caregivers drawn from the evidence-based classroom practices in the foundational reading practice guide. Keating is certified in version 4.1 group design standards and version 4.1 single-case design standards.

Seth Morgan, M.A., systems analyst at Mathematica, served as the deputy evidence lead for this practice guide. Morgan has been a certified WWC reviewer since 2014, reviewing group design studies under multiple topic areas. He co-authored the *WWC-OREGANO Literacy Fact Sheet*, a supplemental IES product providing a brief overview of key WWC activities and findings under the topic area. Morgan is certified in version 4.1 group design standards and in version 4.1 single-case design standards.

Kelly Haymond, M.A., research associate at IRG, served as a deputy evidence coordinator. As a certified WWC reviewer since 2008, Haymond has reviewed group design studies for eight practice guides, three topic areas, two REL research digests, and two meta-analyses. She is currently certified in 4.1 group design and RDD evidence standards. For the updated English learner practice guide, she served as the evidence lead, overseeing reviews of more than 70 studies. Her research interests include universal screening measures and advanced quantitative methods and have yielded publications in peer-reviewed journals such as *Journal of Educational Psychology, Exceptional Children*, and *Elementary School Journal*.

Acknowledgement

The panel would like to thank the team of WWC-certified reviewers from IRG and Mathematica for their contributions to this practice guide. They would also like to thank the following staff from IRG: Samantha Spallone for managing the project, Joseph Dimino for his assistance in coding the studies, Pam Foremski and Christopher Tran for screening and managing reviews, Sarah Krowka for her assistance with the drafting of the appendices, and Jonathan Cohen for his editorial assistance. In addition, they would also like to thank the following Mathematica staff: Clare Fisher for her support with project management and coordinating the review process, Shannon Monahan for providing quality assurance, and Laura Sarnoski for her help with the production of the practice guide

Appendix E: Disclosure of Potential Conflicts of Interest

Practice guide panels are composed of individuals who are nationally recognized experts on the topics about which they are making recommendations. The Institute of Education Sciences (IES) expects the experts to be involved professionally in a variety of other matters that might relate to their work as a panelist. Panel members are asked to disclose these professional activities and institute deliberative processes that encourage critical examination of their views as they relate to the content of the practice guide. The potential influence of the panel members' professional activities is further muted by the requirement that they ground their recommendations in evidence that is documented in the practice guide. In addition, before all practice guides are published, they undergo an independent external peer review focusing on whether the evidence related to the recommendations in the guide has been presented appropriately.

The professional activities reported by each panel or staff member that appear to be most closely associated with the panel recommendations are noted below.

Panelists

Dr. Lynn S. Fuchs (Chair) co-authored articles that were reviewed and used for evidence for this practice guide. She developed the interventions that were examined in articles that contributed to evidence.

Dr. John Woodward was the one of the developers of the TransMath® curricula, which is commercially available. An adapted version of TransMath® was examined in one study that contributes to the evidence.

Dr. Nancy C. Jordan co-authored articles that were reviewed and used for evidence for this practice guide. She developed the interventions that were examined in articles that contributed to evidence.

Dr. Barbara Dougherty co-authored articles referenced in this guide.

Dr. Karen Karp co-authored articles referenced in this guide.

Dr. Ben Clarke co-authored articles that were reviewed and used for evidence for this practice guide. He developed the interventions that were examined in articles that contributed to evidence. He is eligible to receive royalties from the University of Oregon's distribution and licensing of certain ROOTS-based works. The ROOTS intervention was reviewed for this practice guide.

Staff

Dr. Russell Gersten co-authored articles that were reviewed and used for evidence for this practice guide.

Dr. Madhavi Jayanthi co-authored articles that were reviewed and used for evidence for this practice guide.

Dr. Robin F. Schumacher co-authored articles that were reviewed and used for evidence for this practice guide. She played a role in the development of some interventions that were examined in articles that contributed to evidence.

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Notes

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[4] Berkeley et al., 2020.

[5] Gersten, Beckmann, et al., 2009.

[6] Clarke et al., 2015; Steedly et al., 2008.

[7] Steedly et al., 2008.

[8] Steedly et al., 2008.

[9] **Fien et al., 2016.**

[10] **Barbieri et al., 2019**; Steedly et al., 2008.

[11] **Barbieri et al., 2019; Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Clarke et al., 2017; Darch et al., 1984; Doabler et al., 2016; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Fuchs, et al., 2008; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2016; Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014; Tournaki, 2003; Wang et al., 2019; Watt & Therrien, 2016.**

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[14] Clarke et al., 2015; **Fuchs, Malone, et al., 2019**; Sawyer, 2006; Steedly et al., 2008.

[15] Schumacher et al., 2018.

[16] **Fuchs, Malone, et al., 2019; Watt & Therrien, 2016.**

[17] **Barbieri et al., 2019**; Clarke, Doabler, Smolkowski, Kurtz-Nelson, et al., 2016; **Darch et al., 1984; Dyson et al., 2015; Fuchs, Seethaler, et al., 2019.**

[18] Doabler et al., 2015; **Fuchs, Seethaler, et al., 2019**; Gersten, Beckmann, et al., 2009; **Jayanthi et al., 2018**; Steedly et al., 2008.

[19] Clarke et al., 2015.

[20] Clarke et al., 2015; Fyfe et al., 2012; **Gersten et al., 2015**; Moreno, 2004.

[21] **Fuchs, Malone, et al., 2019; Watt & Therrien, 2016.**

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[28] **Clarke et al., 2017**; Dunston & Tyminski, 2013; Fuchs & Fuchs, 2001; Monroe & Orme, 2002.

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[30] NCTM, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010.

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[34] **Bryant et al., 2016; Doabler et al., 2016; Fuchs, Malone, et al., 2019; Smith et al., 2013.**

[35] Bryant et al., 2003.

[36] Bay-Williams & Livers, 2009; Bryant et al., 2003; **Powell & Driver, 2015.**

[37] Bay-Williams & Livers, 2009.

[38] Beck et al., 2002; Pierce & Fontaine, 2009; **Powell & Driver, 2015.**

[39] Dunston & Tyminski, 2013; Roberts & Truxaw, 2013.

[40] Bay-Williams & Livers, 2009; Bryant et al., 2003; Dunston & Tyminski, 2013; Monroe & Orme, 2002; Pierce & Fontaine, 2009; **Powell & Driver, 2015.**

[41] Bay-Williams & Livers, 2009; Monroe & Orme, 2002.

[42] Bush et al., 2020; Karp et al., 2020.

[43] Bush et al., 2020; Karp et al., 2020.

[44] Adapted from Karp et al., 2014.

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[52] Bush et al., 2020; Karp et al., 2020.

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[60] Jitendra et al., 2016.

[61] Jitendra et al., 2016; Witzel, 2005.

[62] Jitendra et al., 2016; Witzel, 2005; Witzel et al., 2003.

[63] **Dyson et al., 2015; Fuchs, Malone, et al., 2019.**

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[65] Witzel, 2005; Witzel et al., 2003.

[66] Witzel et al., 2003.

[67] Jitendra et al., 2016.

[68] Jitendra et al., 2016.

[69] **Watt & Therrien, 2016.**

[70] Jitendra et al., 2016.

[71] Bush et al., 2020; Karp et al., 2020.

[72] Bush et al., 2020; Karp et al., 2020.

[73] **Fuchs, Schumacher, et al., 2016**; Geary et al., 2008; Lannin et al., 2020; Rittle-Johnson et al., 2001.

[74] **Barbieri et al., 2019; Jayanthi et al., 2018.**

[75] **Dyson et al., 2018; Geary et al., 2008; Lannin et al., 2020; National Governors Association Center for Best Pr**actices & Council of Chief State School Officers, 2010.

[76] National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010 (Standard: 3.MD.a.1).

[77] National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010 (Standard: 5.G.a.1).

[78] National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010 (Standards: 2.MD.d.9; 3.MD.b.4; 4.MD.b.4; 5.MD.b.2).

[79] National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010 (Standard: 4.MD.a.2).

[80] National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010 (Standard: 5.G.a.1)

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[83] Geary et al., 2008; Lannin et al., 2020.

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[86] **Barbieri et al., 2019; Fuchs, Malone, et al., 2019; Gersten et al., 2015.**

[87] Fuson, 2009.

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[98] Geary et al., 2008.

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[100] Lannin et al., 2020.

[101] **Fuchs, Seethaler, et al., 2019; Jitendra, Dupuis, et al., 2013; Powell, Fuchs, et al., 2015.**

[102] Batty et al., 2010.

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[115] **Fuchs, Seethaler, et al., 2019; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Swanson, 2014; Wang et al., 2019.**

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[119] **Fuchs, Seethaler, et al., 2019; Swanson, Orosco, et al., 2014.**

[120] **Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jitendra, Rodriguez, et al., 2013.**

[121] **Fuchs et al., 2010; Fuchs, Seethaler, et al., 2019; Wang et al., 2019.**

[122] Gersten, Chard, et al., 2009; Kroesbergen et al., 2004.

[123] Karp et al., 2019.

[124] **Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013.**

[125] **Fuchs, Fuchs, et al., 2008; Fuchs, Schumacher, et al., 2016.**

[126] Baroody et al., 2009; Geary et al., 2007; Geary et al., 2012; Jordan et al., 2003.

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[128] Baroody et al., 2009; **Dyson et al., 2015; Fuchs et al., 2010; Kanive et al., 2014;** Ketterlin-Geller et al., 2008.

[129] Baroody et al., 2009; **Fuchs, Geary, et al., 2013; Fuchs, Powell, et al., 2008.**

[130] Baroody et al., 2009; **Fuchs et al., 2009; Fuchs et al., 2010.**

[131] **Barbieri et al., 2019; Bryant et al., 2011; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Tournaki, 2003; Wang et al., 2019.**

[132] **Bryant et al., 2011; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Tournaki, 2003; Wang et al., 2019.**

[133] **Barbieri et al., 2019; Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Fuchs, Seethaler, et al., 2008; Gersten et al., 2015; Powell, Fuchs, et al., 2015.**

[134] **Dyson et al., 2015; Fuchs, Geary, et al., 2013.**

[135] **Powell et al., 2009.**

[136] **Dyson et al., 2015.**

[137] **Dyson et al., 2015; Fuchs et al., 2009; Fuchs et al., 2010.**

[138] **Dyson et al., 2015; Fuchs et al., 2009; Fuchs et al., 2010.**

[139] Baroody et al., 2009.

[140] **Fuchs et al., 2014**; Fuchs, Fuchs, et al., 2013.

[141] Study reviews occurred using the WWC Version 4.0 Standards Handbook, available at **https://ies.ed.gov/ncee/wwc/Handbooks**, and the version 4.0 practice guide review protocol available at **https://ies.ed.gov/ncee/wwc/Document/275**.

[142] Following WWC guidelines, improved outcomes are indicated by a positive, statistically significant effect from a meta-analytic effect size calculated separately for each relevant outcome domain. For information on how the WWC calculates these effect sizes, see the WWC Version 4.1 Procedures Handbook at **https://ies.ed.gov/ncee/wwc/Handbooks**.

[143] For more information, see the WWC Frequently Asked Questions page at **https://ies.ed.gov/ncee/wwc/FAQ**.

[144] See Table IV.4 from the WWC Version 4.1 Procedures Handbook.

[145] Three studies (Baroody et al., 2012; Beirne-Smith, 1991; Wilson & Sindelar, 1991) that meet standards were not used to provide evidence as the topic or the experimental contrast was not aligned with the framing of the recommendations or as the main outcome measure did not meet WWC standards.

[146] Hedges & Vevea, 1998.

[147] If multiple contrasts from a study are entered into a meta-analysis, participants from experimental conditions that are common across contrasts will be counted twice, resulting in effect sizes that are statistically dependent. This dependence in a meta-analysis can create a serious threat to the validity of the results.

[148] **Barbieri et al., 2019; Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Clarke et al., 2017; Darch et al., 1984; Doabler et al., 2016; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Fuchs, et al., 2008; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014; Tournaki, 2003; Wang et al., 2019; Watt & Therrien, 2016.**

[149] **Bryant et al., 2011; Clarke et al., 2014; Clarke et al., 2017; Darch et al., 1984; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Fuchs, et al., 2008; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Tournaki, 2003; Wang et al., 2019.**

[150] **Barbieri et al., 2019; Bryant et al., 2016; Doabler et al., 2016; Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Fuchs, Seethaler, et al., 2008; Gersten et al., 2015; Powell, Fuchs, et al., 2015; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014; Watt & Therrien, 2016.**

[151] Five studies (**Fuchs, Fuchs, et al., 2008; Fuchs, Seethaler, et al., 2008; Jayanthi et al., 2018; Swanson, Lussier, et al., 2013; Swanson, Orosco, et al., 2014**) did not report the number of schools in the sample; therefore, the total number of schools comes from the 38 studies that report the number of schools.

[152] **Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Clarke et al., 2017; Fuchs, Powell, et al., 2008; Fuchs, Seethaler, et al., 2008; Doabler et al., 2016; Dyson et al., 2015; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Fuchs, et al., 2008; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Malone et al., 2019; Powell & Driver, 2015; Powell et al., 2009; Powell, Fuchs, et al., 2015; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014; Wang et al., 2019.**

[153] **Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Clarke et al., 2017; Darch et al., 1984; Doabler et al., 2016; Dyson et al., 2015; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Fuchs, et al., 2008; Fuchs, Geary, et al., 2013; Fuchs, Powell, et al., 2008; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Kanive et al., 2014; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014; Tournaki, 2003.**

[154] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2018; Malone et al., 2019; Watt & Therrien, 2016.**

[155] **Fuchs, Malone, et al., 2019; Wang et al., 2019.**

[156] **Barbieri et al., 2019; Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Clarke et al., 2017; Darch et al., 1984; Doabler et al., 2016; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Fuchs, et al., 2008; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014; Tournaki, 2003; Wang et al., 2019; Watt & Therrien, 2016.**

[157] **Kanive et al., 2014.**

[158] **Barbieri et al., 2019; Bryant et al., 2011; Clarke et al., 2017; Darch et al., 1984; Doabler et al., 2016; Dyson et al., 2015; Dyson et al., 2018; Fuchs, Schumacher et al., 2013; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Powell, Driver, et al., 2015; Tournaki, 2003.**

[159] **Bryant et al., 2016; Clarke et al., 2017; Doabler et al., 2016; Dyson et al., 2015; Fuchs et al., 2009; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Malone et al., 2019; Powell & Driver, 2015; Smith et al., 2013.**

[160] **Clarke et al., 2017; Dyson et al., 2015; Fuchs et al., 2009; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Malone et al., 2019; Powell & Driver, 2015.**

[161] **Bryant et al., 2016; Doabler et al., 2016; Fuchs, Malone, et al., 2019; Smith et al., 2013.**

[162] One study (**Jayanthi et al., 2018**) did not report the number of schools in the sample; therefore, the total number of schools comes from the 15 studies that report the number of schools.

[163] **Bryant et al., 2016; Clarke et al., 2017; Doabler et al., 2016; Dyson et al., 2015; Fuchs et al., 2009; Fuchs, Geary, et al., 2013; Fuchs, Seethaler, et al., 2019; Powell & Driver, 2015; Smith et al., 2013.**

[164] **Fuchs et al., 2014; Fuchs, Malone, et al., 2019; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2018; Malone et al., 2019.**

[165] **Fuchs, Malone, et al., 2019.**

[166] **Bryant et al., 2016; Dyson et al., 2015; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Malone et al., 2019; Powell & Driver, 2015.**

[167] **Bryant et al., 2016; Dyson et al., 2015; Fuchs et al., 2009; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Malone et al., 2019; Powell & Driver, 2015.**

[168] **Clarke et al., 2017; Doabler et al., 2016; Fuchs et al., 2009; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Malone et al., 2019; Powell & Driver, 2015; Smith et al., 2013.**

[169] **Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Jayanthi et al., 2018.**

[170] **Clarke et al., 2017; Doabler et al., 2016; Dyson et al., 2015; Fuchs, Schumacher, et al., 2013.**

[171] **Powell & Driver, 2015.**

[172] **Barbieri et al., 2019; Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Clarke et al., 2017; Doabler et al., 2016; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Kanive et al., 2014; Malone et al., 2019; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Smith et al., 2013; Wang et al., 2019; Watt & Therrien, 2016.**

[173] **Bryant et al., 2011; Clarke et al., 2014; Clarke et al., 2017; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Kanive et al., 2014; Malone et al., 2019; Powell & Driver, 2015; Powell, Driver, et al., 2015; Wang et al., 2019.**

[174] **Barbieri et al., 2019; Bryant et al., 2016; Doabler et al., 2016; Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Gersten et al., 2015; Powell, Fuchs, et al., 2015; Smith et al., 2013; Watt & Therrien, 2016.**

[175] One study (**Jayanthi et al., 2018**) did not report the number of schools in the sample; therefore, the total number of schools comes from the 27 studies that report the number of schools.

[176] **Barbieri et al., 2019; Clarke et al., 2017; Doabler et al., 2016; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Malone et al., 2019; Powell, Fuchs, et al., 2015.**

[177] **Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Clarke et al., 2017; Doabler et al., 2016; Dyson et al., 2015; Fien et al., 2016; Fuchs et al., 2005; Fuchs, Geary, et al., 2013; Fuchs, Powell, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Kanive et al., 2014; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Smith et al., 2013.**

[178] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2018; Malone et al., 2019; Watt & Therrien, 2016.**

[179] **Fuchs, Malone, et al., 2019; Wang et al., 2019.**

[180] **Barbieri et al., 2019; Bryant et al., 2011; Bryant et al., 2016; Clarke et al., 2014; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Malone et al., 2019; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Smith et al., 2013; Wang et al., 2019; Watt & Therrien, 2016.**

[181] **Clarke et al., 2017; Doabler et al., 2016; Dyson et al., 2015; Fuchs et al., 2005; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2018; Kanive et al., 2014; Malone et al., 2019; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Wang et al., 2019; Watt & Therrien, 2016.**

[182] **Baroody et al., 2012; Dyson et al., 2015; Fuchs et al., 2014; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2018; Malone et al., 2019; Watt & Therrien, 2016.**

[183] **Barbieri et al., 2019; Bryant et al., 2011; Clark et al., 2017; Doabler et al., 2016; Dyson et al., 2015; Dyson et al., 2018; Fuchs, Schumacher, et al., 2013; Powell, Driver, et al., 2015.**

[184] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Malone et al., 2019; Powell et al., 2009; Wang et al., 2019.**

[185] **Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jayanthi et al., 2018; Malone et al., 2019; Powell et al., 2009; Wang et al., 2019.**

[186] **Barbieri et al., 2019; Fuchs, Malone, et al., 2019; Gersten et al., 2015.**

[187] One study (**Jayanthi et al., 2018**) did not report the number of schools in the sample; therefore, the total number of schools comes from the 13 studies that report the number of schools.

[188] **Fuchs, Geary, et al., 2013; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Powell et al., 2009.**

[189] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2018; Malone et al., 2019.**

[190] **Fuchs, Malone, et al., 2019; Wang et al., 2019.**

[191] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Jayanthi et al., 2018; Malone et al., 2019; Wang et al., 2019.**

[192] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Jayanthi et al., 2018; Malone et al., 2019; Wang et al., 2019.**

[193] **Jayanthi et al., 2018; Powell et al., 2009.**

[194] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs, Schumacher, et al., 2013.**

[195] **Darch et al., 1984; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Fuchs, et al., 2008; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Malone et al., 2019; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014; Wang et al., 2019.**

[196] **Darch et al., 1984; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Fuchs, et al., 2008; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Malone et al., 2019; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Wang et al., 2019.**

[197] **Fuchs, Seethaler, et al., 2008; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014.**

[198] Five studies (**Darch et al., 1984; Fuchs, Fuchs, et al., 2008; Fuchs, Seethaler, et al., 2008; Swanson, Lussier, et al., 2013; Swanson, Orosco, et al., 2014**) did not report the number of schools in the sample; therefore, the total number of schools comes from the 13 studies that reported the number of schools.

[199] **Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Malone et al., 2019; Wang et al., 2019.**

[200] **Darch et al., 1984; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Fuchs, et al., 2008; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, Orosco, et al., 2014.**

[201] **Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Malone et al., 2019.**

[202] **Wang et al., 2019.**

[203] **Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Fuchs, et al., 2008; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Malone et al., 2019; Wang et al., 2019.**

[204] **Fuchs, Fuchs, et al., 2008; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Jitendra et al., 1998; Jitendra, Rodriguez, et al., 2013; Wang et al., 2019.**

[205] **Darch et al., 1984; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014; Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Orosco, et al., 2014.**

[206] **Darch et al., 1984.**

[207] **Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014.**

[208] **Swanson, 2014; Swanson, Lussier, et al., 2013; Swanson, Orosco, et al., 2014.**

[209] **Darch et al., 1984; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Fuchs, et al., 2008; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013; Malone et al., 2019; Wang et al., 2019.**

[210] **Darch et al., 1984; Jitendra et al., 1998; Jitendra, Dupuis, et al., 2013; Jitendra, Rodriguez, et al., 2013.**

[211] **Fuchs, Seethaler, et al., 2019; Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014.**

[212] **Fuchs, Seethaler, et al., 2019.**

[213] **Swanson, Moran, et al., 2013; Swanson, Moran, et al., 2014.**

[214] **Barbieri et al., 2019; Bryant et al., 2011; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Tournaki, 2003; Wang et al., 2019.**

[215] **Bryant et al., 2011; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Tournaki, 2003; Wang et al., 2019.**

[216] **Barbieri et al., 2019; Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Fuchs, Seethaler, et al., 2008; Gersten et al., 2015; Powell, Fuchs, et al., 2015.**

[217] Two studies (**Fuchs, Seethaler, et al., 2008; Tournaki, 2003**) did not report the number of schools in their samples; therefore, the total number of schools comes from the 25 studies that report the number of schools.

[218] **Bryant et al., 2011; Dyson et al., 2015; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Fuchs, et al., 2015; Wang et al., 2019.**

[219] **Bryant et al., 2011; Dyson et al., 2015; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Geary, et al., 2013; Fuchs, Powell, et al., 2008; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Kanive et al., 2014; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Tournaki, 2003.**

[220] **Barbieri et al., 2019; Dyson et al., 2018; Fuchs et al., 2014; Fuchs, Malone, et al., 2016; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Malone et al., 2019.**

[221] **Fuchs, Malone, et al., 2019; Wang et al., 2019.**

[222] **Barbieri et al., 2019; Bryant et al., 2011; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs, Geary, et al., 2013; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Kanive et al., 2014; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Tournaki, 2003.**

[223] **Fuchs et al., 2014; Fuchs, Malone, et al., 2019; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Malone et al., 2019.**

[224] **Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Powell, Fuchs, et al., 2015; Wang et al., 2019.**

[225] **Barbieri et al., 2019; Dyson et al., 2018.**

[226] **Bryant et al., 2011; Dyson et al., 2015; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2019; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2019; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Tournaki, 2003; Wang et al., 2019.**

[227] **Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Wang et al., 2019.**

[228] **Barbieri et al., 2019; Dyson et al., 2015; Dyson et al., 2018; Fien et al., 2016; Fuchs et al., 2005; Fuchs et al., 2006; Fuchs et al., 2009; Fuchs et al., 2010; Fuchs et al., 2014; Fuchs, Geary, et al., 2013; Fuchs, Malone, et al., 2019; Fuchs, Malone, et al., 2016; Fuchs, Powell, et al., 2008; Fuchs, Schumacher, et al., 2013; Fuchs, Schumacher, et al., 2016; Fuchs, Seethaler, et al., 2008; Fuchs, Seethaler, et al., 2019; Gersten et al., 2015; Kanive et al., 2014; Malone et al., 2019; Powell et al., 2009; Powell & Driver, 2015; Powell, Driver, et al., 2015; Powell, Fuchs, et al., 2015; Tournaki, 2003; Wang et al., 2019.**

[229] **Barbieri et al., 2019; Bryant et al., 2011; Dyson et al., 2015; Dyson et al., 2018; Fuchs, Schumacher, et al., 2013; Powell, Driver, et al., 2015; Tournaki, 2003.**

[230] **Tournaki, 2003.**

[231] **Fuchs et al., 2010.**

[232] **Dyson et al., 2015.**

[233] **Fuchs, Geary, et al., 2013.**

[234] **Fuchs et al., 2014.**

[235] **Powell et al., 2009.**